Revision Guide for Chapter 17

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I can show my understanding of effects, ideas and relationships by describing and explaining cases involving:

the use of particle accelerators to produce beams of high energy particles for scattering (collision) experiments (knowledge of the construction details of accelerators not required) Revision Notes: accelerators Summary Diagrams: The linear accelerator (chapter 16), Principle of the synchrotron accelerator (chapter 16)	
evidence from scattering for a small massive nucleus within the atom Revision Notes: alpha scattering Summary Diagrams: Alpha particle scattering experiment, Rutherford's picture of alpha particle scattering, Distance of closest approach	
evidence for discrete energy levels in atoms (e.g. obtained from collisions between electrons and atoms or from line spectra) Revision Notes: energy level Summary Diagrams: Energy levels	
a simple model of an atom based on the quantum behaviour of electrons in a confined space Revision Notes: model of the atom Summary Diagrams: Standing waves in boxes, Colours from electron guitar strings, Energy levels, Standing waves in atoms, Size of the hydrogen atom	
a simple model of the internal structure of nucleons (protons and neutrons) as composed of up and down quarks Revision Notes: quark Summary Diagrams: Quarks and gluons	
pair creation and annihilation using $E_{rest} = mc^2$ Revision Notes: pair production and annihilation, subatomic particles Summary Diagrams: Pair creation and annihilation	

I can use the following words and phrases accurately when describing effects and observations:

energy level, scattering Revision Notes: energy level, alpha scattering Summary Diagrams: Energy levels, Rutherford's picture of alpha particle scattering	
nucleus, proton, neutron, nucleon, electron, positron, neutrino, lepton, quark, gluon, hadron, antiparticle Revision Notes: subatomic particles, proton, neutron, nucleon, electron, positron, neutrino,	

quark, antimatter Summary Diagrams: What the world is made of	

I can sketch and interpret:

diagrams showing the paths of scattered particles Summary Diagrams: Rutherford's picture of alpha particle scattering, Distance of closest approach	
pictures of electron standing waves in simple models of an atom Revision Notes: model of the atom Summary Diagrams: Standing waves in boxes, Colours from electron guitar strings, Energy levels, Standing waves in atoms, Size of the hydrogen atom	

I can make calculations and estimates making use of:

the kinetic and potential energy changes as a charged particle approaches and is scattered by a nucleus or other charged particle Summary Diagrams: Rutherford's picture of alpha particle scattering, Distance of closest approach	
changes of energy and mass in pair creation and annihilation, using $E_{rest} = mc^2$ Revision Notes: mass and energy, relativistic calculations of speed and energy, pair production and annihilation Summary Diagrams: Conserved quantities in electron–positron annihilation, Pair creation and annihilation	
mass, energy and speed of highly accelerated particles, using $E_{rest} = mc^2$ and relativistic factor $\gamma = \frac{E_{total}}{E_{rest}} = \frac{1}{\sqrt{1 - v^2/c^2}}$ Revision Notes: mass and energy, relativistic calculations of speed and energy Summary Diagrams: Relativistic momentum $p = \gamma mv$ (chapter 16), Relativistic energy $E_{total} = \gamma mc^2$ (chapter 16), Energy, momentum and mass (chapter 16)	

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Accelerators

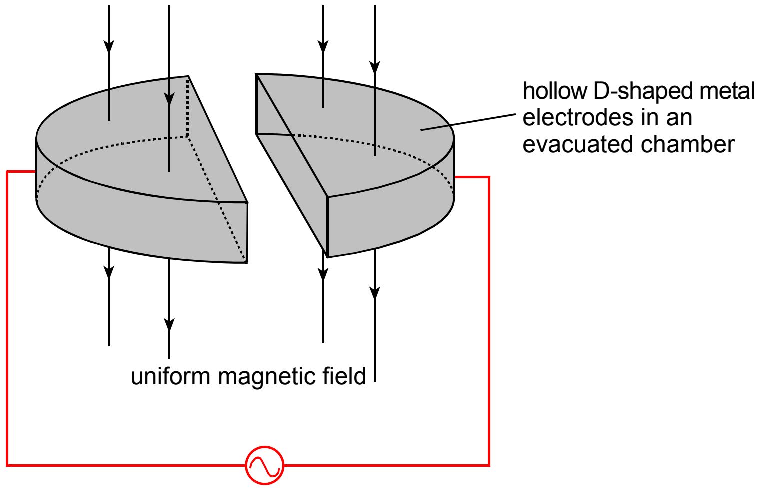
An accelerator is a linear or circular device used to accelerate charged particles. Particles are given energy by electric fields. They are steered using magnetic fields.

A **Van de Graaff accelerator** consists of a large isolated metal dome kept at a high potential by the accumulation of charge from a continuously moving belt. Negative ions created inside the dome in an evacuated tube are thus repelled. The work W done on a particle of charge q is $W = q\ V$, where V is the potential of the dome.

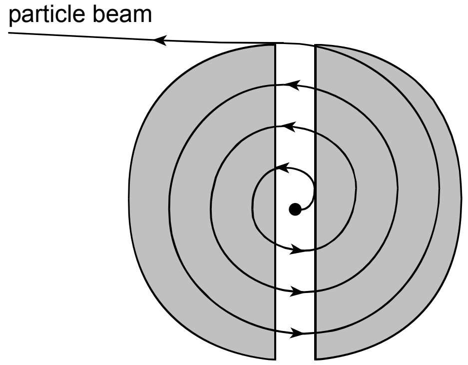
The largest Van de Graaff accelerators can accelerate protons to energies of the order of 20 MeV. Although the maximum energy is low, it is stable and can be accurately controlled, allowing precision investigations of nuclear structure.

A **cyclotron** consists of two hollow evacuated D-shaped metal electrodes. A uniform magnetic field is directed at right angles to the electrodes. As a result, charged particles released at the centre are forced to move round in a circular path, crossing between the electrodes every half turn. A radio-frequency alternating p.d. between the electrodes accelerates the charged particles as they cross the gap between the electrodes. The charged particles spiral out from the centre, increasing in energy every half-cycle.

The cyclotron



construction



particle paths

The following equations apply if the speed of the particles remains much less than the speed of light. The magnetic force on a charged particle q is equal to $B\,q\,v$, where v is the particle's speed and B is the magnetic flux density. Thus

$$Bqv = \frac{mv^2}{r}$$

where m is the particle's mass and r is the radius of the particle orbit.

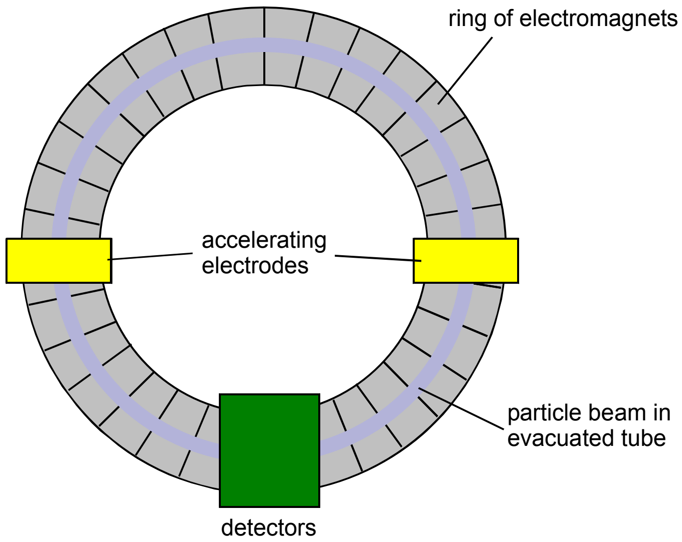
Thus the momentum of a particle is $m\,v = B\,q\,r$ and the frequency of rotation is

$$f = \frac{v}{2\pi r} = \frac{Bq}{2\pi m}.$$

This is independent of radius r and is the constant frequency of the alternating p.d.

Relativistic effects limit the maximum energy a cyclotron can give a particle. At speeds approaching the speed of light the momentum of a particle is larger than the classical value mv. The frequency of orbit in the magnetic field is no longer constant, so the alternating accelerating potential difference is no longer synchronised with the transit of a particle between the two electrodes.

Synchrotron Accelerator



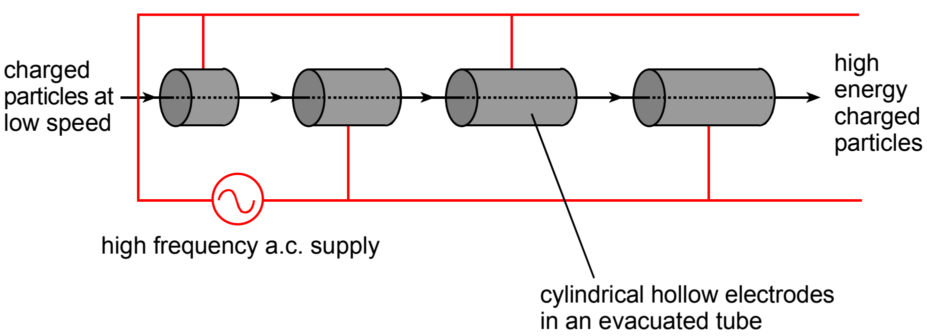
The **synchrotron** makes particles travel at a fixed radius, adjusting the magnetic field as they accelerate to keep them on this fixed path. The frequency of the alternating accelerating potential difference is also adjusted as the particles accelerate, to synchronise with their time of orbit.

The machine consists of an evacuated tube in the form of a ring with a large number of electromagnets around the ring. Pairs of electrodes at several positions along the ring are used to accelerate charged particles as they pass through the electrodes. The electromagnets provide a uniform magnetic field which keeps the charged particles on a circular path of fixed radius.

In a collider, pulses of particles and antiparticles circulate in opposite directions in the synchrotron, before they are brought together to collide head-on.

A **linear accelerator** consists of a long series of electrodes connected alternately to a source of alternating p.d. The electrodes are hollow coaxial cylinders in a long evacuated tube. Charged particles released at one end of the tube are accelerated to the nearest electrode. Because the alternating p.d. reverses polarity, the particles are repelled as they leave this electrode and are now attracted to the next electrode. Thus the charged particles gain energy each time they pass between electrodes.

The linear accelerator



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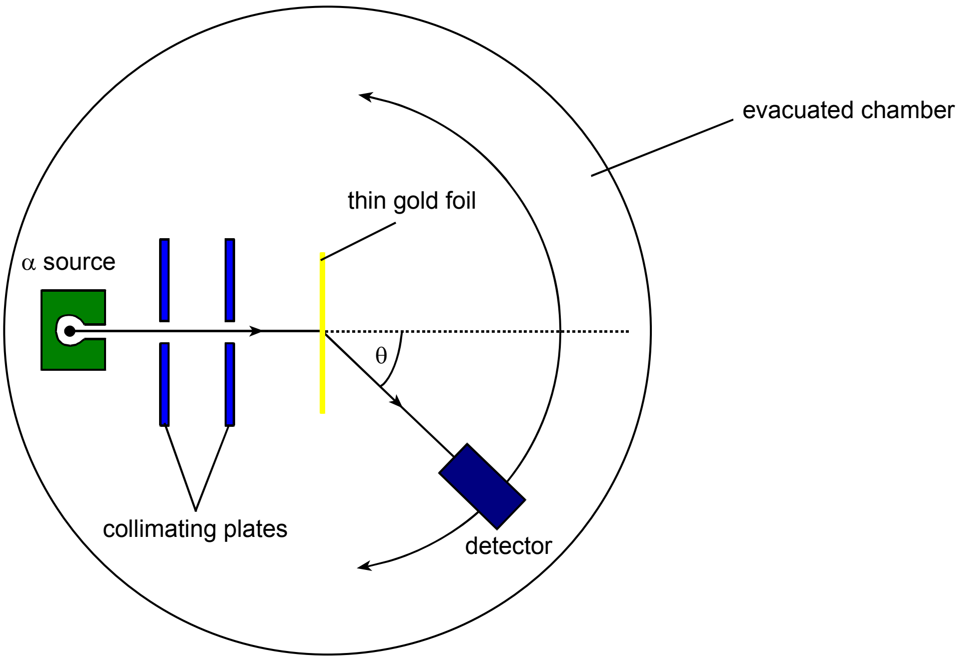
Alpha scattering

Rutherford, working with Geiger and Marsden, discovered that most of the alpha particles in a narrow beam directed at a thin metal foil passed through the foil.

α scattering



They measured the number of particles deflected through different angles and found that a small number were deflected through angles in excess of 90°. Rutherford explained these results by picturing an atom as having a small massive positively charged nucleus.

The fraction of particles scattered at different angles could be explained by assuming that the alpha particles and nucleus are positively charged and so repel one another with an electrical inverse square law force (Coulomb's Law).

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Energy level

Confined quantum objects exist in discrete quantum states, each with a definite energy. The term **energy level** refers to the energy of one or more such quantum states (different states can have the same energy).

The existence of discrete energy levels in atoms has been confirmed in electron collision experiments using gas-filled electron tubes. The gas atoms exchange energy with the electrons in discrete amounts corresponding to differences in energy levels of the atoms.

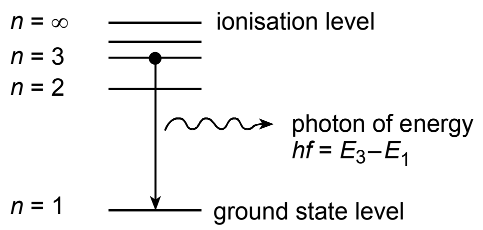
Evidence of discrete energy levels in atoms also comes from the existence of sharp line spectra. A line emission spectrum is seen if the light from a glowing gas or vapour is passed through a narrow slit and observed after it has been refracted by a prism or diffracted by a diffraction grating. The spectral line is just the image of the slit.

The energy of a photon $E = h f = h c / \lambda$, where f is the frequency of the light, c is the speed of light and λ is its wavelength. If an electron goes from energy level E_2 to a lower energy level E_1, the emitted photon has energy $h f = E_2 - E_1$.

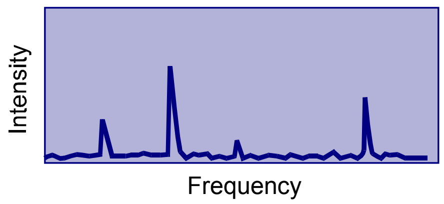
The energy levels of an atom may be deduced by measuring the wavelength of each line in the spectrum then calculating the photon energies corresponding to those lines. These energies correspond to the *difference* in energy between two energy levels in the atom

Energy levels and line spectra

(a) Energy levels



(b) Spectrum



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Model of the atom

A simple model of the atom explains why the electrons have discrete energy levels.

The quantum properties of the electron are responsible for limiting its energy in the atom to certain discrete energy levels. Any quantum particle confined to a limited region of space can exist only in one of a number of distinct quantum states, each with a specific energy.

One way of thinking about this is to associate wave behaviour with the quantum particles. A particle is assigned a de Broglie wavelength, $\lambda = h / m v$, where m is its mass, v is its velocity and h is the Planck constant.

An electron trapped in an atom can be thought of as a standing wave in a box such that the wave 'fits' into the box exactly, like standing waves fit on a vibrating string of fixed length.

Consider a model atom in which an electron is trapped in a rectangular well of width L. Standing waves fit into the well if a whole number of half wavelengths fit across the well. Hence $n \lambda = 2L$ where n is a whole number.

De Broglie's hypothesis therefore gives the electron's momentum $m v = h / \lambda = n h / 2L$. Therefore, the kinetic energy of an electron in the well is:

$$\frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{n^2h^2}{2m(2L)^2}.$$

Thus in this model, the energy of the electron takes discrete values, varying as n^2.

This simple model explains why electrons are at well-defined energy levels in the atom, but it gets the variation of energy with number n quite wrong. Optical spectra measurements indicate that the energy levels in a hydrogen atom follow a $1/n^2$ rule rather than an n^2 rule.

A much better model of the atom is obtained by considering the quantum behaviour of electrons in the correct shape of 'box', which is the $1 / r$ potential of the charged nucleus.

The mathematics of this model, first developed by Schrödinger in 1926, generates energy levels in very good agreement with the energy levels of the hydrogen atom.

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Quark

Quarks are the building blocks of protons and neutrons, and other fundamental particles.

The nucleons in everyday matter are built from two kinds of quark, each with an associated antiquark:

The **up** quark ($+ ^2/_3\, e$) and the **down** quark ($- ^1/_3\, e$).

A proton, charge $+1e$, is made of two up quarks and one down quark **uud**. A neutron, charge 0, is made of one up quark and two down quarks **udd**. A **meson** consists of a quark and an antiquark. For example, a π meson consists of an up or a down quark and a down or up antiquark.

The first direct evidence for quarks was obtained when it was discovered that very high-energy electrons in a beam were scattered from a stationary target as if there were point-like scattering centres in each proton or neutron.

Quarks do not exist in isolation.

Beta decay

β^- decay occurs in neutron-rich nuclei as a result of a down quark changing to an up quark (udd → uud) and emitting a W^-, which decays into an electron (i.e. a β^- particle) and an antineutrino.

β^+ decay occurs in proton-rich nuclei as a result of an up quark changing to a down quark (uud → udd) and emitting a W^+, which decays into a positron (i.e. a β^+ particle) and a neutrino.

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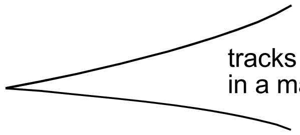
Pair production and annihilation

The positron is the antiparticle of the electron. It differs from the electron in carrying an electric charge of $+ e$ instead of $- e$. The masses of the two are identical. One point of view in quantum mechanics regards positrons as simply electrons moving backwards in time.

A gamma-ray photon of energy in excess of around 1 MeV is capable of creating an electron and a positron. Energy and momentum must always be conserved in a pair production event. The photon energy must exceed the combined rest energy $E_{rest} = mc^2$ of the electron and of the positron, which is about 0.5 MeV for each (actual value 0.505 MeV). To conserve momentum, the creation event must take place close to a nucleus which recoils, carrying away momentum.

Pair production



A positron and an electron annihilate each other when they collide, releasing two gamma photons to conserve momentum and energy. The energy of each gamma photon is half the total energy of the electron and positron. For example, if a positron of energy 1 MeV was annihilated by an electron at rest, the total energy would be approximately 2 MeV including the rest energy of each particle. Hence the energy of each gamma photon would be 1 MeV.

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Subatomic particles

Subatomic particles divide into two main groups:

Leptons: particles not affected by the strong interaction, including electrons, positrons, neutrinos and antineutrinos.

Hadrons: composite particles, made of quarks, held together by the exchange of gluons (strong interaction).

Whereas protons and neutrons contain three quarks, intermediate particles such as pions are made of one quark and one antiquark.

A further fundamental distinction is between bosons and fermions.

A **boson** is the general name given to particles (such as photons and gluons) which carry the interaction between other particles. Bosons have integer spin, and do not obey the exclusion principle. (Some particles made of fermions – e.g. certain nuclei – also have integer spin.)

A **fermion** is the general name given to particles (such as electrons and protons) which function as particles of matter. Fermions have a half integer spin, and obey the exclusion principle (no two particles can be in the same quantum state).

The bosons such as the photon which 'carry' the forces or interactions between matter particles are called **exchange particles**. In quantum physics, all interactions are understood in terms of the exchange of such particles.

It used to be said that there are four different fundamental kinds of interaction between particles: gravity, electromagnetism, the weak interaction and the strong interaction. Electromagnetism is due to the exchange of massless virtual photons between electrically charged bodies. The weak nuclear interaction is responsible for beta decay. However, electromagnetism and the weak interaction have now been brought together into one unified theory. The strong nuclear force is the residual effect of the exchange of gluons between the quarks in a nucleon. There are hopes, not yet fulfilled in 2008, of bringing all the interactions together into one unified theory.

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Proton

Protons are positively charged particles in the nucleus of every atom.

The proton is a positively charged particle of mass 1.007 28 u. Its electric charge is equal and opposite to the charge of the electron. Attempts to detect the decay of protons have so far failed, and it must be regarded as either stable or with a lifetime many times longer than the life of the Universe.

The proton and the neutron are the building blocks of the nucleus, jointly called nucleons. Neutrons and protons in the nucleus are bound together by the strong nuclear force, which is an attractive force with a range of no more than 2 to 3 fm. The strong nuclear force is strong enough to offset the electrical repulsion of the charged protons in a nucleus. Large nuclei have more neutrons than protons because additional neutrons offset the increased mutual repulsion of the large number of protons.

The atomic number of an element is equal to the number of protons in the nucleus of an atom. The symbol Z is used for proton number. An isotope ${}^{A}_{Z}X$ therefore has a nucleus which consists of Z protons and $N = A - Z$ neutrons since A is the number of neutrons and protons in the nucleus.

Neutrons and protons and other particles which interact through the strong nuclear force are collectively referred to as hadrons. Collider experiments have confirmed the quark model of hadronic matter. The proton consists of two up quarks, each of charge $+ {}^{2}/_{3}\ e$, and a down quark which carries a charge of $- {}^{1}/_{3}\ e$. The neutron consists of one up quark and two down quarks.

High-energy proton beams are used in collider experiments to investigate particles and antiparticles created in collisions between protons and protons or between protons and antiprotons. The antiproton is the antimatter counterpart of the proton with exactly the same properties except its electric charge which is equal and opposite to the charge of the proton.

In nuclear fusion in the Sun, protons fuse in a series of reactions to form helium. When two protons fuse, one of the protons becomes a neutron by emitting a β^+ particle, releasing binding energy and leaving a ${}^{2}_{1}H$ nucleus. Further fusion takes place to form ${}^{3}_{2}He$ nuclei and then ${}^{4}_{2}He$ nuclei. In this way, hydrogen in the Sun is gradually converted to helium and energy is released. The fusion process is very slow, accounting for the long lifetime of stars such as the Sun.

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Neutron

Neutrons are, with protons, the particles that make up atomic nuclei.

Neutrons are electrically uncharged particles which exist together with protons in every nucleus except the nucleus of the hydrogen atom ${}^{1}_{1}H$ which is a single proton. Neutrons and protons are the building blocks of nuclei and are collectively referred to as nucleons.

The neutron is an uncharged particle of mass 1.008 67 u. Free neutrons have a half-life of 12.8 minutes. However, in a stable nucleus, neutrons are stable.

Neutrons and protons in the nucleus are held together in the nucleus by the strong nuclear force which is an attractive force with a range of no more than 2 to 3 fm. The strong nuclear force is strong enough to offset the electrical repulsion between the positively charged protons.

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Nucleon

Because neutrons and protons are similar in many respects they are collectively termed nucleons.

A nucleon is a neutron or a proton.

The nucleon number, also called the mass number *A* of an isotope, is the number of protons and neutrons in each nucleus of the isotope.

An isotope is characterised by the number *Z* of protons and the number *N* of neutrons in each nucleus. The nucleon number of an isotope is $A = N + Z$. The symbol for an isotope is ${}^{A}_{Z}X$, where X is the chemical symbol of the element.

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Electron

The electron is a fundamental particle and a constituent of every atom.

The electron carries a fixed negative charge. It is one of six fundamental particles known as leptons.

The charge of the electron, e , is -1.60×10^{-19} C.

The specific charge of the electron, e / m , is its charge divided by its mass. The value of e / m is 1.76×10^{11} C kg^{-1}.

The energy gained by an electron accelerated through a potential difference V is eV. If its speed v is much less than the speed of light, then $eV = (1/2)\, mv^2$.

Electrons show quantum behaviour. They have an associated **de Broglie wavelength** λ given by $\lambda = h/p$, where h is the Planck constant and p the momentum. At speeds much less than the speed of light, $p = mv$. The higher the momentum of the electrons in a beam, the shorter the associated de Broglie wavelength.

Relationships

The electron gun equation $(1 / 2)\, m\, v^2 = e\, V$

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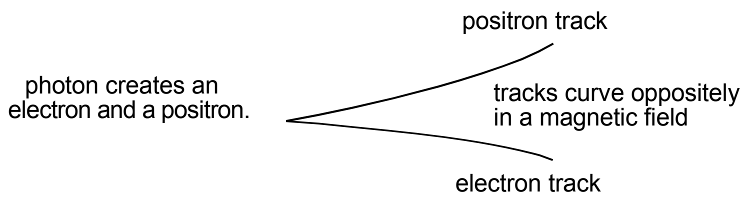
Positron

The positron is a positively charged electron.

The positron is the antiparticle of the electron. It differs from the electron in carrying a charge of $+\,e$ instead of $-\,e$. The masses of the two are identical. One point of view in quantum mechanics regards positrons as simply electrons moving backwards in time.

A gamma-ray photon of energy in excess of around 1 MeV is capable of creating an electron and a positron. Energy and momentum must always be conserved in a pair production event. The photon energy must exceed the combined rest energy of the electron and of the positron which is about 0.5 MeV for each.

Pair production



A positron and an electron annihilate each other when they collide, releasing two gamma photons to conserve momentum and energy.

Positrons are emitted by proton-rich nuclei as a result of a proton changing to a neutron in the nucleus, together with the emission of a neutrino. Positrons may be accelerated to high energies using an accelerator. Synchrotrons are used to store high-energy positrons by making them circulate round the synchrotron. In this way, high-energy positrons and electrons travelling in opposite directions can be brought into collision, annihilating each other to produce photons or other particles.

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Neutrino

A neutrino is a neutral very nearly mass-less particle involved in beta decay.

A neutrino is a lepton, that is a particle that interacts through the weak nuclear force. It has a very low probability of interaction with any other particle. It is electrically uncharged. Its mass was at first thought to be zero; if so it travels at the speed of light. However a very small non-zero mass now seems probable.

The neutrino and its antimatter counterpart, the antineutrino, are created when a nucleus emits an electron and an antineutrino or a positron and a neutrino. Neutrinos and antineutrinos are produced in large quantities in a nuclear reactor.

Neutrinos and antineutrinos are produced in vast quantities in the core of a star as a result of the fusion reactions that take place. They leave the star at high speed, scarcely affected by the outer layers of the star, and they spread out and travel across the Universe. Finally, neutrinos and antineutrinos have been left throughout the Universe by processes in the first few moments of the Big Bang. Estimates of their number put the density of neutrinos and antineutrinos throughout the Universe at an average of over 100 000 per litre. They pass continually through your body essentially without interacting, being uncharged and because of the weakness of the weak interaction.

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Antimatter

For each particle of matter there is an equivalent antiparticle. A few particles (e.g. photons) are their own antiparticles.

Antimatter consists of antiparticles. An antiparticle and particle pair can be produced from a photon of high-energy radiation which ceases to exist as a result. A particle and antiparticle pair have:

1. equal but opposite spin to its particle counterpart;
2. equal but opposite electric charge (or both are uncharged);
3. equal mass (rest energy).

The positron is the antiparticle of the electron.

Two key processes involving antiparticles and particles are:

1. **Pair production**, in which a high-energy photon produces a particle and its antiparticle. This can only occur if the photon energy $h\,f$ is greater than or equal to $2\,m\,c^2$, where m is the mass of the particle, with rest energy $m\,c^2$ for each particle of the pair. More generally, particles are always created in particle–antiparticle pairs. The masses of particles and their antiparticles are identical. All other properties, such as electric charge, spin, lepton or baryon number, are equal but opposite in sign. These properties are therefore conserved in pair production.
2. **Annihilation**, in which a particle and a corresponding antiparticle collide and annihilate each other, producing two photons of total momentum and total energy equal to the initial momentum and energy of the particle and antiparticle, including their combined rest energy $2\,mc^2$. Because properties such as electric charge, spin, and lepton or baryon number are equal but opposite for particles and their antiparticles, these properties are conserved in particle annihilation.

In the theory of special relativity, the rest energy, that is the mass, of particles is just a part of the total energy with $E_{rest} = mc^2$. Thus energy can materialise as particle-antiparticle pairs, having rest energy (mass) greater than zero. Similarly, a particle-antiparticle pair can dematerialise, with their rest energy carried away by for example a pair of photons of zero mass.

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Mass and energy

Mass and energy are linked together in the theory of relativity.

The theory of relativity changes the meaning of mass, making the mass a part of the total energy of a system of objects. For example, the energy of a photon can be used to create an electron-positron pair with mass 0.51 MeV / c^2 each.

Mass and momentum

In classical Newtonian mechanics, the ratio of two masses is the inverse of the ratio of the velocity changes each undergoes in any collision between the two. Mass is in this case related to the difficulty of changing the motion of objects. Another way of saying the same thing is that the momentum of an object is $p = m\,v$.

In the mechanics of the special theory of relativity, the fundamental relation between momentum p , speed v and mass m is different. It is:

$$p = \gamma m v$$

with

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

At low speeds, with $v << c$, where γ is approximately equal to 1, this reduces to the Newtonian value $p = mv$.

Energy

The relationships between energy, mass and speed also change. The quantity

$$E_{total} = \gamma m c^2$$

gives the total energy of the moving object. This now includes energy the particle has at rest (i.e. traveling with you), since when $v = 0$, $\gamma = 1$ and:

$E_{rest} = mc^2$

This is the meaning of the famous equation ***E* = *mc*²**. The mass of an object (scaled by the factor c^2) can be regarded as the **rest energy** of the object. If mass is measured in energy units, the factor c^2 is not needed. For example, the mass of an electron is close to 0.51 MeV.

Kinetic energy
The total energy is the sum of rest energy and kinetic energy, so that:

$E_{kinetic} = E_{total} - E_{rest}$

This means that the kinetic energy is given by:

$E_{kinetic} = (\gamma - 1)mc^2$

At low speeds, with $v << c$, it turns out that γ - 1 is given to a good approximation by:

$(\gamma - 1) = \frac{1}{2}(v^2 / c^2)$

so that the kinetic energy has the well-known Newtonian value:

$E_{kinetic} = \frac{1}{2}mv^2$

High energy approximations
Particle accelerators such as the Large Hadron Collider are capable of accelerating particles to a total energy many thousands of times larger than their rest energy. In this case, the high energy approximations to the relativistic equations become very simple.

At any energy, since $E_{total} = \gamma mc^2$ and $E_{rest} = mc^2$, the ratio of total energy to rest energy is just the relativistic factor γ:

$$\gamma = \frac{E_{total}}{E_{rest}}$$

This gives a very simple way to find γ, and so the effect of time dilation, for particles in such an accelerator.

Since the rest energy is only a very small part of the total energy,

$E_{kinetic} \approx E_{total}$

the relationship between energy and momentum also becomes very simple. Since $v \approx c$, the momentum can be written:

$p \approx \gamma mc$

and since the total energy is given by

$E_{total} = \gamma mc^2$

their ratio is simply:

$\frac{E_{total}}{p} \approx c$, giving $E_{total} \approx pc$

This relationship is exactly true for photons or other particles of zero rest mass, which always travel at speed *c*.

Differences with Newtonian theory
The relativistic equations cover a wider range of phenomena than the classical relationships do.

Change of mass equivalent to the change in rest energy is significant in nuclear reactions where extremely strong forces confine protons and neutrons to the nucleus. Nuclear rest energy changes are typically of the order of MeV per nucleon, about a million times larger than chemical energy changes. The change of mass for an energy change of 1 MeV is therefore comparable with the mass of an electron.

Changes of mass associated with change in rest energy in chemical reactions or in gravitational changes near the Earth are small and usually undetectable compared with the masses of the particles involved. For example, a 1 kg mass would need to gain 64 MJ of potential energy to leave the Earth completely. The corresponding change in mass is insignificant (7×10^{-10} kg = 64 MJ / c^2). A typical chemical reaction involves energy change of the order of an electron volt (= 1.6×10^{-19} J). The mass change is about 10^{-36} kg (= 10^{-19} J / c^2), much smaller than the mass of an electron.

Approximate and exact equations
The table below shows the relativistic equations relating energy, momentum, mass and speed. These are valid at all speeds *v*. It also shows the approximations which are valid at low speeds $v << c$, at very high speeds $v \approx c$, and in the special case where $m = 0$ and $v = c$.

Conditions	Relativistic factor γ	Total energy	Rest energy	Kinetic energy	Momentum
$m > 0$ *v* any value $< c$ any massive particle	$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$ $\gamma = \frac{E_{total}}{E_{rest}}$	$E_{total} = \gamma mc^2$	$E_{rest} = mc^2$	$E_{kinetic} = (\gamma - 1)mc^2$	$p = \gamma mv$
$m > 0$ $v << c$ Newtonian	$\gamma \approx 1$	$E_{total} \approx mc^2$	$E_{rest} = mc^2$	$E_{kinetic} \approx \frac{1}{2}mv^2$	$p \approx mv$
$m > 0$ $v \approx c$ ultra-relativistic	$\gamma = \frac{E_{total}}{E_{rest}}$	$E_{total} = \gamma mc^2$	$E_{rest} = mc^2$	$E_{kinetic} \approx E_{total}$	$p \approx \gamma mc$ $p \approx \frac{E_{total}}{c}$
$m = 0$ $v = c$ photons	γ is undefined	$E = hf$	$E_{rest} = 0$	$E = E_{kinetic} = E_{total}$	$p = \frac{E}{c}$

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Relativistic calculations of energy and speed

Calculating the speed of an accelerated particle, given its kinetic energy
If the speed is much less than that of light, Newton's laws are good approximations.

Newtonian calculation:
Since the kinetic energy $E_K = (1/2)mv^2$, the speed *v* is given by:

$$v^2 \quad \frac{2E_K}{m}$$

In an accelerator in which a particle of charge q is accelerated through a potential difference V, the kinetic energy is given by:

$E_K = qV$,

Thus:

$$v^2 \quad \frac{2qV}{m}$$

The Newtonian calculation seems to give an 'absolute' speed, not a ratio v/c.

Relativistic calculation:
A relativistic calculation mustn't give an 'absolute speed'. It can only give the speed of the particle as a fraction of the speed of light. The total energy E_{total} of the particle has to be compared with its rest energy mc^2. For an electron, the rest energy corresponding to a mass of 9.1×10^{-31} kg is 0.51 MeV. A convenient relativistic expression is:

$$\frac{E_{total}}{E_{rest}} = \gamma$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Since the total energy $E_{total} = mc^2 + qV$, then:

$$\gamma = 1 + \frac{qV}{mc^2}$$

This expression gives a good way to see how far the relativistic calculation will depart from the Newtonian approximation. The Newtonian calculation is satisfactory only if γ is close to 1.

The ratio v / c can be calculated from γ:

$$v/c = \sqrt{1 - 1/\gamma^2} \;.$$

A rule of thumb
As long as the accelerator energy qV is much less than the rest energy, the factor γ is close to 1, and v is much less than c. The Newtonian equations are then a good approximation. To keep γ close to 1, say up to 1.1, the accelerator energy qV must be less than 1/10 the rest energy. So for electrons, rest energy 0.51 MeV, accelerating potential differences up to about 50 kV give speeds fairly close to the Newtonian approximation. This is a handy rule of thumb.

Accelerating voltage / kV	Speed of electrons (Newton)	$\gamma = 1 + qV/mc^2$ $mc^2 = 0.51$ MeV	Speed of electrons (Einstein)	Error in speed
10	0.198c	1.019	0.195c	1.5%
50	0.442c	1.097	0.412c	7%
100	0.625c	1.195	0.548c	14%
500	1.4c	$1.97 \cong 2$	0.86c	62%
5000	4.4c	10.7	0.99c	>300%

An example: a cosmic ray crosses the Galaxy in 30 seconds

A proton of energy 10^{20} eV is the highest energy cosmic ray particle yet observed (2008). How long does such a proton take to cross the entire Milky Way galaxy, diameter of the order 10^5 light years?

The rest energy (mass) of a proton is about 1 GeV/c^2, or 10^9 eV/c^2. Then:

$$E_{total} = \gamma mc^2$$

with

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Inserting values: E_{total} = 10^{20} eV/c^2 and m = 10^9 eV/c^2 gives:

$$\gamma = \frac{10^{20}\ \text{eV}}{10^9\ \text{eV}} = 10^{11}$$

The proton, travelling at very close to the speed of light, would take 10^5 years to cross the galaxy of diameter 10^5 light years. But to the proton, the time required will be its wristwatch time τ where:

$t = \gamma\tau$

$$\tau = \frac{t}{\gamma} = \frac{10^5\ \text{year}}{10^{11}} \cong \frac{3 \times 10^{12}\ \text{s}}{10^{11}} = 30\ \text{s}$$

The wristwatch time for the proton to cross the whole Galaxy is half a minute. From its point of view, the diameter of the galaxy is shrunk by a factor 10^{11}, to a mere 10^5 km.

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Summary Diagrams

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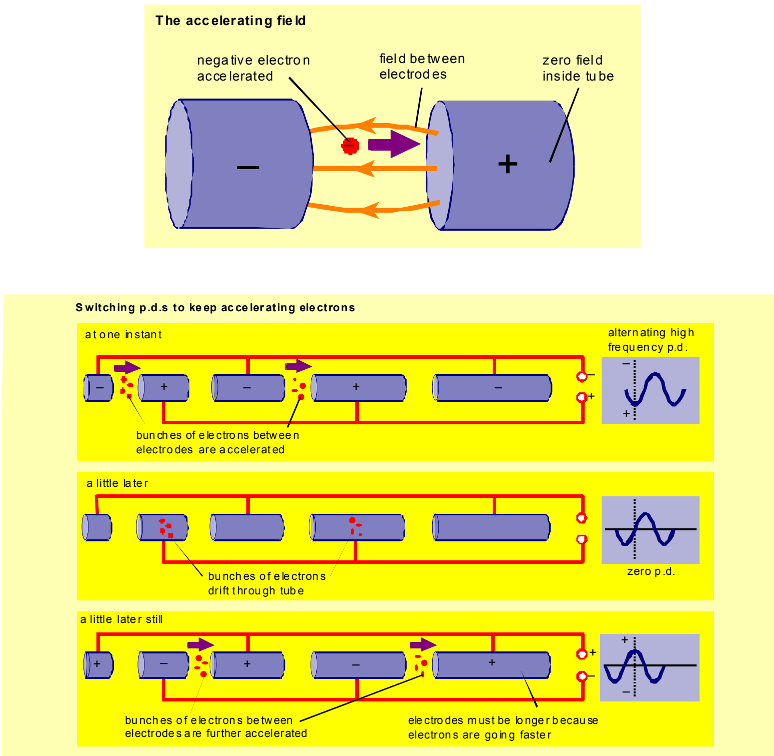
The linear accelerator (from Chapter 16)

The principle of a linear accelerator



The alternating p.d. switches back and forth, the electrons accelerating as they pass between electrodes

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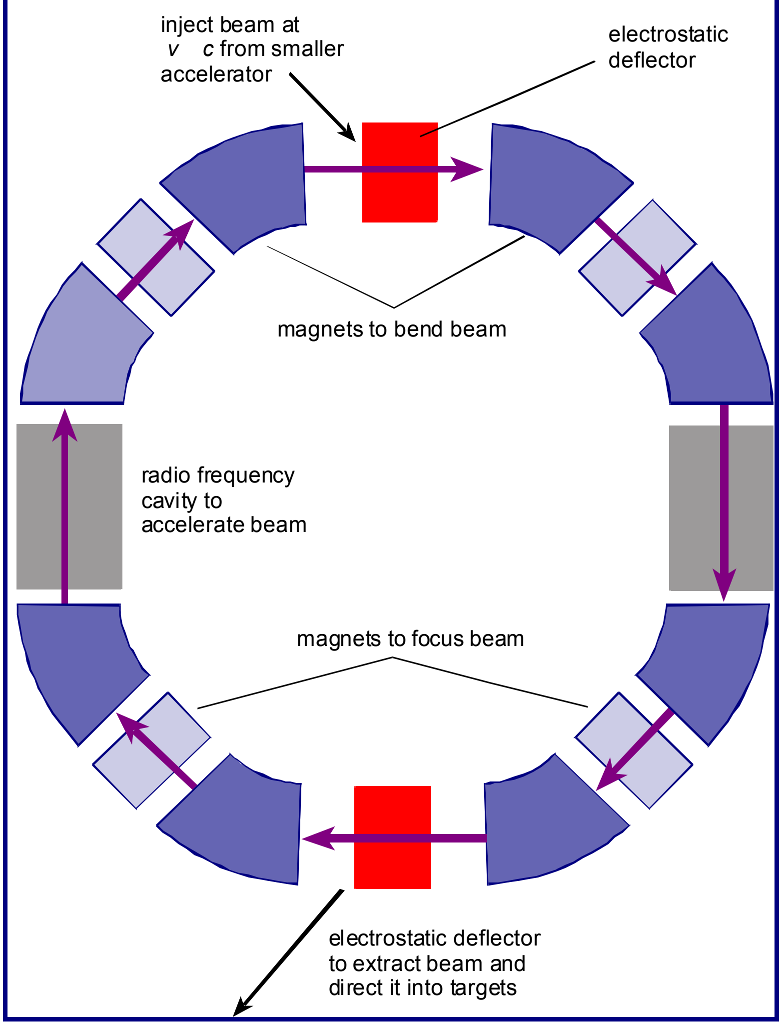
Principle of the synchrotron accelerator (from Chapter 16)

The principle of a synchrotron accelerator



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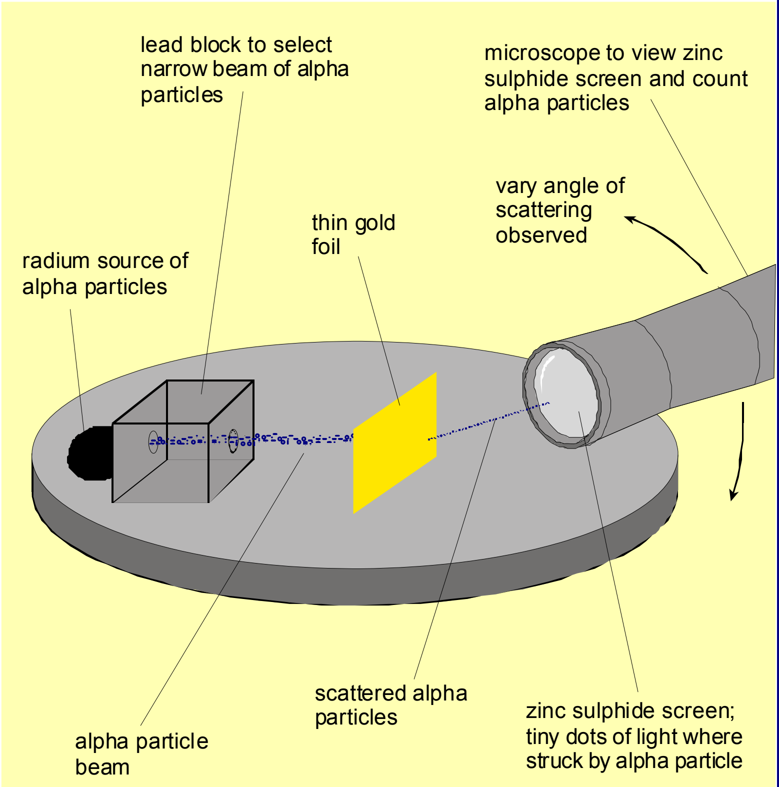
Alpha particle scattering experiment

Rutherford's scattering experiment



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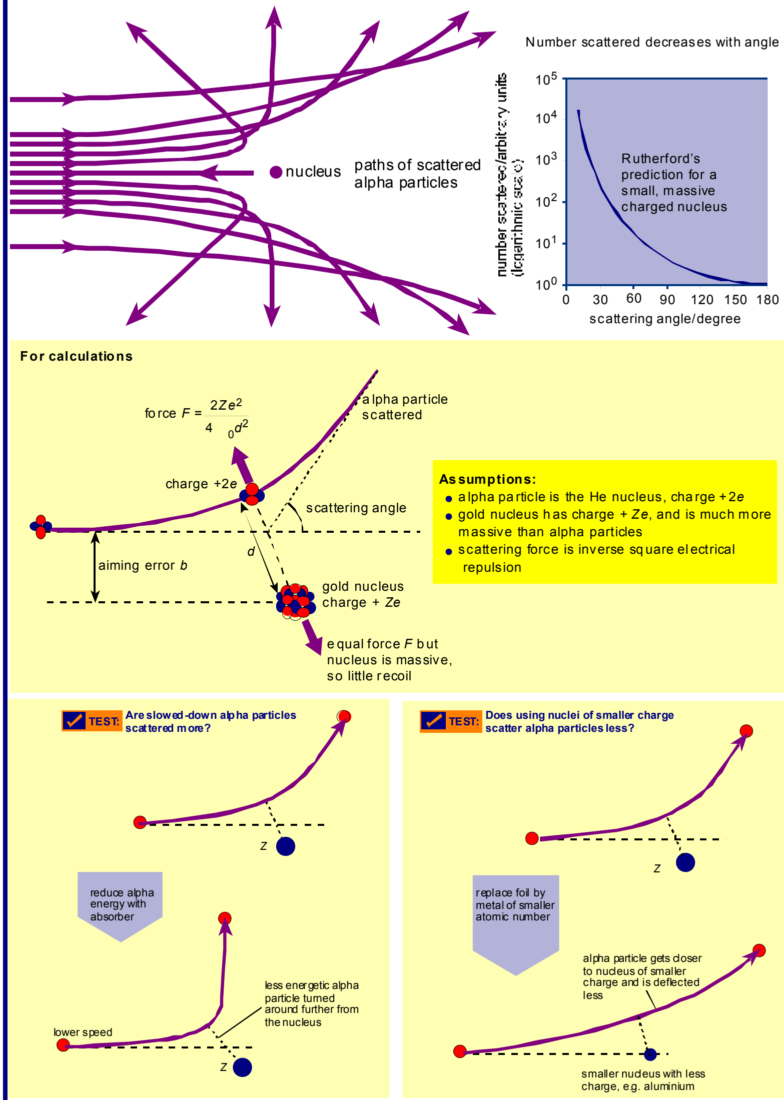
Rutherford’s picture of alpha particle scattering

Rutherford’s picture of alpha scattering



Careful investigation of alpha scattering supported the nuclear model of the atom

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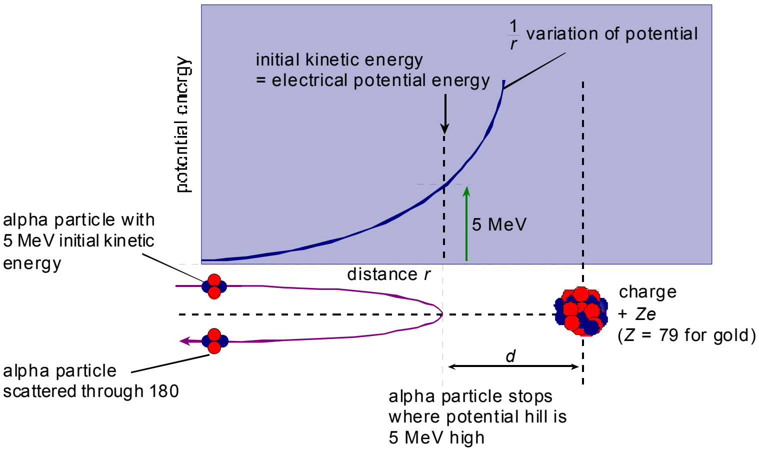
Distance of closest approach

Distance of closest approach



Where does the alpha particle stop?

Initial kinetic energy

= 5 MeV

= 5 $\quad 10^{6}$ eV $\quad$ 1.6 $\quad 10^{-19}$ J eV^{-1}

= 8.0 $\quad 10^{-13}$ J

Electrical potential energy

$$V = \frac{+2\,Ze^2}{4 \quad _0 d}$$

$Z = 79$, $e = 1.6 \quad 10^{-19}$ C,

$_0 = 8.9 \quad 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

Alpha particle stops where

initial kinetic energy = electrical potential energy

$$8.0 \quad 10^{-13}\ \text{J} = \frac{+2\,Ze^2}{4 \quad _0 d}$$

substitute values of Z, e, $_0$:

$d = 4.5 \quad 10^{-14}$ m

The radius of a gold nucleus must be less than 10^{-14} m. Atoms are 10000 times larger than their nuclei.

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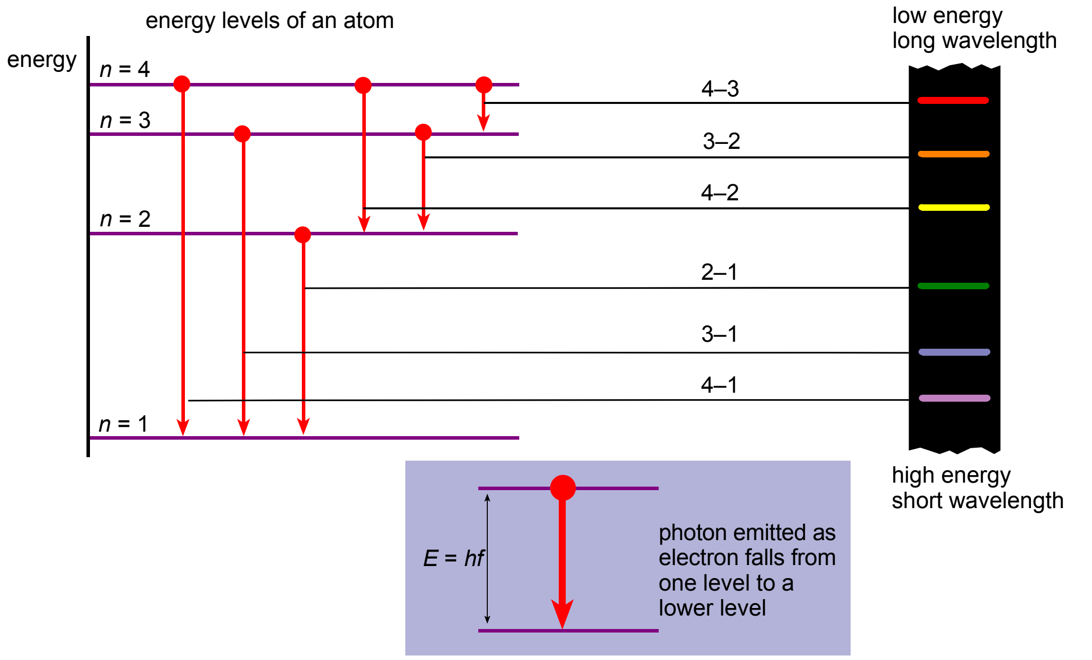
Spectra and energy levels

Spectral lines and energy levels



Spectral lines map energy levels
***E* = *hf* is the energy difference between two levels**

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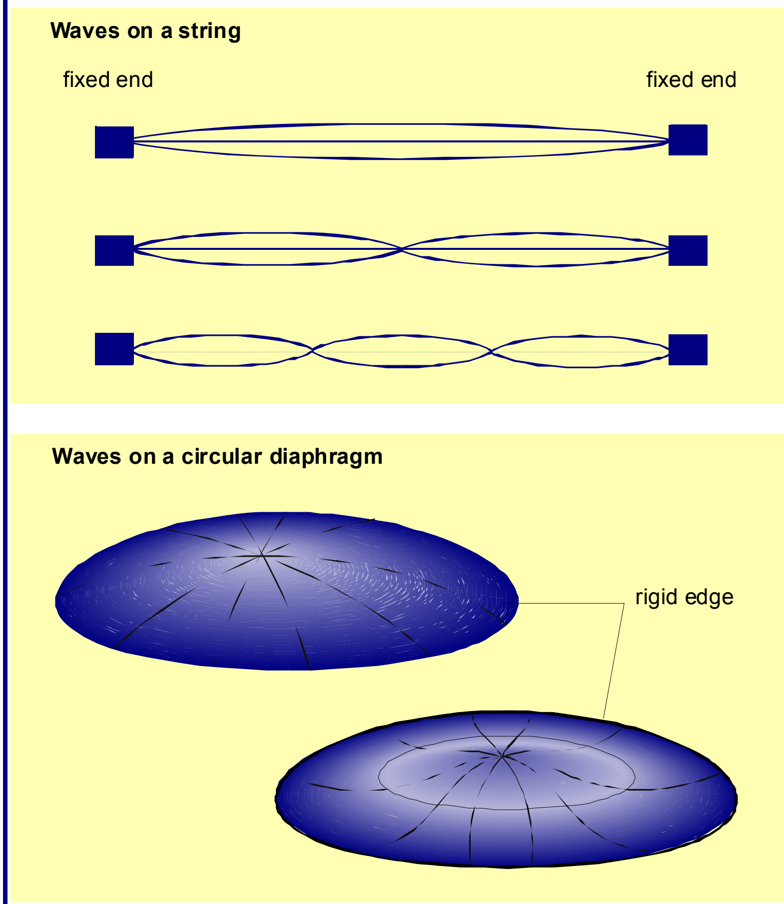
Standing waves in boxes

Standing waves in boxes



Only certain field patterns are possible because the waves must fit inside the box

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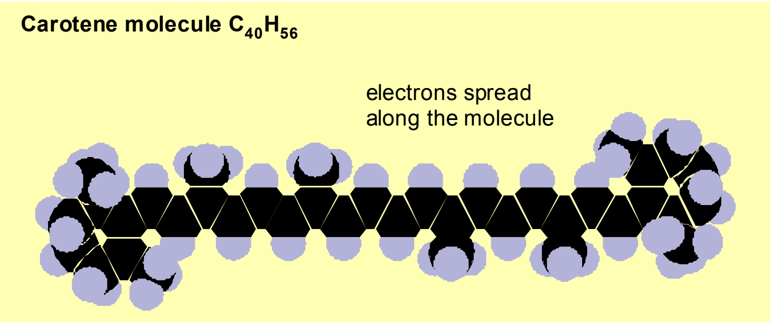
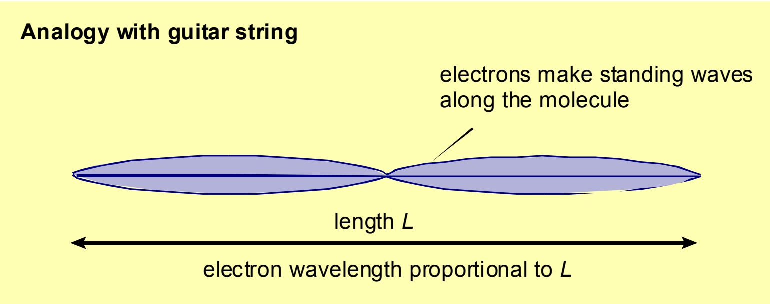
Colours from electron guitar strings

Molecular guitar strings





Long molecules absorb visible wavelengths of light

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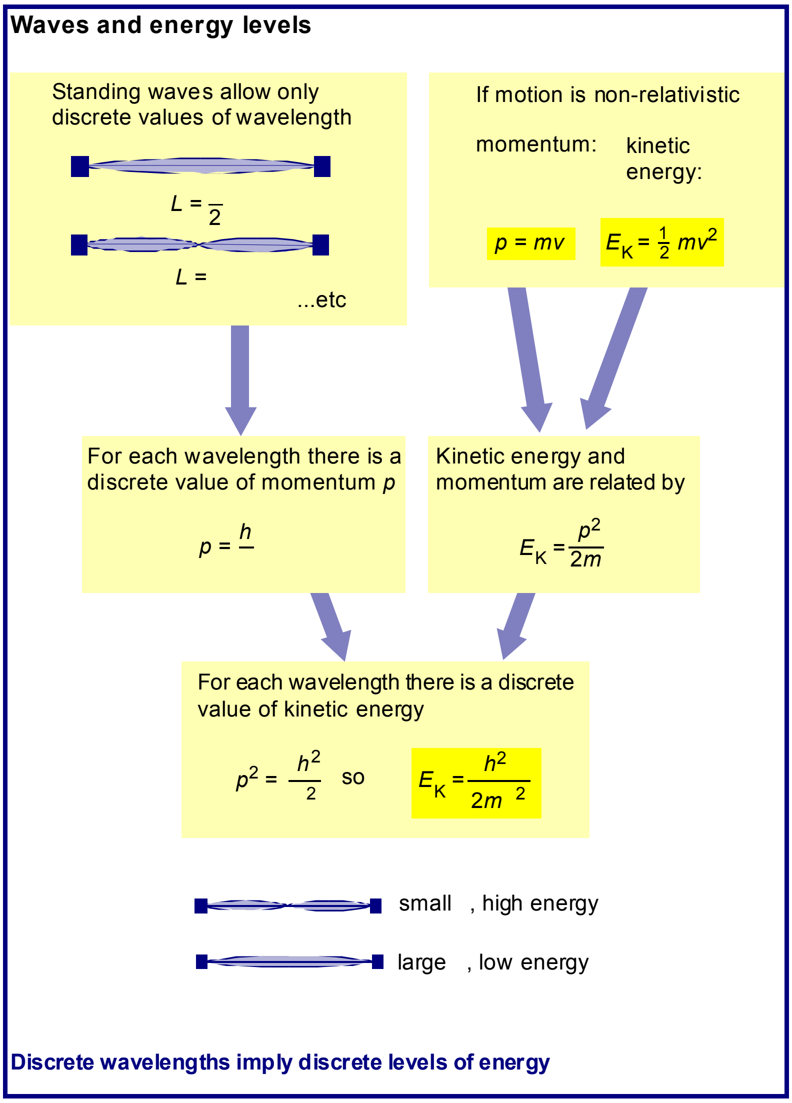
Energy levels

Standing waves and energy levels



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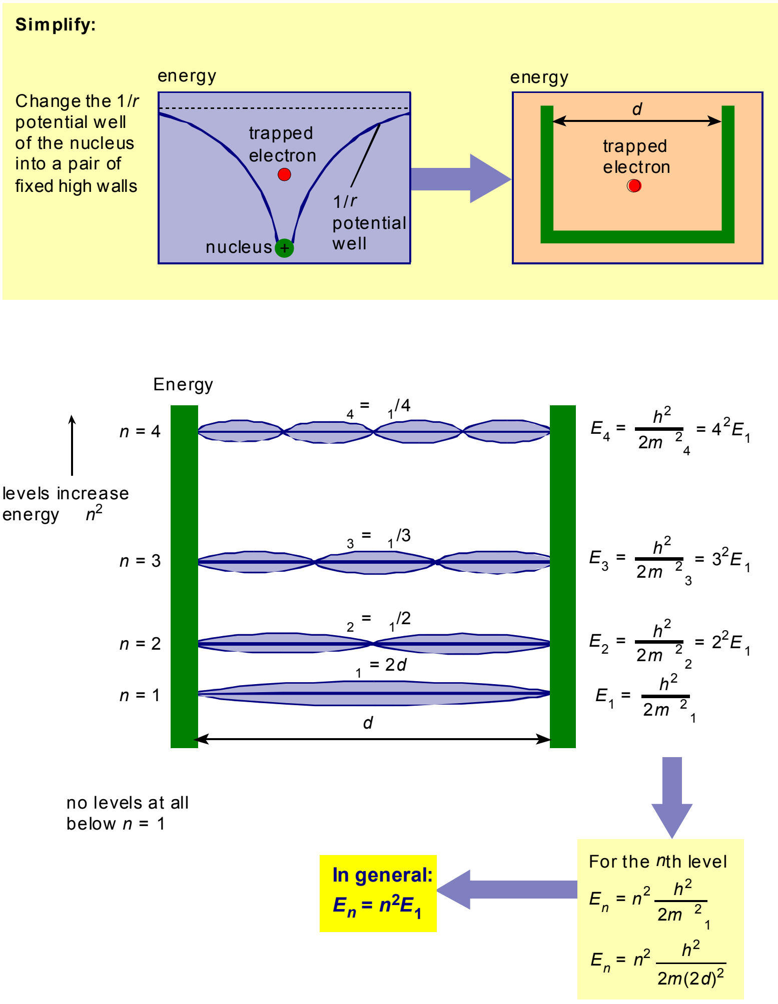
Standing waves in atoms

Standing waves and energy levels

A guitar–string atom



Each level has a quantum number *n*. The energy depends on the quantum number.

Note that in a real one-electron atom (hydrogen) the energy of the levels varies as $1/n^2$, not as n^2.

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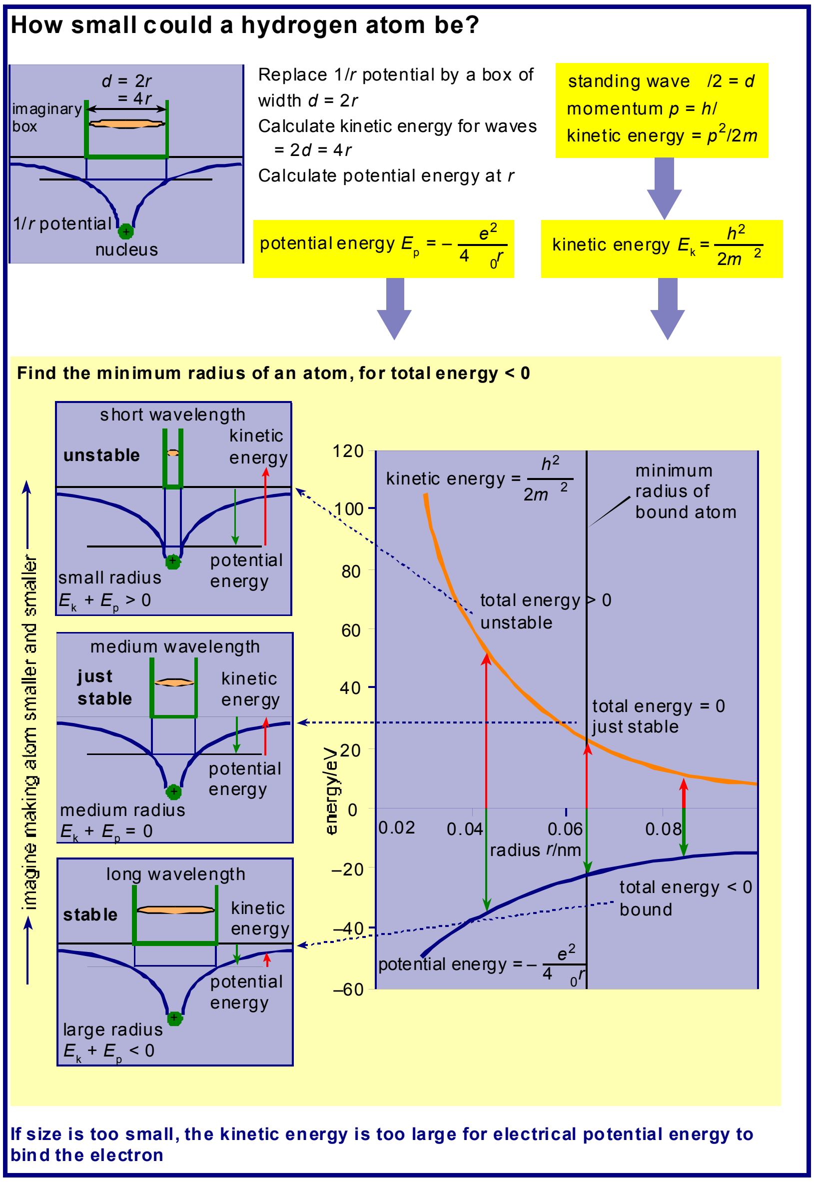
Size of the hydrogen atom



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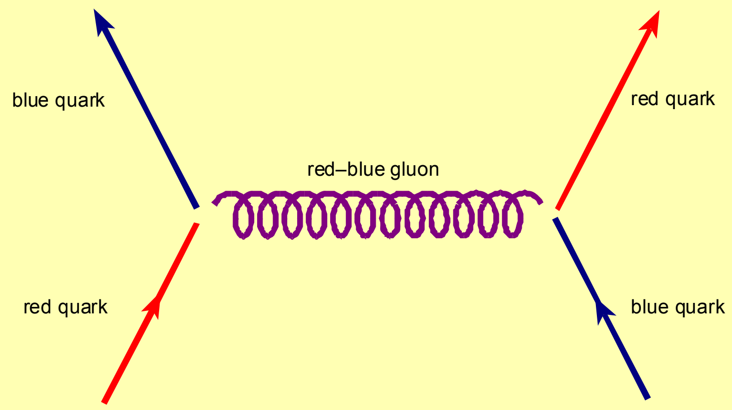
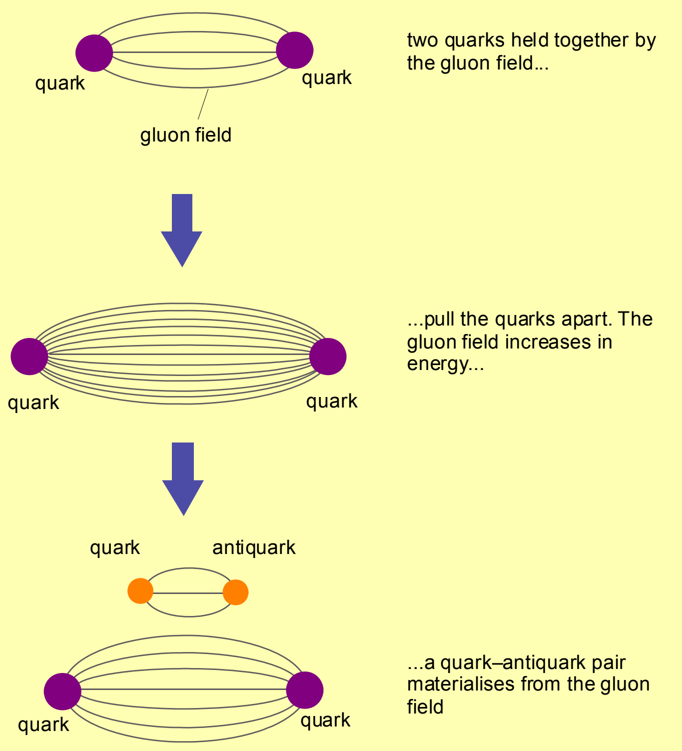
Quarks and gluons

Quark–gluon interaction



Quarks interact by exchanging gluons, which change the quark colours. Here a red quark and a blue quark exchange a red–blue gluon. The red quark becomes blue and the blue quark becomes red. The quarks exchange energy and momentum.

Quarks pulled apart make more quarks



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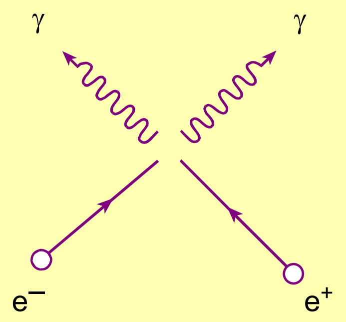
Pair creation and annihilation

Pair annihilation and creation

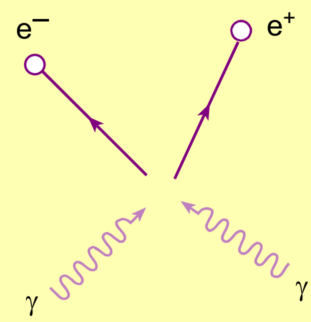
Annihilation

gamma energy
= 2 × 0.511 MeV plus
kinetic energy of electrons



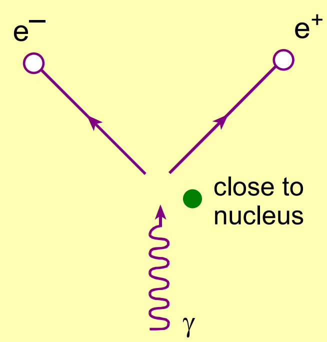
Creation?

extremely rare
(cannot bring two
identical photons
together)



Pair creation

gamma energy
= 2 × 0.511 MeV (minimum)
nucleus carries away
momentum, to conserve
momentum and energy



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What the world is made of

The particles of the everyday world
Everything you touch around you is made of just these particles:

The world around you	Leptons	charge	rest energy / MeV	Quarks	charge	rest energy / MeV
	e^- electron	–1	0.511	u up	$+\frac{2}{3}$	6?
	$_e$ neutrino	0	0?	d down	$-\frac{1}{3}$	10?

A complete picture of your world should include their antiparticles too:

The world around you	Leptons	charge	rest energy / MeV	Quarks	charge	rest energy / MeV
particles	e^- electron	–1	0.511	u up	$+\frac{2}{3}$	6?
	$_e$ neutrino	0	0?	d down	$-\frac{1}{3}$	10?
antiparticles	e^+ positron	+1	0.511	$\bar{u}$ anti-up	$-\frac{2}{3}$	6?
	$\bar{}_e$ antineutrino	0	0?	$\bar{d}$ anti-down	$+\frac{1}{3}$	10?

To account for all known matter, the pattern of a pair of leptons and a pair of quarks repeats three times:

Generation	Leptons	charge	rest energy / MeV	Quarks	charge	rest energy / MeV
1 The world around you	e^- electron	–1	0.511	u up	$+\frac{2}{3}$	6?
	$_e$ neutrino	0	0?	d down	$-\frac{1}{3}$	10?
2	$^-$ muon	–1	106	s strange	$-\frac{1}{3}$	200?
	muon-neutrino	0	0?	c charmed	$+\frac{2}{3}$	1500?
3	$^-$ tau	–1	1780	b bottom	$-\frac{1}{3}$	5000?
	tau-neutrino	0	0?	t top	$+\frac{2}{3}$	90 000?

The other particles that make up the world are the bosons, the carriers of interactions:

interaction	force carrier	electric charge	rest energy / GeV	explains
electromagnetism	photon	0	0	Everyday interactions including all chemistry
weak interaction	Z^0 W^+ W^-	0 +1 –1	93 81 81	Radioactive decays; changing particle nature
strong interaction	8 different 'colour combinations' of gluons	0	0	What holds nucleons and mesons together
gravity	'graviton'	0	0	Conjectured, but not detected

The hunt is on at the LHC (Large Hadron Collider) for another particle, the Higgs boson, which is thought to be responsible for particles having mass.

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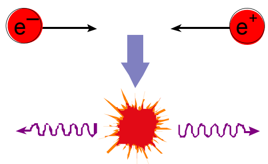
Conserved quantities in electron-positron annihilation

Conserved quantities

Simplify:
assume head-on
collision with
equal speeds



Energy is conserved

total energy before	=	total energy after
= kinetic energy of particles + rest energy of particles minimum value of energy before is rest energy: = 2 mc^2 = 2 0.511 MeV		energy after is energy of gamma photons = 2 0.511 MeV

Momentum is conserved

total linear momentum before	=	total linear momentum after
 same mass; equal and opposite velocities total momentum before = 0		 energy E, momentum $p = E/c$ photons identical, momentums opposite total momentum = 0

Electric charge is conserved

total charge before	=	total charge after
charge $(-e) + (+e) = 0$		charge 0 + 0 = 0

Energy, momentum and electric charge are always conserved in electron–positron annihilation

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Relativistic momentum $p = \gamma mv$ (from Chapter 16)

Einstein redefines momentum

Problem:

time *t* depends on relative motion because of time dilation (chapter 12)

Newton's definition of momentum

$p = mv$

$p = m \frac{x}{t}$

Einstein's solution:

Replace *t* by , the change in wristwatch time , which does not depend on relative motion

from time dilation:

$= \frac{1}{\sqrt{1-v^2/c^2}}$

t =

substitute for

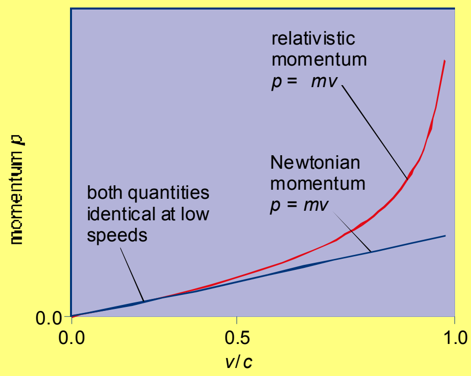
$\frac{x}{t} = v$

Einstein's new definition of momentum

$p = m \frac{x}{}$

$p = m \frac{x}{t}$

$p = mv$



relativistic momentum

$p = mv$

Relativistic momentum *p* = *mv* increases faster than Newtonian momentum *mv* as *v* increases towards *c*

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Relativistic energy $E_{total} = \gamma mc^2$ (from Chapter 16)

Einstein rethinks energy

relativistic idea:

Space and time are related. Treat variables x and ct similarly.
Being at rest means moving in wristwatch time .

so invent

relativistic momentum for 'movement in wristwatch time'

Just write ct in place of x

from time dilation $\frac{x}{\ }$ =

multiply p_0 by c, getting a quantity E having units of energy (momentum × speed)

relativistic momentum

for component of movement in space

$p = m \frac{x}{\ }$

relativistic momentum

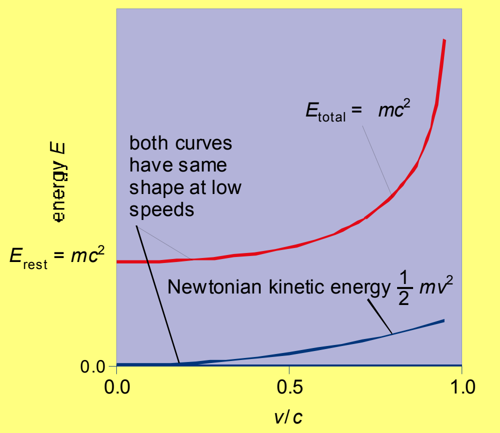
for component of movement in time

$p_0 = m \frac{ct}{\ }$

$p_0 = mc \frac{t}{\ }$

$p_0 = \ mc$

$p_0c = E = \ mc^2$



interpret energy $E = \ mc^2$

1 particle at rest: v = 0 and = 1 → $E_{rest} = mc^2$ → particle has rest energy

2 particle moving at speed v energy mc^2 greater than rest energy → $E_{kinetic} = \ mc^2 - mc^2 = (\ -1)mc^2$

3 at low speeds $v << c$ kinetic energy has same value as for Newton → $E_{kinetic} = (\ -1)mc^2 \approx \frac{1}{2}mv^2$ (see graph)

$E_{total} = \ mc^2$

interpret E as total energy = kinetic energy + rest energy

relativistic energy

$E_{total} = \ mc^2$

$E_{rest} = mc^2$

$= \frac{E_{total}}{E_{rest}}$

Total energy = mc^2. Rest energy = mc^2. Total energy = kinetic energy + rest energy.

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Energy, momentum and mass (from Chapter 16)

Energy, momentum and mass

Key relationships

relativistic factor $= \frac{1}{\sqrt{1-v^2/c^2}}$

momentum $p = mv$

total energy $E_{total} = mc^2$

rest energy $E_{rest} = mc^2$

$E_{kinetic} = E_{total} - E_{rest}$

$= \frac{E_{total}}{E_{rest}}$

low speeds $v << c$ ≈ 1

energy

$E_{total} \approx E_{rest} = mc^2$

kinetic energy small compared to total energy

$E_{kinetic} \approx \frac{1}{2} mv^2$

large rest energy nearly equal to total energy

rest energy $E_{rest} = mc^2$

$E_{rest} >> E_{kinetic}$

momentum

since $= 1$ momentum $p = mv \approx mv$

$p \approx mv$

Kinetic energy small compared to rest energy

same rest energy scaled down

high speeds v c $>> 1$

energy

$E_{total} = mc^2$

$E_{kinetic} = E_{total} - E_{rest} = (-1)mc^2$

large kinetic energy nearly equal to total energy

$E_{kinetic} \approx E_{total} >> E_{rest}$

rest energy $E_{rest} = mc^2$

rest energy small compared to total energy

momentum

since $v \approx c$ momentum $p = mv$

since $E_{total} = mc^2$ $p \times \frac{E_{total}}{c}$

Kinetic energy large compared to rest energy

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