

# Revision Guide for Quality of Measurement

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# Revision Checklist

I can show my ability to make better measurements by:

<p>identifying and estimating the largest source of percentage uncertainty in measurements with sensors and other instruments</p> <p>Revision Notes: <a href="#">measuring with instruments</a>, <a href="#">accuracy and precision</a>, <a href="#">average</a>, <a href="#">uncertainty</a></p>	
<p>taking account of properties of sensors and instruments: resolution, sensitivity, stability, response time, and calibration, systematic and zero error</p> <p>Revision Notes: <a href="#">measuring with instruments</a>, <a href="#">resolution</a>, <a href="#">sensitivity</a>, <a href="#">response time</a>, calibration, <a href="#">uncertainty</a>, systematic error</p>	
<p>'plot and look': using dot-plots or histograms of repeated measurements to look at the distribution of values and to identify and remove outliers, and then to estimate mean and uncertainty of values (uncertainty may be taken as <math>\pm 1/2</math> range)</p> <p>Revision Notes: <a href="#">average</a>, <a href="#">plot and look</a>, <a href="#">uncertainty</a>, <a href="#">random variation</a></p>	
<p>'plot and look': plotting graphs including uncertainty bars, using them to estimate uncertainty in gradient or intercept</p> <p>Revision Notes: <a href="#">plot and look</a>, <a href="#">graphs</a>, <a href="#">working with graphs</a></p>	
<p>suggesting ways to reduce the largest source of percentage uncertainty in an experiment</p> <p>Revision Notes: <a href="#">measuring with instruments</a>, <a href="#">accuracy and precision</a>, <a href="#">uncertainty</a></p>	
<p>identifying possible sources of systematic and zero error in measurements, and suggesting ways to eliminate or reduce them</p> <p>Revision Notes: <a href="#">measuring with instruments</a>, <a href="#">calibration</a>, <a href="#">systematic error</a></p>	
<p>being critical about measurements, looking for ways of cross-checking results</p> <p>Revision Notes: <a href="#">measuring with instruments</a>, <a href="#">accuracy and precision</a>, <a href="#">uncertainty</a>, <a href="#">systematic error</a></p>	
<p>analysing data carefully and thoroughly, using graphs and other plots to detect and display their important features</p> <p>Revision Notes: <a href="#">accuracy and precision</a>, <a href="#">average</a>, <a href="#">uncertainty</a>, <a href="#">systematic error</a>, <a href="#">graphs</a>, <a href="#">working with graphs</a>, <a href="#">random variation</a></p>	

# Revision Notes

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## Measuring with instruments

### Measurements matter

For a physicist, making good measurements matters. In a hospital, the measurement may detect a serious health problem. In industry, it may make sure that a component fits properly. In research, it may show that an accepted idea needs to be reconsidered. A physicist is always asking 'How might I do better?', and taking action to improve a measurement or to decide how far to trust it. So you should be aiming to:

- develop a sense of pride in measuring as well as possible given the tools you have, and to be clear about how well the job has been done
- become better able to experiment well, and to recognise the limitations of instruments
- become better at handling data, particularly in looking at uncertainty in measurement
- learn to look for important sources of uncertainty and attempt to reduce them
- consider possible systematic errors and try to remove them.

### Focus on the instruments

There are two main ways to estimate the uncertainty of a measurement:

- repeat it many times and make an estimate from the variation you get
- look at the process of measurement used, and inspect and test the instruments used.

You should focus mainly on the second way, on the process of measuring and on the qualities of the instruments you have. This points the way to how to improve your measurement. The main reason for being interested in the quality of a measurement is to see how to do better.

### Properties of instruments

The essential qualities and limitations of sensors and measuring instruments are:

- resolution: the smallest detectable change in input
- sensitivity: the ratio of output to input
- stability (repeatability, reproducibility): the extent to which repeated measurements give the same result, including any gradual change with time (drift)
- response time: the time interval between a change in input and the corresponding change in output
- zero error: the output for zero input
- noise: variations, which may be random, superimposed on a signal
- calibration: determining the relation between output and true input value, including linearity of the relationship.

### Estimating uncertainty

The best way to improve a measurement is to identify the largest source of uncertainty and take steps to reduce it. Thus the main focus in thinking about uncertainties is:

- identifying and estimating **the most important source of uncertainty** in a measurement.

This can be estimated in several ways:

- from the resolution of the instrument concerned. For example, the readout of a digital instrument ought not to be trusted to better than  $\pm 1$  in the last digit
- from the stability of the instrument, or by making deliberate small changes in conditions (a tap on the bench, maybe) that might anyway occur, to see what difference they make
- by trying another instrument, even if supposedly identical, to see how the values they give compare
- from the range of some repeated measurements.

When comparing uncertainties in different quantities, it is the percentage uncertainties that need to be compared, to identify the largest.

### Why results vary

There are different kinds of variation, uncertainty or error:

- inherent variation in the measured quantity (for example, fluctuations in wind speed; variation in the value amongst a set of nominally identical commercial resistors)
- small (maybe random) uncontrollable variations in conditions, including noise, leading to uncertainty
- simple mistakes, for example misreading a scale, or 'one-off' accidental errors, which need to be detected and removed; 'outliers' often turn out to be due to such mistakes
- systematic error or bias; a problem with the design of the experiment which can only be removed either by improving the design or by calculating its likely magnitude and allowing for it; this may show up as an intercept on a suitable graph, prompting students to consider how it arose
- a genuine outlying value, whose departure from the overall variation has some physical cause, which may well be of interest in itself.

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## Accuracy and precision

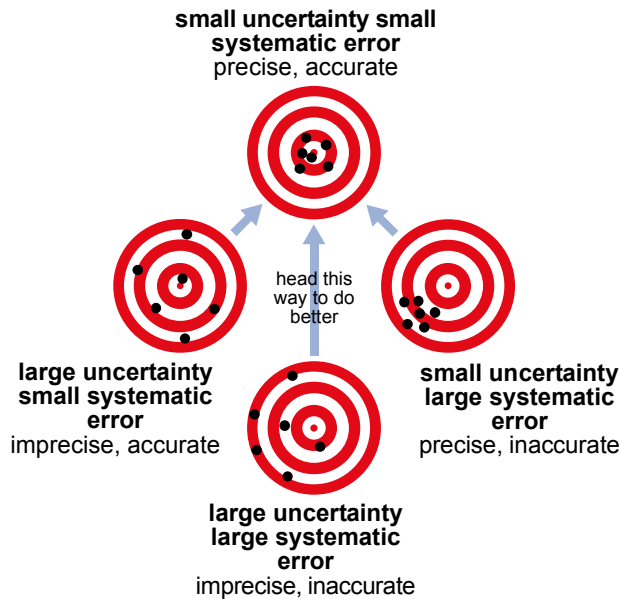
A measurement is accurate if it is close to the true value. A measurement is precise if values cluster closely, with small uncertainty.

A watch with an accuracy of 0.1% could be up to five minutes astray within a few days of being set. A space probe with a trajectory accurate to 0.01 % could be more than 30 km off target at the Moon.

Think of the true value as like the bullseye on a target, and measurements as like arrows or darts aimed at the bullseye.

## Uncertainty and systematic error

Think of measurements as shots on a target. Imagine the 'true value' is at the centre of the target



An accurate set of measurements is like a set of hits that centre on the bullseye. In the diagram above at the top, the hits also cluster close together. The uncertainty is small. This is a measurement that gives the true result rather precisely.

On the left, the accuracy is still good (the hits centre on the bullseye) but they are more scattered. The uncertainty is higher. This is a measurement where the average still gives the true result, but that result is not known very precisely.

On the right, the hits are all away from the bullseye, so the accuracy is poor. But they cluster close together, so the uncertainty is low. This is a measurement that has a systematic error, giving a result different from the true result, but where other variations are small.

Finally, at the bottom, the accuracy is poor (systematic error) and the uncertainty is large.

A statement of the result of a measurement needs to contain two distinct estimates:

1. The best available estimate of the value being measured.
2. The best available estimate of the range within which the true value lies.

Note that both are statements of belief based on evidence, not of fact.

For example, a few years ago discussion of the 'age-scale' of the Universe put it at 14 plus or minus 2 thousand million years. Earlier estimates gave considerably smaller values but with larger ranges of uncertainty. The current (2008) estimate is  $13.7 \pm 0.2$  Gy. This new value lies within the range of uncertainty for the previous value, so physicists think the estimate has been improved in precision but has not fundamentally changed.

Fundamental physical constants such as the charge of the electron have been measured to an astonishing small uncertainty. For example, the charge of the electron is  $1.602\,173\,335 \times 10^{-19}$  C to an uncertainty of  $0.000\,000\,005 \times 10^{-19}$  C, better than nine significant figures.

There are several different reasons why a recorded result may differ from the true value:

1. **Constant systematic bias**, such as a zero error in an instrument, or an effect which has not been allowed for.

Constant systematic errors are very difficult to deal with, because their effects are only observable if they can be removed. To remove systematic error is simply to do a better experiment. A clock running slow or fast is an example of systematic instrument error. The effect of temperature on the resistance of a strain gauge is an example of systematic experimental error.

2. **Varying systematic bias**, or drift, in which the behaviour of an instrument changes with time, or an outside influence changes.

Drift in the sensitivity of an instrument, such as an oscilloscope, is quite common in electronic instrumentation. It can be detected if measured values show a systematic variation with time. Another example: the measured values of the speed of light in a pipe buried in the ground varied regularly twice a day. The cause was traced to the tide coming in on the nearby sea-shore, and compressing the ground, shortening the pipe a little.

3. **Limited resolution of an instrument.** For example the reading of a digital voltmeter may change from say 1.25 V to 1.26 V with no intermediate values. The true potential difference lies in the 0.01 V range 1.25 V to 1.26 V.

All instruments have limited resolution: the smallest change in input which can be detected. Even if all of a set of repeated readings are the same, the true value is not exactly equal to the recorded value. It lies somewhere between the two nearest values which can be distinguished.

4. **Accidental momentary effects**, such as a 'spike' in an electrical supply, or something hitting the apparatus, which produce isolated wrong values, or 'outliers'.

Accidental momentary errors, caused by some untoward event, are very common. They can often be traced by identifying results that are very different from others, or which depart from a general trend. The only remedy is to repeat them, discarding them if further measurements strongly suggest that they are wrong. Such values should never be included in any average of measurements, or be used when fitting a line or curve.

5. **Human errors**, such as misreading an instrument, which produce isolated false recorded values.

Human errors in reading or recording data do occur, such as placing a decimal point wrongly, or using the wrong scale of an instrument. They can often be identified by noticing the kinds of mistake it is easy to make. They should be removed from the data, replacing them by repeated check observations.

6. **Random fluctuations**, for example noise in a signal, or the combined effect of many unconnected minor sources of variation, which alter the measured value unpredictably from moment to moment.

Truly random variations in measurements are rather rare, though a number of unconnected small influences on the experiment may have a net effect similar to random variation. But because there are well worked out mathematical methods for dealing with random variations, much emphasis is often given to them in discussion of the estimation of the uncertainty of a measurement. These methods can usually safely be used when inspection of the data suggests that variations around an average or a fitted line or curve are small and unsystematic. It is important to look at visual plots of the variations in data before deciding how to estimate uncertainties.

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## Average

Just as people vary in height and weight, so physical objects and measurements may vary. Averaging has to do with identifying a 'central' or 'typical' representative value.

There are three kinds of useful average:

1. The arithmetic mean, or the sum of the values divided by their number (often called 'the average').
2. The median, or middle value.
3. The mode, or the most frequently occurring value.

For a set of  $N$  readings or values,  $x_1, x_2, x_3, x_4, \dots, x_N$

The **mean** value:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + \dots + x_N}{N} = \frac{\sum x_n}{N}$$

Mean values are important in measurement, for example when the diameter of a wire is to be determined from several measurements at different points along the wire.

The **median** value is the middle value of the set when the values are arranged in order of magnitude. If the number of readings is an odd number there are as many values above the median as below it. If the number of readings is an even number, the median is the mean of the middle two values. Median values are important in the processing of digital images, for example removing noise by replacing a byte value of a pixel with the median of it and its neighbours.

The **mode** is the value that occurs most frequently.

Before deciding whether to use the arithmetic mean, median or mode as representative of a batch of numbers, it is essential to look at how the values are spread or distributed, and to consider the reason for wanting a representative value. The values can be plotted for inspection using dots on a line (dot-plot): "plot and look".

Consider for example the measured values of a set of 100 electrical resistors, all supposedly the same, but known to vary a little from one to another in manufacture. If a visual plot shows two groups, one varying around a high resistance and one varying around a low resistance, there is evidence that in fact the set consists of two different kinds of resistors. It would be absurd to calculate the arithmetic mean or the median at all. There might even be no resistor having this value. The values of the two modes could be reported as a first step.

Now suppose that the set of resistances cluster around a central value, but with a few very extreme values present. These might be due to the occasional faulty resistor, perhaps open or short circuit. Amongst 100 resistors all roughly  $100 \Omega$ , but with two of resistance  $100\,000 \Omega$ , the arithmetic mean resistance is around  $2000 \Omega$ , nowhere near any value present and completely unrepresentative. The median value, the resistance in the middle if the values are put in order, will be close to  $100 \Omega$  and will be the best and safest measure in this case. Alternatively, remove the unrepresentative values (outliers) before calculating the mean.

In general, the rules for finding average or representative values are:

1. Always check first with a visual plot of the values: "plot and look".
2. If there appears to be more than one group of values, report only modal values and try to separate the groups.
3. If there are possible 'outlying' (remarkably high or low) values, or the distribution of values is very asymmetrical, use the median in preference to the arithmetic mean, or remove outliers before calculating the mean.
4. Use the arithmetic mean when values appear to be clustered symmetrically about the central value.

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## Uncertainty

The uncertainty of an experimental result is the range of values within which the true value may reasonably be believed to lie. To estimate the uncertainty, the following steps are needed.

1. Removing from the data **outlying** values which are reasonably suspected of being in serious error, for example because of human error in recording them correctly, or because of an unusual external influence, such as a sudden change of supply voltage. Such values should not be included in any later averaging of results or attempts to fit a line or curve to relationships between measurements.
2. Estimating the possible magnitude of any **systematic error**. An example of a constant systematic error is the increase in the effective length of a pendulum because the string's support is able to move a little as the pendulum swings. The sign of the error is known (in effect increasing the length) and it may be possible to set an upper limit on its magnitude by observation. Analysis of such systematic errors points the way to improving the experiment.
3. Assessing the **resolution** of each instrument involved, that is, the smallest change it can detect. Measurements from it cannot be known to less than the range of values it does not distinguish.
4. Assessing the magnitude of other small, possibly random, unknown effects on each measured quantity, which may include human factors such as varying speed of reaction. Evidence of this may come from the spread of values of the measurement conducted under what are as far as possible identical conditions. The purpose of repeating measurements is to decide how far it appears to be possible to hold conditions identical.
5. Determining the combined effect of possible **uncertainty** in the result due to the limited resolution of instruments (3 above) and uncontrollable variation (4 above).

To improve a measurement, it is essential to identify the largest source of uncertainty. This tells you where to invest effort to reduce the uncertainty of the result.

Having eliminated accidental errors, and allowed for systematic errors, the range of values within which the true result may be believed to lie can be estimated from (a) consideration of the resolution of the instruments involved and (b) evidence from repeated measurements of the variability of measured values.

Most experiments involve measurements of more than one physical quantity, which are combined to obtain the final result. For example, the length  $L$  and time of swing  $T$  of a simple pendulum may be used to determine the local acceleration of free fall,  $g$ , using

$$T = 2\pi \sqrt{\frac{L}{g}}$$

so that

$$g = \frac{4\pi^2 L}{T^2}.$$

The range in which the value of each quantity may lie needs to be estimated. To do so, first consider the resolution of the instrument involved – say ruler and stopwatch. The uncertainty of a single measurement cannot be better than the resolution of the instrument. But it may be worse. Repeated measurements under supposedly the same conditions may show small and perhaps random variations.

If you have repeated measurements, 'plot and look', to see how the values vary. A simple estimate of the variation is the spread =  $\pm \frac{1}{2}$  range.

A simple way to see the effect of uncertainties in each measured quantity on the final result is to recalculate the final result, but adding or subtracting from the values of variables the maximum possible variation of each about its central value. This is pessimistic because it is



unlikely that 'worst case' values all occur together. However, pessimism may well be the best policy: physicists have historically tended to underestimate uncertainties rather than overestimate them. The range within which the value of a quantity may reasonably be believed to lie may be reduced somewhat by making many equivalent measurements, and averaging them. If there are  $N$  independent but equivalent measurements, with range  $R$ , then the range of their average is likely to be approximately  $R$  divided by the factor  $\sqrt{N}$ . These benefits are not automatic, because in collecting many measurements conditions may vary.

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## Resolution

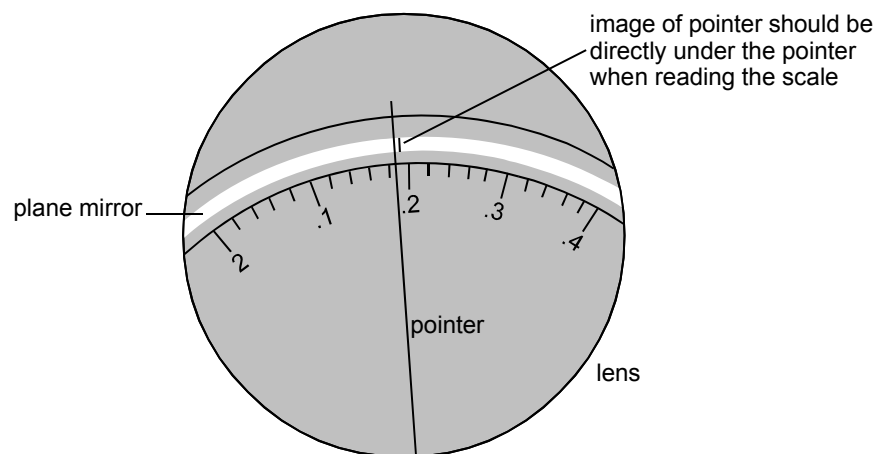
The term resolution can apply to both instruments and images.

The resolution of an instrument is the smallest change of the input that can be detected at the output.

The output of a digital instrument is a numerical display. The resolution is the smallest change of input the instrument can display. For example, a digital voltmeter that gives a three-digit read-out such as 1.35 V has a resolution of 0.01 V since the smallest change in p.d. it can display is 0.01 V.

For an analogue instrument, the output is the position of a pointer on a scale. Its resolution is the smallest change in input that can be detected as a movement of the pointer. The resolution of an analogue instrument can be improved using a magnifying lens to observe movement of the pointer.

### Reading a scale



The resolution of an image is the scale of the smallest detail that can be distinguished. The size of the pixels sets a limit to the resolution of a digital image. In an ultrasound system, the pixel dimensions may correspond to about one millimetre in the object imaged. A high-quality CCD may have an array about 10 mm  $\times$  10 mm consisting of more than 2000  $\times$  2000 light-sensitive elements, each about 5  $\mu\text{m}$  in width. In a big close-up picture of a face 200 mm across, the width of each pixel would correspond to 1 / 10 mm in the object photographed.

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## Sensitivity

The sensitivity of a measuring instrument is the change of its output divided by the corresponding change in input.

A temperature sensor whose output changes by 100 mV for a change of 2 K in its input has a sensitivity of 50 mV per kelvin.

A silicon photocell with an output of 500 mV when illuminated by light of intensity 1000 lux has a sensitivity of 0.5 mV per lux.

A very sensitive instrument gives a large change of output for a given change of input.

In a linear instrument, the change of output is directly proportional to the change of the input. Thus a graph of output against input would be a straight line through the origin. The gradient of the line is equal to the sensitivity, which is constant. Thus a linear instrument has a sensitivity that is independent of the input.

If the change of output is not proportional to the change of the input, the graph would be a curve. In this case, the sensitivity would vary with input. Many instruments, such as light meters, have a logarithmic dependence of output on light input.

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## Response time

Response time is the time taken by a system to change after a signal initiates the change.

In a **temperature-control system**, the response time is the time taken for the system to respond after its temperature changes. For example, a home heating system with a response time that is too long would not start to warm the building as soon as its temperature fell below the desired level.

In an electronic measuring instrument, the response time is the time taken by the instrument to give a reading following a change in its input. If the response time is too long, the instrument would not measure changing inputs reliably. If the response time is too short, the instrument might respond to unwanted changes in input.

Reasons for slow response times include the inertia of moving parts and the thermal capacity of temperature sensors.

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## Calibration

A measuring instrument needs to be calibrated to make sure its readings are accurate.

Calibration determines the relation between the input and the output of an instrument. This is done by measuring known quantities, or by comparison with an already calibrated instrument. For example, an electronic top pan balance is calibrated by using precisely known masses. If the readings differ from what they should be, then the instrument needs to be recalibrated.

Important terms used in the calibration of an instrument include:

The **zero reading** which should be zero when the quantity to be measured is zero. Electrical instruments are prone to drift off-zero and need to be checked for zero before use.

A **calibration graph**, which is a graph to show how the output changes as the input varies.

**Linearity**, which is where the output increases in equal steps when the input increases in equal steps. If the output is zero when the input is zero, the output is then directly proportional to the input, and its calibration graph will be a straight line through the origin. An instrument with a linear scale is usually easier to use than an instrument with a non-linear scale. However, with the advent of digital instruments, linearity has become less important. Given

the output, the instrument simply looks up the correct value of the input to record, in a 'look-up' table. The 'look-up' table is the equivalent of a calibration graph.

The **resolution** of the instrument, which is the smallest change of the input that can be detected at the output.

The **sensitivity** of the instrument, which is the ratio of change in output for a given change in input. If the calibration graph is curved, then the sensitivity - the slope of the graph - varies across the range.

The **reproducibility** of its measurements, which is the extent to which it gives the same output for a given input, at different times or in different places. Reproducibility thus includes zero drift and changes in sensitivity.

Most instruments are calibrated using secondary standards which themselves are calibrated from primary standards in specialist laboratories.

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### Systematic error

Systematic error is any error that biases a measurement away from the true value.

All measurements are prone to systematic error. A systematic error is any biasing effect, in the environment, methods of observation or instruments used, which introduces error into an experiment. For example, the length of a pendulum will be in error if slight movement of the support, which effectively lengthens the string, is not prevented, or allowed for.

Incorrect zeroing of an instrument leading to a **zero error** is an example of systematic error in instrumentation. It is important to check the zero reading during an experiment as well as at the start.

Systematic errors can change during an experiment. In this case, measurements show trends with time rather than varying randomly about a mean. The instrument is said to show **drift** (e.g. if it warms up while being used).

Systematic errors can be reduced by checking instruments against known standards. They can also be detected by measuring already known quantities.

The problem with a systematic error is that you may not know how big it is, or even that it exists. The history of physics is littered with examples of undetected systematic errors. The only way to deal with a systematic error is to identify its cause and either calculate it and remove it, or do a better measurement which eliminates or reduces it.

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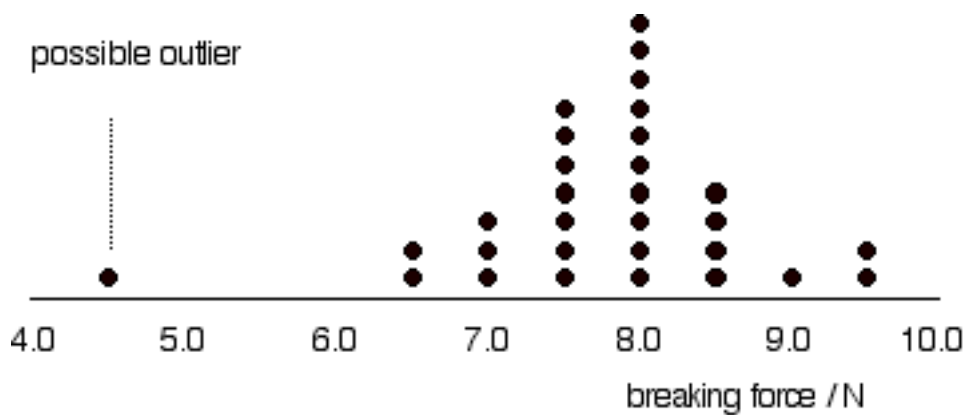
### Plot and look

Whenever you repeat measurements, make a simple plot to see how they vary. Do this before you try to find the average and spread of the results.

Here are some measurements of the breaking strength of strips of paper, found by pulling on the strips using a newton-meter. Each of 10 students tested three strips. The breaking force was measured to the nearest 0.5 N.

Student	strip 1	strip 2	strip 3	
A	7.5	8.0	8.5	
B	8.0	7.5	7.5	
C	8.0	8.0	8.0	
D	8.0	9.5	7.0	
E	6.5	7.0	6.5	possible low-reading newton meter
F	8.0	7.5	9.5	
G	9.0	8.5	7.5	
H	7.5	7.5	8.5	
I	7.0	8.0	8.5	
J	8.0	4.5	8.0	
		possible outlier		
		might be a faulty strip		
		e.g. with torn edge		

A simple way to plot such a batch of results is to make a dotplot. The dotplot for these results looks like this.



It is clear that most of the results cluster around about 8.0 N. However, the dotplot shows that one value, at 4.5 N, looks rather different from the rest. It may possibly be an outlier.

Before deciding what to do about such a result, you need to consider:

- whether there is a possible explanation of the unusual value
- whether the result is far enough from the others to justify treating it as an outlier.

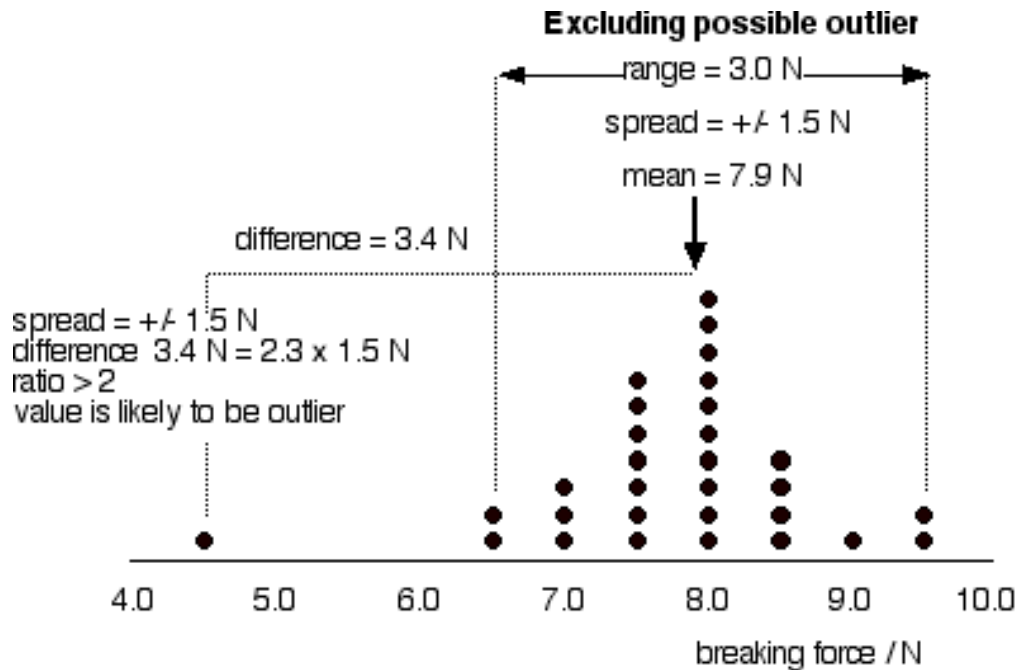
In this case, there is a possible explanation. It is easy to make a small accidental notch on the edge of a strip when cutting it. Stress can concentrate at such a notch, making the strip break at a smaller force. It would be a good idea to check this by looking carefully at the torn strip.

Now find out how big the discrepancy is, compared with the spread of the other results:

1. exclude the potential outlying value, for the moment
2. find the mean of the remaining results
3. find the range of the remaining results
4. calculate the spread of the remaining results, from  $\text{spread} = \pm \frac{1}{2} \text{range}$
5. find the ratio of the difference between the possible outlying value and the mean, to the spread.

If the possible outlier is more than 2 x spread from the mean, there is further reason to treat it as outlying.

Here are the results of doing this, for the data on tearing paper strips.



Notice that including the outlier in these results does not very much affect the mean, which becomes 7.8 N instead of 7.9 N. But it does make a big difference to the range and spread. The range becomes 5.0 N instead of 3.0 N, and the spread becomes  $\pm 2.5$  N instead of  $\pm 1.5$  N.

Given the spread of the results, it does not seem appropriate to give the final value to more than one significant figure. The spread of any set of results can rarely be estimated to better than one significant figure. Thus the final result might now be:

breaking force =  $8 \pm 2$  N

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## Random variation

Random variation has to do with small unpredictable variations in quantities.

Truly random variation may be rather rare. However, variations due to a number of minor and unrelated causes often combine to produce a result that appears to vary randomly.

Random variation can be due to uncontrollable changes in the apparatus or the environment or due to poor technique on the part of an observer. They can be reduced by redesigning the apparatus or by controlling the environment (e.g. the temperature). Even so, random variation can still remain. The experimenter then needs to use the extent of such variation to assess the range within which the true result may reasonably be believed to lie.

First, accidental variations with known causes need to have been eliminated, and known systematic errors should have been allowed for. Then, variations of measurements around an average value are often treated as random.

The simplest approach is to suppose that the true result lies somewhere in the range covered by the random variation. This is a bit pessimistic, since it is more likely that the true result lies fairly near the middle of the range than near the extremes.

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## Graphs

Graphs visually display patterns in data and relationships between quantities.

A graph is a line defined by points representing pairs of data values plotted along perpendicular axes corresponding to the ranges of the data values.

Many experiments are about finding a link between two variable quantities. If a mathematical relationship between them is suggested, the suggestion can be tested by seeing how well the graph of the experimental measurements corresponds to the graph of the mathematical relationship, at least over the range of values of data taken. For example, a graph of the tension in a spring against the extension of the spring is expected to be a straight line through the origin if the spring obeys Hooke's law, namely tension = constant  $\times$  extension. But if the spring is stretched further, the graph of the experimental results is likely to become curved, indicating that Hooke's law is no longer valid in this region.

Many plots of data yield curved rather than straight-line graphs. By re-expressing one or both variables, it may be possible to produce a graph which is expected to be a straight line, which is easier to test for a good fit. Some of the curves and related mathematical relationships met in physics are described below:

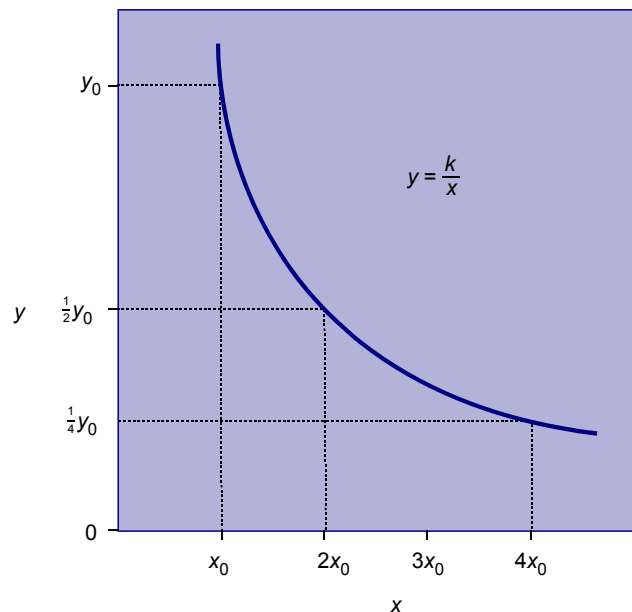
Inverse curves are asymptotic at both axes. The mathematical form of relationship for an inverse curve is

$$y = \frac{k}{x^n}$$

where  $n$  is a positive number and  $k$  is a constant.

$$n = 1: y = k / x$$

**An inverse curve**



### Examples:

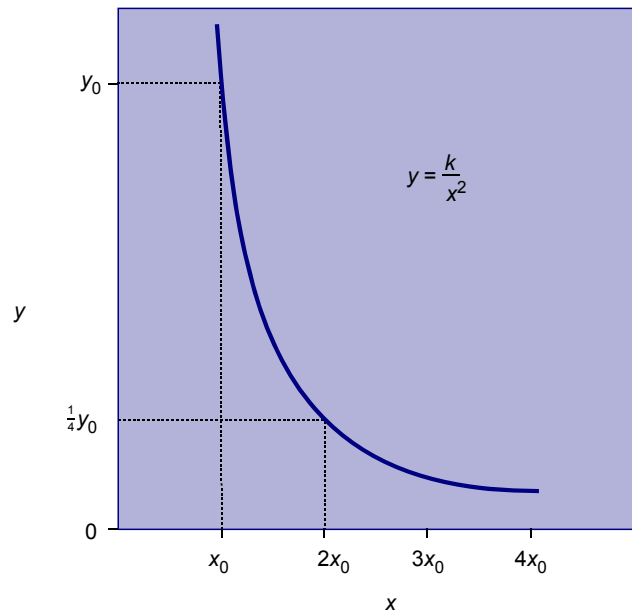
Pressure = constant / volume for a fixed amount of gas at constant temperature.

Gravitational potential = constant / distance for an object near a spherical planet.

Electrostatic potential = constant / distance for a point charge near a large charge.

$$n = 2: y = k / x^2$$

**An inverse square curve**



**Examples:**

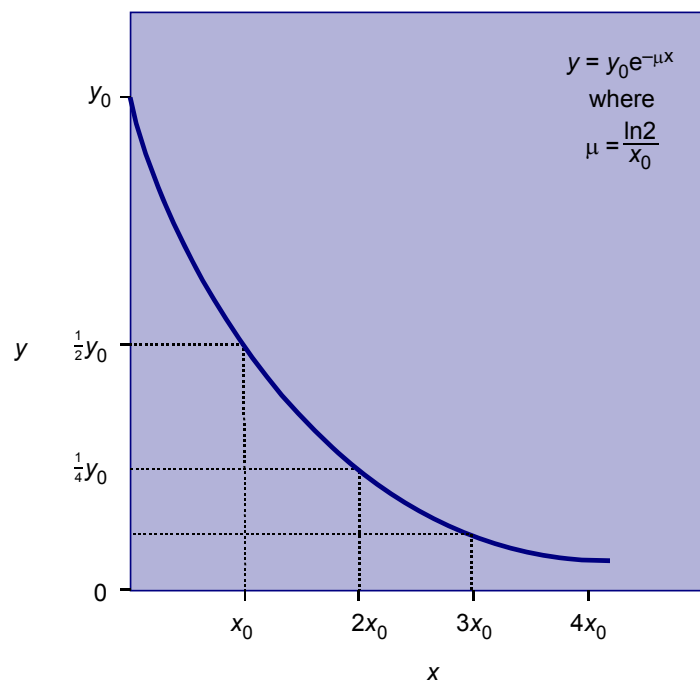
Gravitational force between two points masses = constant / distance<sup>2</sup>.

Electric field intensity = constant / distance<sup>2</sup>.

Intensity of gamma radiation from a point source = constant / distance<sup>2</sup>.

**Exponential decay**

**Exponential decrease**



Exponential decay curves are asymptotic along one axis but not along the other axis.  
Exponential decay curves fit the relationship

$$I = I_0 e^{-ct}$$

where  $I_0$  is the intensity at  $t = 0$  and  $c$  is a constant.

**Examples:**

Radioactive decay

$$N = N_0 e^{-\lambda t}$$

Capacitor decay

$$Q = Q_0 e^{-t/CR}$$

Absorption of x-rays and gamma rays by matter

$$I = I_0 e^{-\alpha x}$$

In general, to establish a relationship between two variables or to find the value of a constant in an equation, the results are processed to search for a straight-line relationship. This is because a straight line is much easier to recognise than a specific type of curve. To test a proposed relationship between two variables, the variables are re-expressed if possible to yield the equation for a straight line  $y = m x + c$ . For example, a graph of  $y =$  pressure against  $x = 1 /$  volume should give a straight line through the origin, thus confirming that the gas under test obeys Boyle's law. In the case of a test for an exponential decay curve of the form

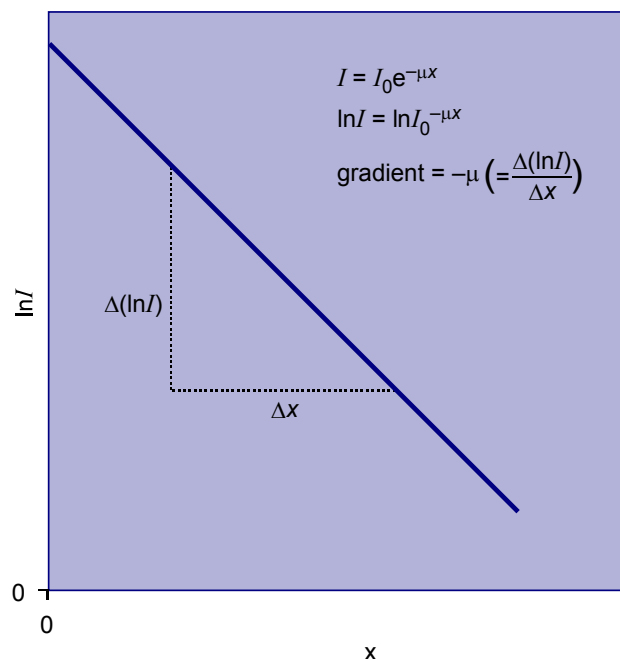
$$I = I_0 e^{-\alpha x}$$

the variable  $I$  is re-expressed as its natural logarithm  $\ln I$ , giving

$$\ln I = \ln I_0 - \alpha x.$$

A graph of  $\ln I$  against  $x$  is now expected to be a straight line.

$\ln I$  against  $x$  for  $I = I_0 e^{-\mu x}$



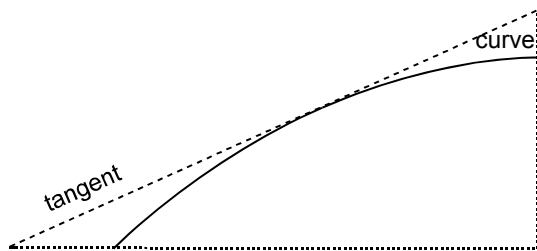


Graphs are a means of communication. To communicate clearly using graphs follow these rules:

1. Always choose a scale for each axis so that the points spread over at least 50% of each axis.
2. Obtain more points by making more measurements where a line curves sharply.
3. Label each axis with the name and symbol of the relevant quantity, and state the appropriate unit of measurement (e.g. pressure  $p$  / kPa).
4. Prefer graph areas which are wider than they are long ('landscape' rather than 'portrait').
5. Put as much information as possible on the graph, for example labeling points informatively.
6. When using a computer to generate graphs, always try several different formats and shapes. Choose the one which most vividly displays the story you want the graph to tell.
7. Label every graph with a caption which conveys the story it tells, for example 'Spring obeys Hooke's law up to 20% strain', not 'extension against strain for a spring'.

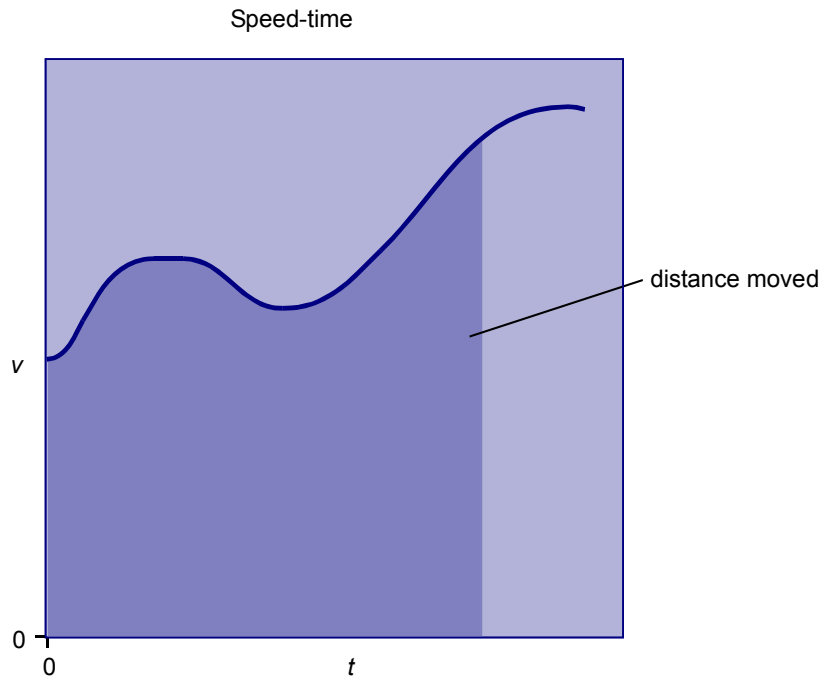
To measure the **gradient** of a curve at a point on the curve, draw the tangent to the curve as shown below and measure the gradient of the tangent by drawing a large 'gradient triangle'. To measure the gradient of a straight line, draw a large gradient triangle with the line itself as the hypotenuse of the triangle then measure the gradient.

### Drawing a tangent

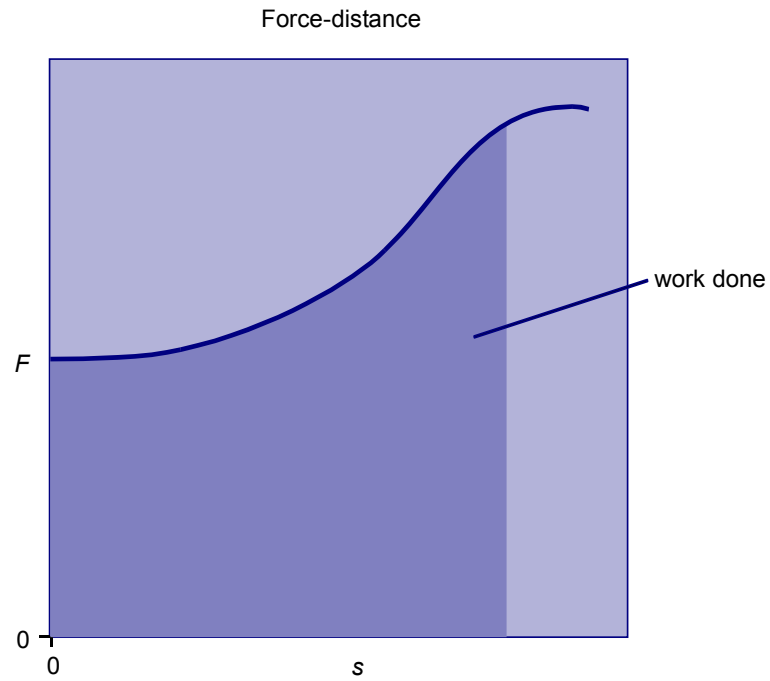


The **area under a curve** is usually measured by counting the grid squares, including parts of squares over half size as whole squares and disregarding parts of squares less than half size. Graphs in physics where areas are useful include:

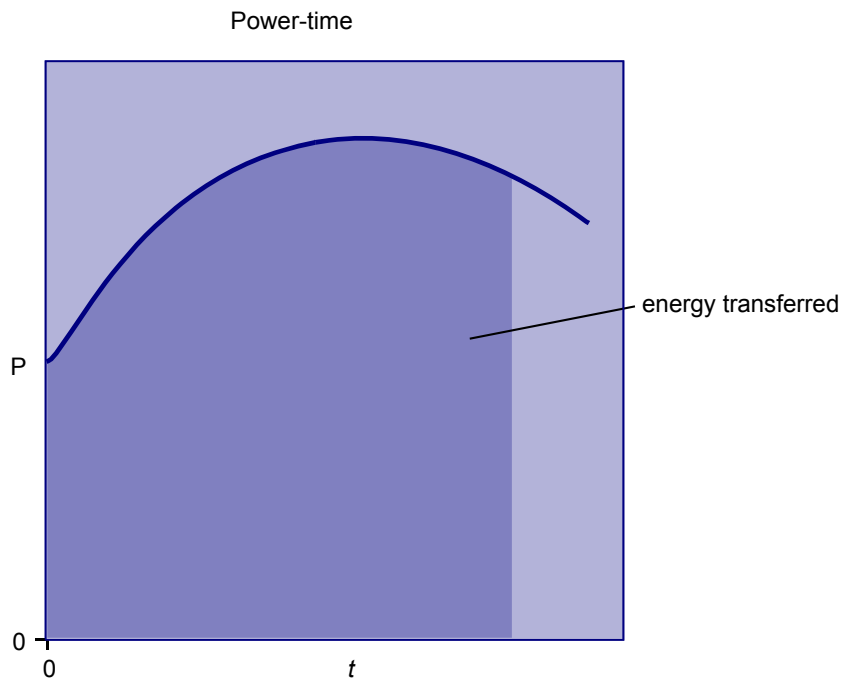
speed–time graphs (where area represents distance moved)



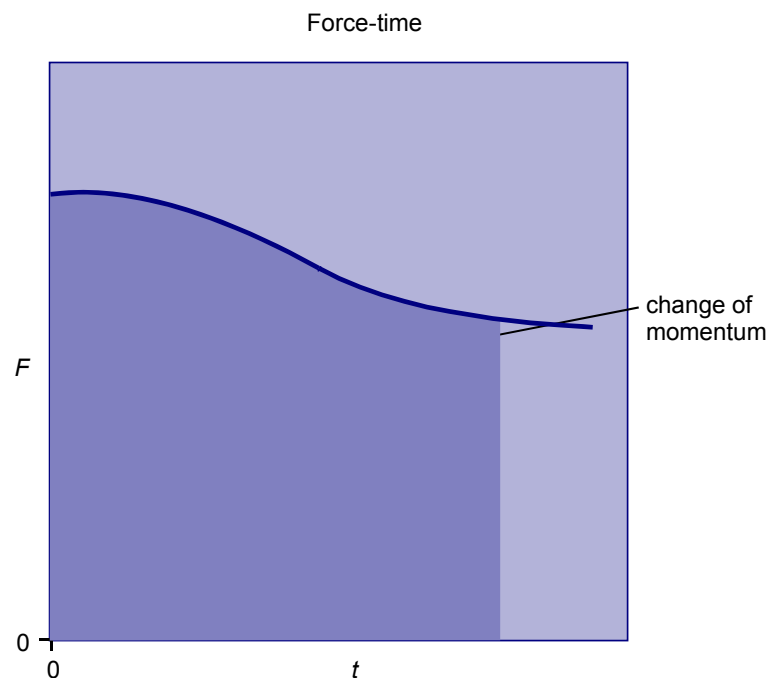
force–distance graphs (where area represents work done or energy transferred)



power–time graphs (where area represents energy transferred)



force–time graphs (where area represents change of momentum)



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## Working with graphs

A good graph lets you see patterns in data that you can't see just by looking at the numbers. So graphs are an essential **working tool**. A good graph also helps you to communicate your results quickly, effectively and visually. So graphs are also an essential **presentation tool**.

Always have at least two versions of every graph: one for working on and one for the final presentation. Your working graph needs finely spaced grid lines for accurate plotting and reading off values (for example, slope and intercept). Your presentation graph needs just enough grid lines to be easy to read. Expect to have to try several versions of your presentation graph before you find the best form for it.

### Working graphs

The job of your working graph is to store information, to let you see patterns in data, and to help you draw conclusions, for example about slope and intercept.

- Whenever possible, 'plot as you go'. This lets you quickly spot mistakes and to decide at what intervals to take measurements.
- Plot points with vertical crosses ('plus sign' shape). This most easily lets you get the position on each axis correct.
- Always choose a scale for each axis so that the points spread over at least 50% of the axis. But keep the scale simple, to avoid plotting errors.
- Think about whether you need to include the zero values on the axes, or not.
- Label the axes clearly, with quantity and unit (e.g. pressure  $p$  / kPa).
- Give each point 'uncertainty bars', indicating the range of values within which you believe the true value to lie.
- Obtain more points by making more measurements where a line curves sharply.
- Use the working graph to measure slopes or intercepts, taking account of uncertainties in the values.

### Presentation graphs

The job of your presentation graph is to tell a story about the results as clearly and effectively as possible.

- When using a computer to generate graphs, always try several different formats and shapes. Choose the one that most vividly displays the story you want the graph to tell.
- Prefer graph areas that are wider than they are tall ('landscape' rather than 'portrait'). A ratio of width to height of 3:2 is often good.
- Label each axis with the name and symbol of the relevant quantity, and state the appropriate unit of measurement (e.g. pressure  $p$  / kPa).
- Show 'uncertainty bars' for each point.
- Give every graph a caption that conveys the story it tells. For example:  
'Spring obeys Hooke's law up to 20% strain',  
**not**  
'Extension against load for a spring'.

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