Revision Guide for Chapter 16

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Revision Checklist

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I can show my understanding of effects, ideas and relationships by describing and explaining cases involving:

a uniform electric field $E = V/d$ (measured in volts per metre) Revision Notes: electric field Summary Diagrams: The electric field between parallel plates, Two ways of describing electrical forces, Field strength and potential gradient, Field lines and equipotential surfaces	
the electric field of a charged body; the force on a small charged body in an electric field; the inverse square law for the field due to a small (point or spherical) charged object Revision Notes: electric field, inverse square laws Summary Diagrams: Inverse square law and flux, Radial fields in gravity and electricity, How an electric field deflects an electron beam	
electrical potential energy and electric potential due to a point charge and the $1/r$ relationship for electric potential due to a point charge Revision Notes: electric potential, inverse square laws Summary Diagrams: Force, field, energy and potential, Radial fields in gravity and electricity	
evidence for the discreteness of the charge on an electron Revision Notes: electron Summary Diagrams: Millikan's experiment	
the force qvB on a moving charged particle due to a magnetic field Revision Notes: force on a moving charge Summary Diagrams: How a magnetic field deflects an electron beam, Force on current: force on moving charge, Measuring the momentum of moving charged particles	
relativistic relationships between mass and energy Revision Notes: mass and energy, relativistic calculations of energy and speed Summary Diagrams: The ultimate speed – Bertozzi's demonstration, Relativistic momentum, Relativistic energy, Energy, momentum and mass	

I can use the following words and phrases accurately when describing effects and observations:

electric charge, electric field; electric potential ($J\ C^{-1}$) and electrical potential energy (J); equipotential surface Revision Notes: electric field, inverse square laws, electric potential Summary Diagrams: Force, field, energy and potential	
the **electron volt** used as a unit of energy Revision Notes: electron volt	

I can sketch and interpret:

graphs of **electric force versus distance**, knowing that the area under the curve between two points gives the electric potential difference between the points graphs of **electric potential and electrical potential energy versus distance**, knowing that the tangent to the potential vs distance graph at a point gives the value of the electric field at that point Summary Diagrams: Force, field, energy and potential, Radial fields in gravity and electricity	
diagrams illustrating electric fields (*e.g. uniform and radial*) and the corresponding equipotential surfaces Revision Notes: electric field, inverse square laws Summary Diagrams: Field strength and potential gradient, Field lines and equipotential surfaces	

I can make calculations and estimates making use of:

the force qE on a moving charged particle in a uniform electric field Revision Notes: electric field Summary Diagrams: How an electric field deflects an electron beam	
radial component of electric force due to a point charge $F_{\text{electric}} = \frac{kqQ}{r^2}$ radial component of electric field due to a point charge $E_{\text{electric}} = \frac{F_{\text{electric}}}{q} = \frac{kQ}{r^2}$ Revision Notes: electric field, inverse square laws Summary Diagrams: Force, field, energy and potential	
electric field related to electric potential difference $E_{\text{electric}} = -\frac{dV}{dx}$ and $E_{\text{electric}} = \frac{V}{d}$ for a uniform field Revision Notes: electric field Summary Diagrams: Force, field, energy and potential, Field strength and potential gradient	
electric potential at a point distance r from a point charge: $V = \frac{kQ}{r}$ Revision Notes: electric potential Summary Diagrams: Force, field, energy and potential	

the force *F* on a charge *q* moving at a velocity *v* perpendicular to a magnetic field *B*: $F = q v B$ Revision Notes: force on a moving charge Summary Diagrams: How a magnetic field deflects an electron beam, Force on current: force on moving charge, Measuring the momentum of moving charged particles	

Revision Notes

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Electric field

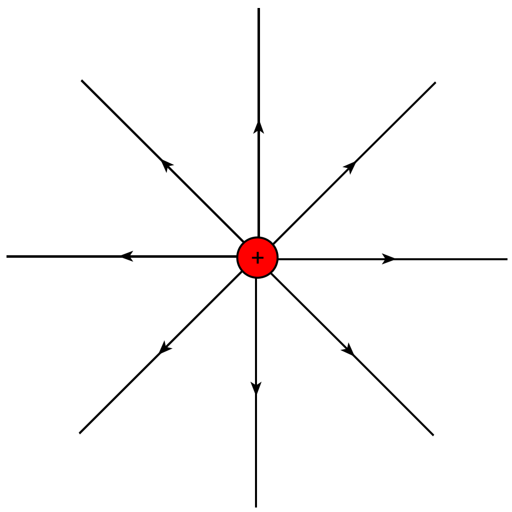
Electric fields have to do with the forces electric charges exert on one another.

Electric fields are important in a wide range of devices ranging from electronic components such as diodes and transistors to particle accelerators.

An electric field occupies the space round a charged object, such that a force acts on any other charged object in that space.

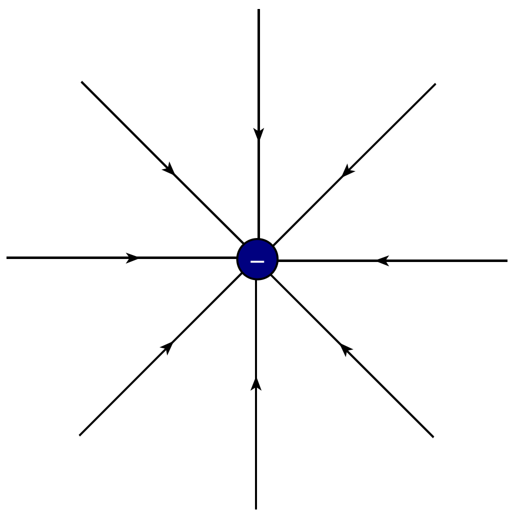
The lines of force of an electric field trace the direction of the force on a positive **point charge**.

Electric field lines



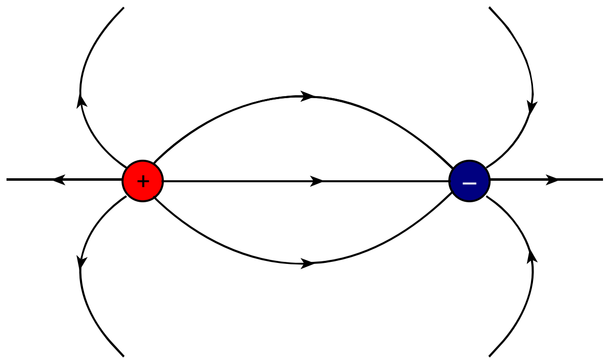
Near a positive point charge

Electric field lines



Near a negative point charge

Electric field lines



Near opposite point charges

The **electric field strength** *E* at a point in an electric field is the force per unit charge acting on a small positive charge at that point. Electric field strength is a vector quantity in the direction of the force on a positive charge.

The SI unit of electric field strength is the newton per coulomb ($N\ C^{-1}$) or equivalently the volt per metre ($V\ m^{-1}$).

The force F on a point charge q at a point in an electric field is given by $F = q E$, where E is the electric field strength at that point.

If a point charge $+q$ is moved a small distance δx along a line of force in the direction of the line, the field acts on the charge with a force qE and therefore does work δW on the charge equal to the force multiplied by the displacement. Hence $\delta W = qE\delta x$ so the potential energy E_P of the charge in the field is changed by an amount $\delta E_P = -qE\delta x$, where the minus sign signifies a reduction. Since the change of potential is given by

$$\delta V = \frac{\delta E_P}{q}$$

then $\delta V = -E\delta x$, so that

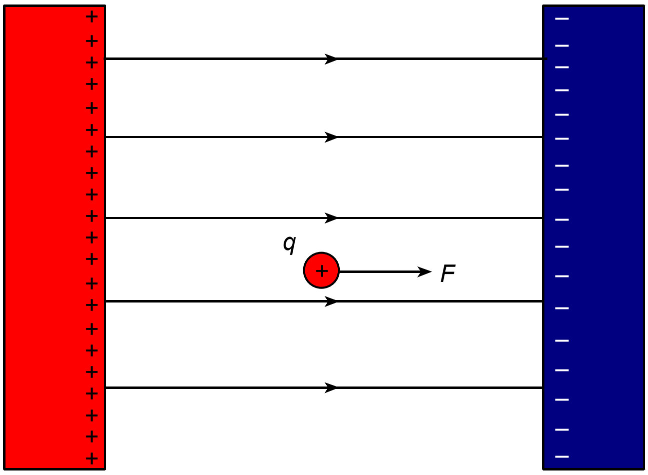
$$E = -\frac{\delta V}{\delta x}.$$

The electric field is thus the negative gradient of the electric potential. In the limit $\delta x \to 0$

$$E = -\frac{dV}{dx}.$$

Thus the larger the magnitude of the potential gradient, the stronger is the electric field strength. The direction of the electric field is **down** the potential gradient. A strong field is indicated by a concentration of lines of force or by equipotential surfaces close together.

Force on a point charge in a uniform field



A **uniform electric field** exists between two oppositely charged parallel conducting plates at fixed separation. The lines of force are parallel to each other and at right angles to the plates. Because the field is uniform, its strength is the same in magnitude and direction everywhere. The potential increases uniformly from the negative to the positive plate along a line of force. For perpendicular distance d between the plates, the potential gradient is constant and equal to V / d , where V is the potential difference between the plates. The electric field strength therefore has magnitude $E = V / d$.

A point charge q at any point in the field experiences a force $q\, V / d$ at any position between the plates.

Relationships
$F = q\, E$ gives the force on a test charge q in an electric field of strength E.

$$E = -\frac{dV}{dx}.$$

$E = V / d$ for the electric field strength between two parallel plates.

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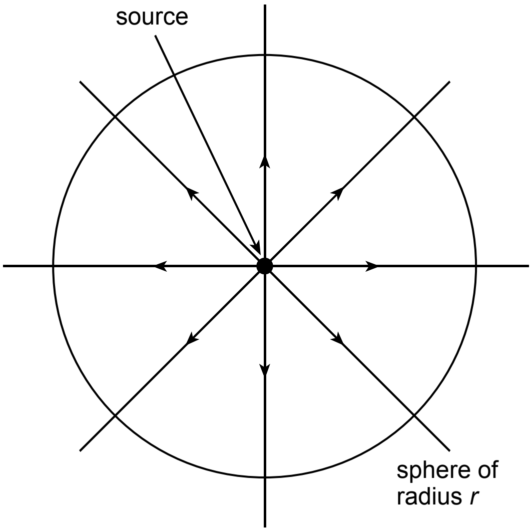
Inverse square laws

Radiation from a point source, and the electric and gravitational fields of point charges and masses respectively all obey inverse square laws of intensity with distance.

An inverse square law is a law in which a physical quantity such as radiation intensity or field strength at a certain position is proportional to the inverse of the square of the distance from a fixed point.

The inverse square law



The following quantities obey an inverse square law:

The **intensity of radiation** I from a point source (provided the radiation is not absorbed by material surrounding the source) is given by

$$I = \frac{W}{4\pi r^2}$$

where r is the distance from the source and W is the rate of emission of energy by the source. The factor 4π arises because all the radiation energy emitted per second passes through a sphere of surface area $4\pi r^2$ at distance r.

The radial component of **electric field strength** E at distance r from a point charge Q in a vacuum

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

The lines of force are radial, spreading out from Q. The inverse square law for the intensity of the field shows that the lines of force may usefully be regarded as continuous, with their number per unit area representing the field intensity, since in this case the lines will cover the area $4\pi r^2$ of a sphere surrounding the charge.

The radial component of **gravitational field strength**, g, at distance r from the centre of a sphere of mass M,

$$g = -\frac{GM}{r^2}.$$

The lines of force are radial. As with the electric field of a point charge, the inverse square law means that the lines of force may be thought of as continuous, representing the field intensity by their number per unit area. Then the r^2 factor may be thought of as related to the surface area of a sphere of radius r which the field has to cover.

Relationships
Radiation intensity

$$I = \frac{W}{4\pi r^2}$$

at distance *r* from a point source of power *W.*

Electric field

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

Gravitational field

$$g = -\frac{GM}{r^2}.$$

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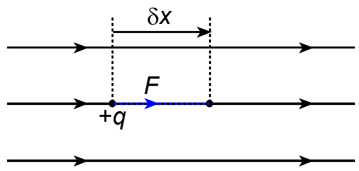
Electric potential

The electric potential at a point is the potential energy per unit charge of a small positive test charge placed at that point. This is the same as the work done per unit positive charge to move a small positive charge from infinity to that point.

The potential energy of a point charge *q* is $E_p = q\,V$, where *V* is the potential at that point. The unit of electric potential is the volt (V), equal to 1 joule per coulomb. Electric potential is a scalar quantity.

Potential gradient



The **potential gradient**, d*V* / d*x*, at a point in an electric field is the rate of change of potential with distance in a certain direction. The electric field strength at a point in an electric field is the negative of the potential gradient at that point:

$$E = -\frac{dV}{dx}.$$

In the radial field at distance *r* from a point charge *Q* the potential *V* is:

$$V = \frac{Q}{4\pi\varepsilon_0 r}.$$

The corresponding electric field strength is:

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{Q}{4\pi\varepsilon_0 r}\right) = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

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Electron

The electron is a fundamental particle and a constituent of every atom.

The electron carries a fixed negative charge. It is one of six fundamental particles known as leptons.

The charge of the electron, e , is -1.60×10^{-19} C.

The specific charge of the electron, e / m , is its charge divided by its mass. The value of e / m is 1.76×10^{11} C kg^{-1}.

The energy gained by an electron accelerated through a potential difference V is eV. If its speed v is much less than the speed of light, then $eV = (1/2)\, mv^2$.

Electrons show quantum behaviour. They have an associated **de Broglie wavelength** λ given by $\lambda = h/p$, where h is the Planck constant and p the momentum. At speeds much less than the speed of light, $p = mv$. The higher the momentum of the electrons in a beam, the shorter the associated de Broglie wavelength.

Relationships
The electron gun equation $(1 / 2)\, m\, v^2 = e\, V$ (for speed v much less than the speed of light).

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Force on a moving charge

The force F on a charged particle moving at speed v in a uniform magnetic field is

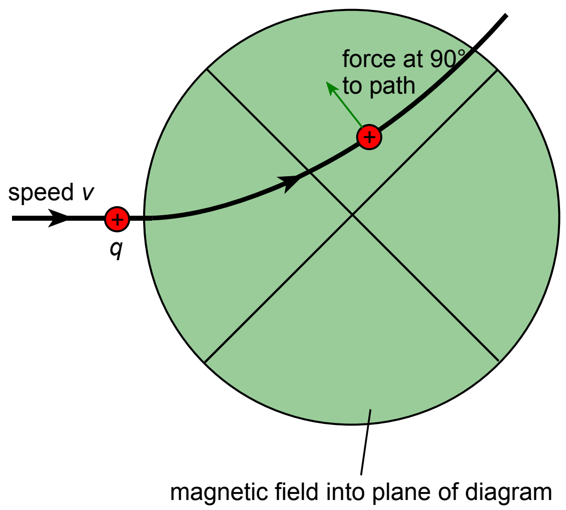
$$F = q\, v\, B \sin\theta$$

where q is the charge of the particle and θ is the angle between its direction of motion and the lines of force of the magnetic field.

The direction of the force is perpendicular to both the direction of motion of the charged particle and the direction of the field. The forces on positively and negatively charged particles are opposite in direction.

A beam of charged particles in a vacuum moving at speed v in a direction perpendicular to the lines of a uniform magnetic field is forced along a circular path because the magnetic force $q\, v\, B$ on each particle is always perpendicular to the direction of motion of the particle.

Force on a moving charge



The radius of curvature of the path of the beam

$$r = \frac{mv}{qB}.$$

This is because the magnetic force causes a centripetal acceleration

$$a = \frac{v^2}{r}.$$

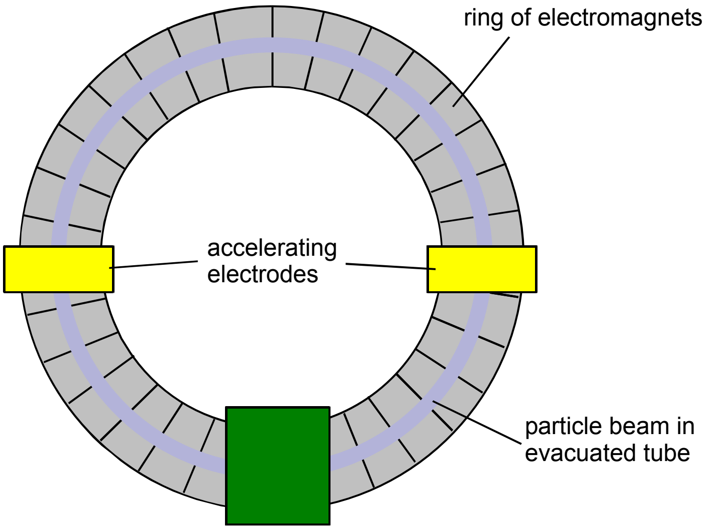
Using $F = ma$ gives

$$qvB = \frac{mv^2}{r}$$

and hence $r = \frac{mv}{qB}$.

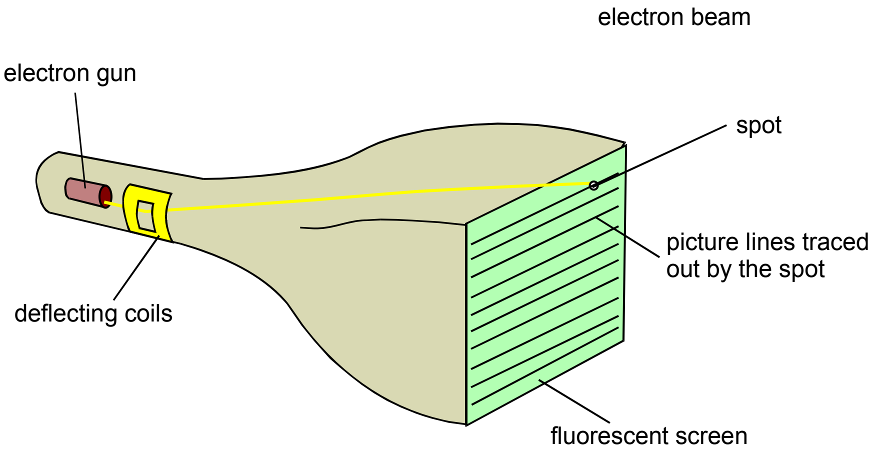
Note that writing the momentum mv as p, the relationship $r = p / B q$ remains correct even for velocities approaching that of light, when the momentum p becomes larger than the Newtonian value $m v$.

The particle accelerator



In a **particle accelerator** or collider, a ring of electromagnets is used to guide high-energy charged particles on a closed circular path. Accelerating electrodes along the path of the beam increase the energy of the particles. The magnetic field strength of the electromagnets is increased as the momentum of the particles increases, keeping the radius of curvature constant.

A TV or oscilloscope tube



In a **TV or oscilloscope tube**, an electron beam is deflected by magnetic coils at the neck of the tube. One set of coils makes the spot move horizontally and a different set of coils makes it move vertically so it traces out a raster of descending horizontal lines once for each image.

In a **mass spectrometer**, a velocity selector is used to ensure that all the particles in the beam have the same speed. An electric field E at right angles to the beam provides a sideways deflecting force Eq on each particle. A magnetic field B (at right angles to the electric field) is used to provide a sideways deflecting force qvB in the opposite direction to that from the electric field. The two forces are equal and sum to zero for just the velocity v given by $E q = qvB$, or $v = E / B$. Only particles with this velocity remain undeflected.

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Mass and energy

Mass and energy are linked together in the theory of relativity.

The theory of relativity changes the meaning of mass, making the mass a part of the total energy of a system of objects. For example, the energy of a photon can be used to create an electron-positron pair with mass 0.51 MeV / c^2 each.

Mass and momentum

In classical Newtonian mechanics, the ratio of two masses is the inverse of the ratio of the velocity changes each undergoes in any collision between the two. Mass is in this case related to the difficulty of changing the motion of objects. Another way of saying the same thing is that the momentum of an object is $p = m\,v$.

In the mechanics of the special theory of relativity, the fundamental relation between momentum p , speed v and mass m is different. It is:

$$p = \gamma m v$$

with

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

At low speeds, with $v << c$, where γ is approximately equal to 1, this reduces to the Newtonian value $p = mv$.

Energy

The relationships between energy, mass and speed also change. The quantity

$$E_{total} = \gamma mc^2$$

gives the total energy of the moving object. This now includes energy the particle has at rest (i.e. traveling with you), since when $v = 0$, $\gamma = 1$ and:

$$E_{rest} = mc^2$$

This is the meaning of the famous equation ***$E = mc^2$***. The mass of an object (scaled by the factor c^2) can be regarded as the **rest energy** of the object. If mass is measured in energy units, the factor c^2 is not needed. For example, the mass of an electron is close to 0.51 MeV.

Kinetic energy
The total energy is the sum of rest energy and kinetic energy, so that:

$$E_{kinetic} = E_{total} - E_{rest}$$

This means that the kinetic energy is given by:

$$E_{kinetic} = (\gamma - 1)mc^2$$

At low speeds, with $v << c$, it turns out that γ - 1 is given to a good approximation by:

$$(\gamma - 1) = \tfrac{1}{2}(v^2 / c^2)$$

so that the kinetic energy has the well-known Newtonian value:

$$E_{kinetic} = \tfrac{1}{2}mv^2$$

High energy approximations
Particle accelerators such as the Large Hadron Collider are capable of accelerating particles to a total energy many thousands of times larger than their rest energy. In this case, the high energy approximations to the relativistic equations become very simple.

At any energy, since $E_{total} = \gamma mc^2$ and $E_{rest} = mc^2$, the ratio of total energy to rest energy is just the relativistic factor γ:

$$\gamma = \frac{E_{total}}{E_{rest}}$$

This gives a very simple way to find γ, and so the effect of time dilation, for particles in such an accelerator.

Since the rest energy is only a very small part of the total energy,

$$E_{kinetic} \approx E_{total}$$

the relationship between energy and momentum also becomes very simple. Since $v \approx c$, the momentum can be written:

$$p \approx \gamma mc$$

and since the total energy is given by

$$E_{total} = \gamma mc^2$$

their ratio is simply:

$$\frac{E_{total}}{p} \approx c \text{, giving } E_{total} \approx pc$$

This relationship is exactly true for photons or other particles of zero rest mass, which always travel at speed c.

Differences with Newtonian theory
The relativistic equations cover a wider range of phenomena than the classical relationships do.

Change of mass equivalent to the change in rest energy is significant in nuclear reactions where extremely strong forces confine protons and neutrons to the nucleus. Nuclear rest energy changes are typically of the order of MeV per nucleon, about a million times larger than chemical energy changes. The change of mass for an energy change of 1 MeV is therefore comparable with the mass of an electron.

Changes of mass associated with change in rest energy in chemical reactions or in gravitational changes near the Earth are small and usually undetectable compared with the masses of the particles involved. For example, a 1 kg mass would need to gain 64 MJ of potential energy to leave the Earth completely. The corresponding change in mass is insignificant (7×10^{-10} kg = 64 MJ / c^2). A typical chemical reaction involves energy change of the order of an electron volt (= 1.6×10^{-19} J). The mass change is about 10^{-36} kg (= 10^{-19} J / c^2), much smaller than the mass of an electron.

Approximate and exact equations
The table below shows the relativistic equations relating energy, momentum, mass and speed. These are valid at all speeds *v*. It also shows the approximations which are valid at low speeds $v << c$, at very high speeds $v \approx c$, and in the special case where $m = 0$ and $v = c$.

Conditions	Relativistic factor γ	Total energy	Rest energy	Kinetic energy	Momentum
$m > 0$ *v* any value $< c$ any massive particle	$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ $\gamma = \frac{E_{total}}{E_{rest}}$	$E_{total} = \gamma mc^2$	$E_{rest} = mc^2$	$E_{kinetic} = (\gamma - 1)mc^2$	$p = \gamma mv$
$m > 0$ $v << c$ Newtonian	$\gamma \approx 1$	$E_{total} \approx mc^2$	$E_{rest} = mc^2$	$E_{kinetic} \approx \frac{1}{2}mv^2$	$p \approx mv$
$m > 0$ $v \approx c$ ultra-relativistic	$\gamma = \frac{E_{total}}{E_{rest}}$	$E_{total} = \gamma mc^2$	$E_{rest} = mc^2$	$E_{kinetic} \approx E_{total}$	$p \approx \gamma mc$ $p \approx \frac{E_{total}}{c}$
$m = 0$ $v = c$ photons	γ is undefined	$E = hf$	$E_{rest} = 0$	$E = E_{kinetic} = E_{total}$	$p = \frac{E}{c}$

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Relativistic calculations of energy and speed

Calculating the speed of an accelerated particle, given its kinetic energy
If the speed is much less than that of light, Newton's laws are good approximations.

Newtonian calculation:
Since the kinetic energy $E_K = (1/2)mv^2$, the speed *v* is given by:

$$v^2 \quad \frac{2E_K}{m}$$

In an accelerator in which a particle of charge q is accelerated through a potential difference V, the kinetic energy is given by:

$E_K = qV$,

Thus:

$$v^2 \quad \frac{2qV}{m}$$

The Newtonian calculation seems to give an 'absolute' speed, not a ratio v/c.

Relativistic calculation:
A relativistic calculation mustn't give an 'absolute speed'. It can only give the speed of the particle as a fraction of the speed of light. The total energy E_{total} of the particle has to be compared with its rest energy mc^2. For an electron, the rest energy corresponding to a mass of 9.1×10^{-31} kg is 0.51 MeV. A convenient relativistic expression is:

$$\frac{E_{total}}{E_{rest}} = \gamma$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Since the total energy $E_{total} = mc^2 + qV$, then:

$$\gamma = 1 + \frac{qV}{mc^2}$$

This expression gives a good way to see how far the relativistic calculation will depart from the Newtonian approximation. The Newtonian calculation is satisfactory only if γ is close to 1.

The ratio v / c can be calculated from γ:

$$v/c = \sqrt{1 - 1/\gamma^2}\ .$$

A rule of thumb
As long as the accelerator energy qV is much less than the rest energy, the factor γ is close to 1, and v is much less than c. The Newtonian equations are then a good approximation. To keep γ close to 1, say up to 1.1, the accelerator energy qV must be less than 1/10 the rest energy. So for electrons, rest energy 0.51 MeV, accelerating potential differences up to about 50 kV give speeds fairly close to the Newtonian approximation. This is a handy rule of thumb.

Accelerating voltage / kV	Speed of electrons (Newton)	$\gamma = 1 + qV/mc^2$ $mc^2 = 0.51$ MeV	Speed of electrons (Einstein)	Error in speed
10	0.198c	1.019	0.195c	1.5%
50	0.442c	1.097	0.412c	7%
100	0.625c	1.195	0.548c	14%

Accelerating voltage / kV	Speed of electrons (Newton)	$\gamma = 1 + qV/mc^2$ $mc^2 = 0.51$ MeV	Speed of electrons (Einstein)	Error in speed
500	1.4c	$1.97 \cong 2$	0.86c	62%
5000	4.4c	10.7	0.99c	>300%

An example: a cosmic ray crosses the Galaxy in 30 seconds
A proton of energy 10^{20} eV is the highest energy cosmic ray particle yet observed (2008). How long does such a proton take to cross the entire Milky Way galaxy, diameter of the order 10^5 light years?

The rest energy (mass) of a proton is about 1 GeV/c^2, or 10^9 eV/c^2. Then:

$$E_{total} = \gamma mc^2$$

with

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

Inserting values: $E_{\text{total}} = 10^{20}$ eV/c^2 and m = 10^9 eV/c^2 gives:

$$\gamma = \frac{10^{20}\ \text{eV}}{10^9\ \text{eV}} = 10^{11}$$

The proton, travelling at very close to the speed of light, would take 10^5 years to cross the galaxy of diameter 10^5 light years. But to the proton, the time required will be its wristwatch time τ where:

$t = \gamma\tau$

$$\tau = \frac{t}{\gamma} = \frac{10^5\ \text{year}}{10^{11}} \cong \frac{3 \times 10^{12}\ \text{s}}{10^{11}} = 30\ \text{s}$$

The wristwatch time for the proton to cross the whole Galaxy is half a minute. From its point of view, the diameter of the galaxy is shrunk by a factor 10^{11}, to a mere 10^5 km.

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Electron volt

The charge of the electron $e = -1.60 \times 10^{-19}$ C. The charge e on the electron was measured in 1915 by Robert Millikan, who invented a method of measuring the charge on individual charged oil droplets. Millikan discovered that the charge on an oil droplet was always a whole number multiple of 1.6×10^{-19} C.

Physicists often measure the energy of charged particles in the unit **electron volt** (eV). This is the work done when an electron is moved through a potential difference of 1 volt. Since the charge of the electron is 1.6×10^{-19} C, then 1 eV = 1.6×10^{-19} J. The energy needed to ionise an atom is of the order of 10 eV. X-rays are produced when electrons with energy of the order 10 keV or more strike a target. The energy of particles from radioactive decay can be of the order 1 MeV.

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Summary Diagrams

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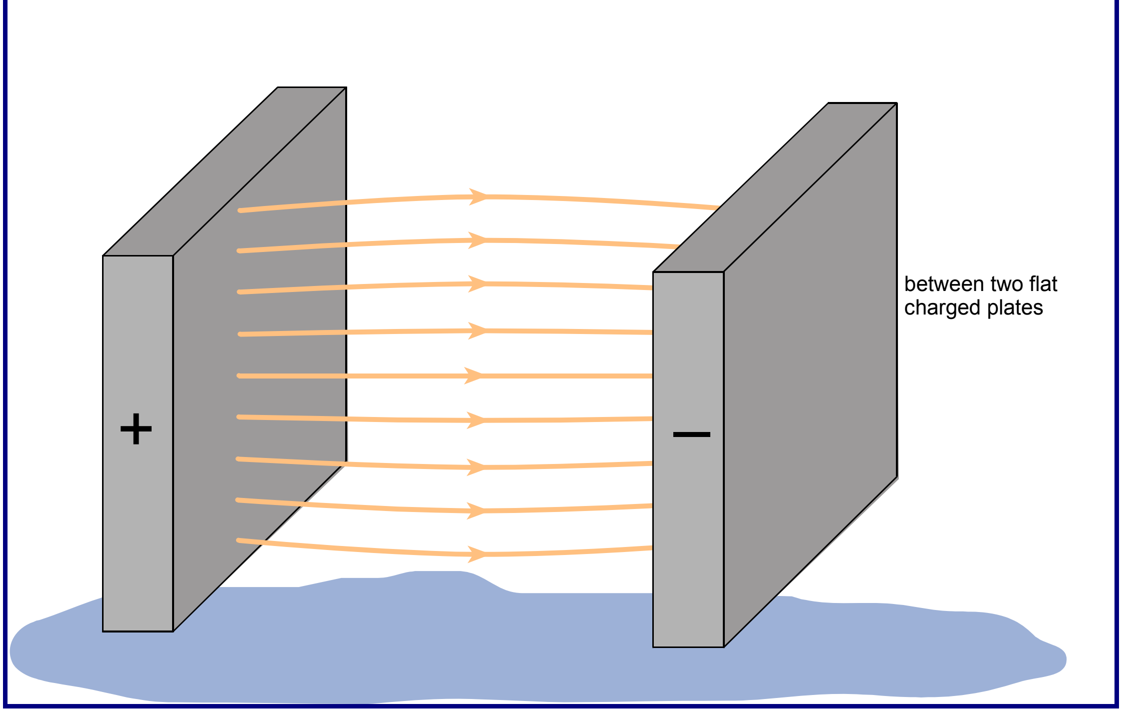
The electric field between parallel plates

Shapes of electric fields



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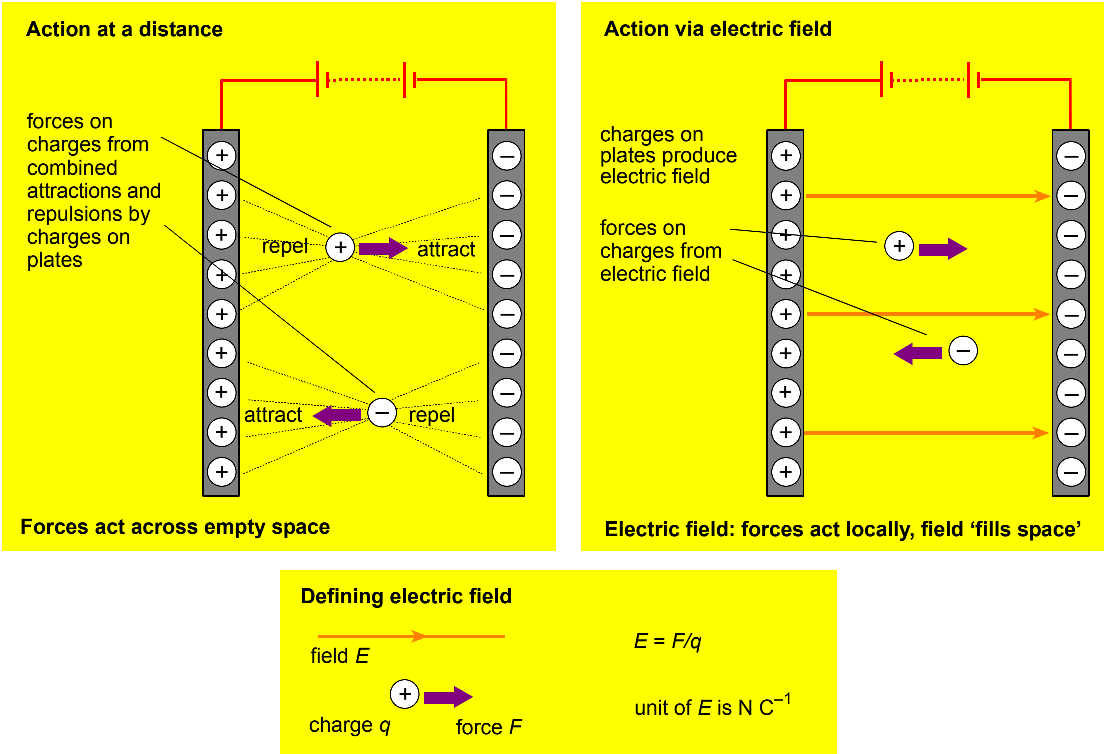
Two ways of describing electric forces

Two ways of describing electrical forces



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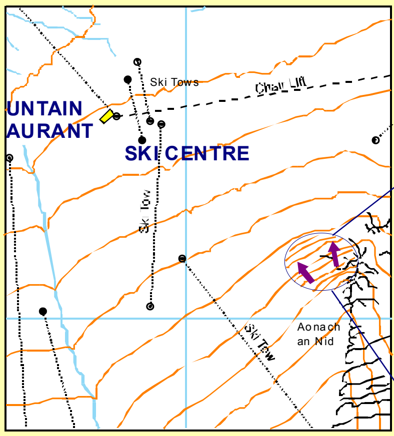
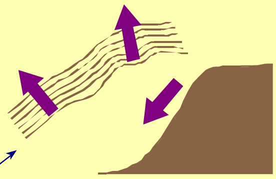
Field strength and potential gradient

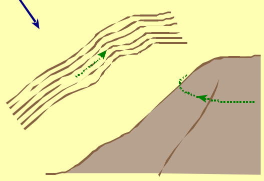
Potential gradients

Contours and slopes





Slope is steep where contours are close. Direction of steepest slope is perpendicular to contours.



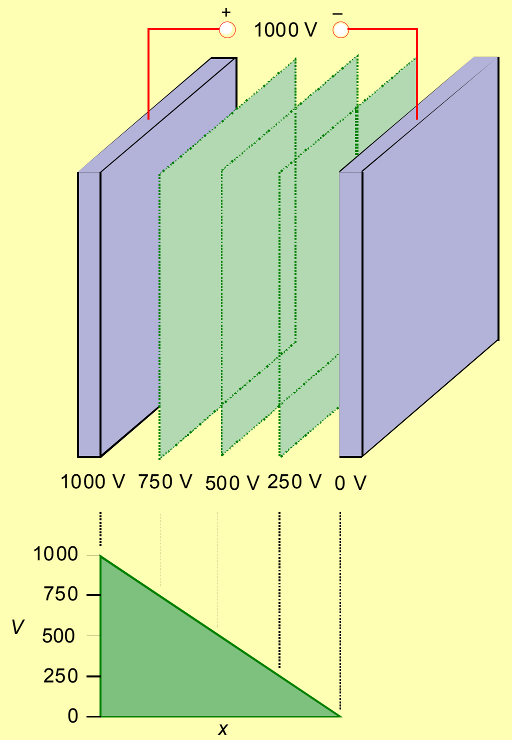
walk along contour to stay at same height

$$\text{slope} = \frac{\text{change in height}}{\text{distance}} = -\frac{h}{x}$$

negative slope is downhill, decreasing height

Potential gradients

Field and potential gradient



$$\text{slope} = \frac{\text{change in potential}}{\text{distance}} = -\frac{V}{x}$$

electric field $E = -\dfrac{V}{x}$

or $E = -\dfrac{dV}{dx}$ if slope varies continuously

negative slope is downhill, decreasing potential

Field strength = –potential gradient

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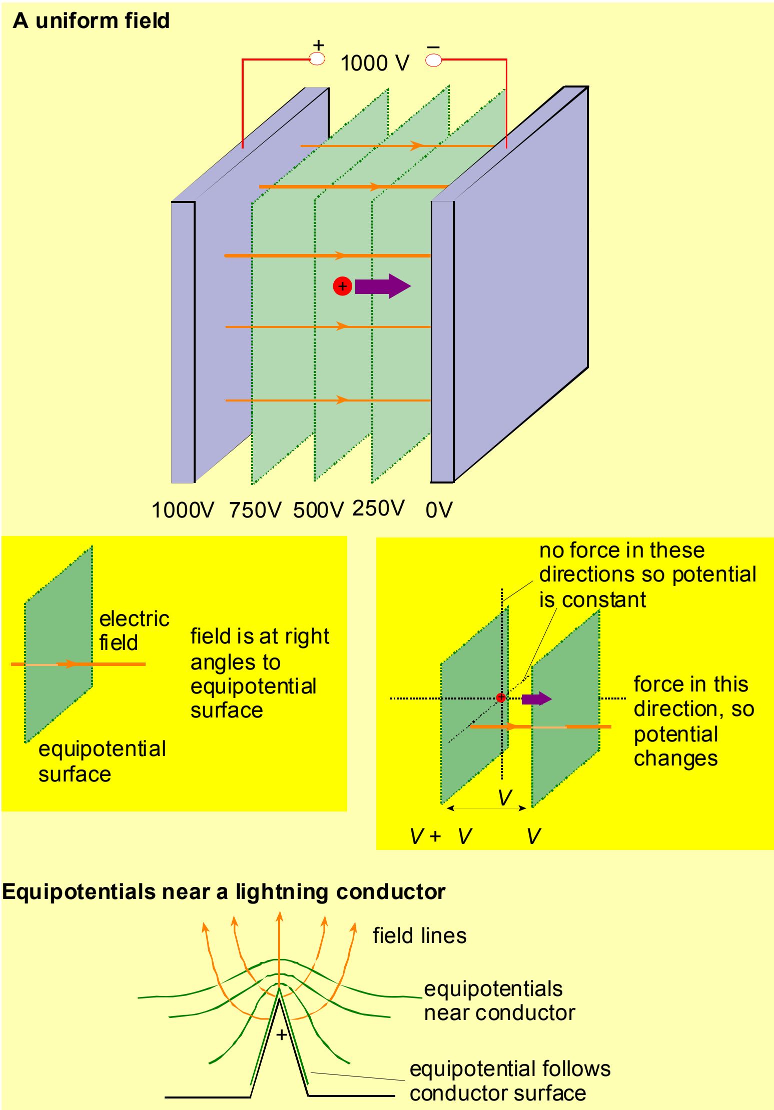
Field lines and equipotential surfaces

Field lines and equipotentials



Field lines are always perpendicular to equipotential surfaces

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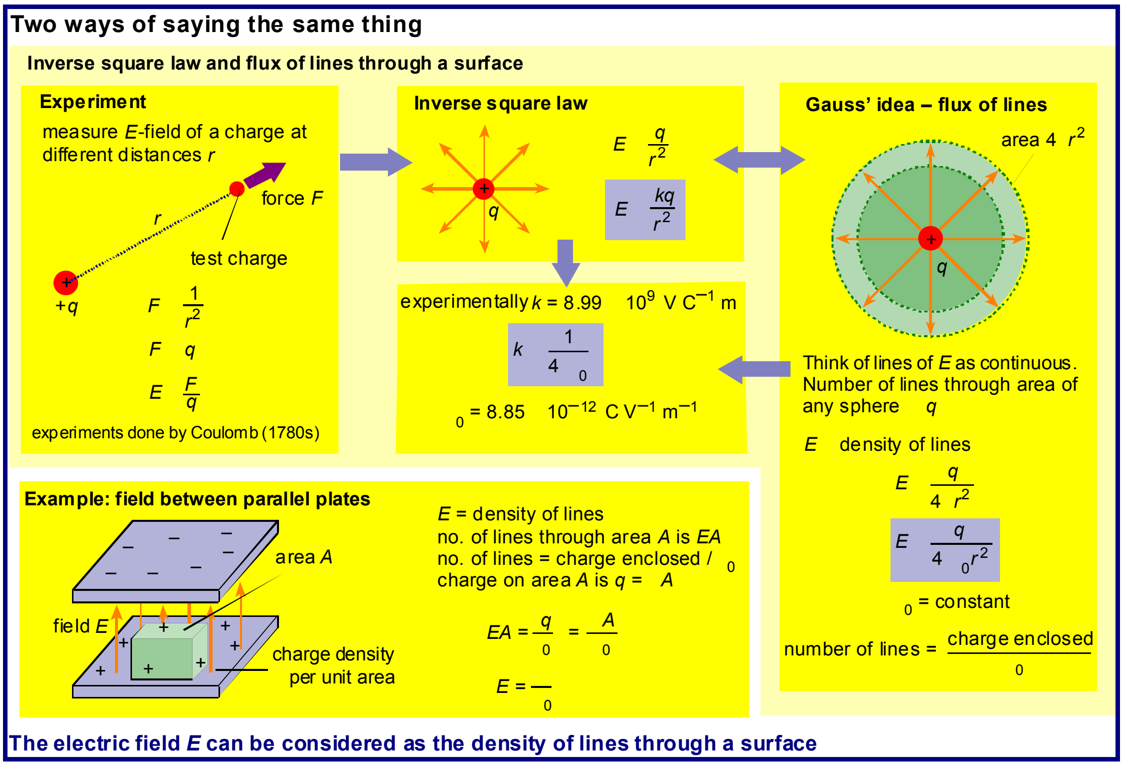
Inverse square law and flux



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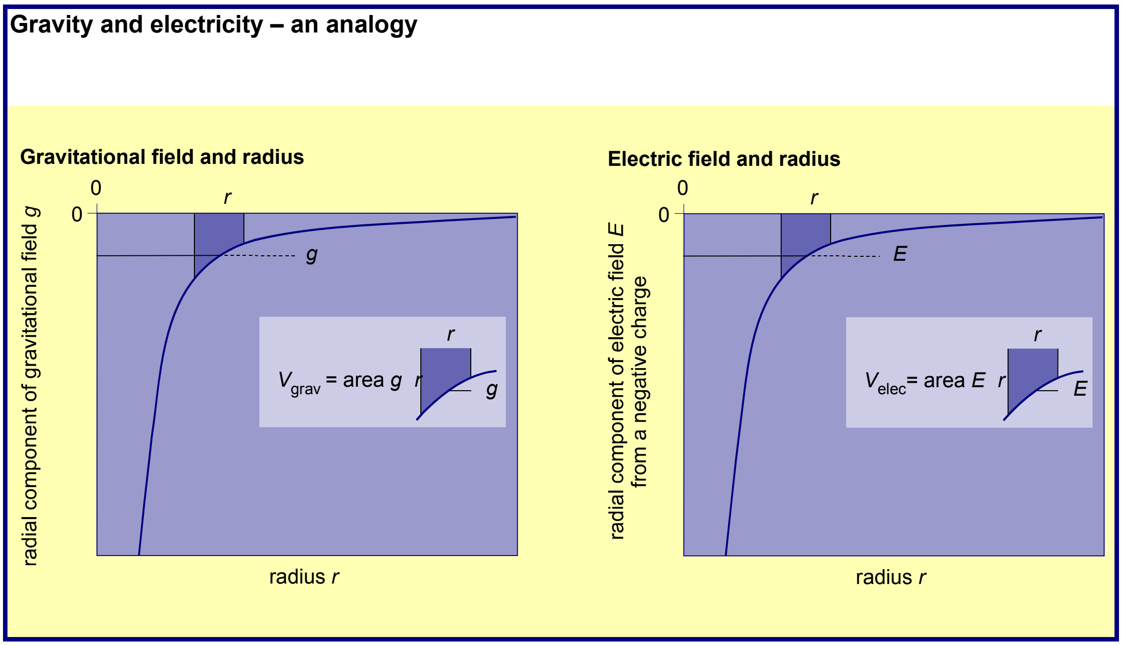
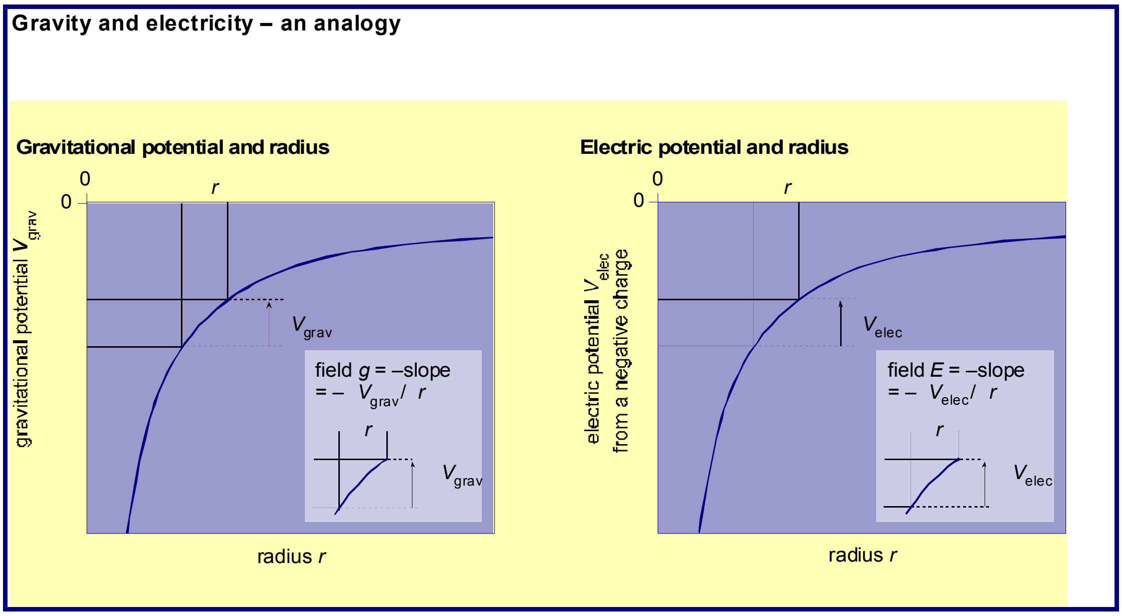
Radial fields in gravity and electricity





Gravity and electricity – an analogy

V_{grav} = area under graph of field g against r

field g = – slope of V_{grav} against r

$$g = \frac{-GM}{r^2}$$

$$V = \frac{-GM}{r}$$

force between masses is attractive

V_{elec} = area under graph of field E against r

field E = – slope of V_{elec} against r

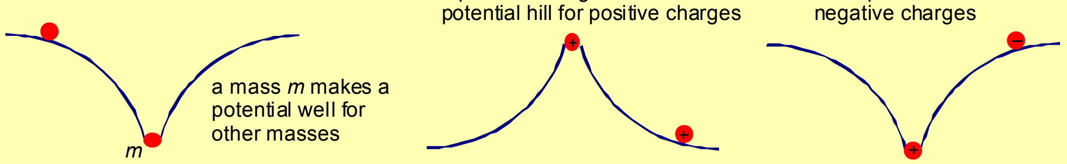
$$E = \frac{kq}{r^2}$$

$$V_{elec} = \frac{kq}{r}$$

$$k = \frac{1}{4\ {}_0}$$

force between like charges is repulsive

Potential wells and hills



Charges make potential energy hills for like charges and potential energy wells for unlike charges

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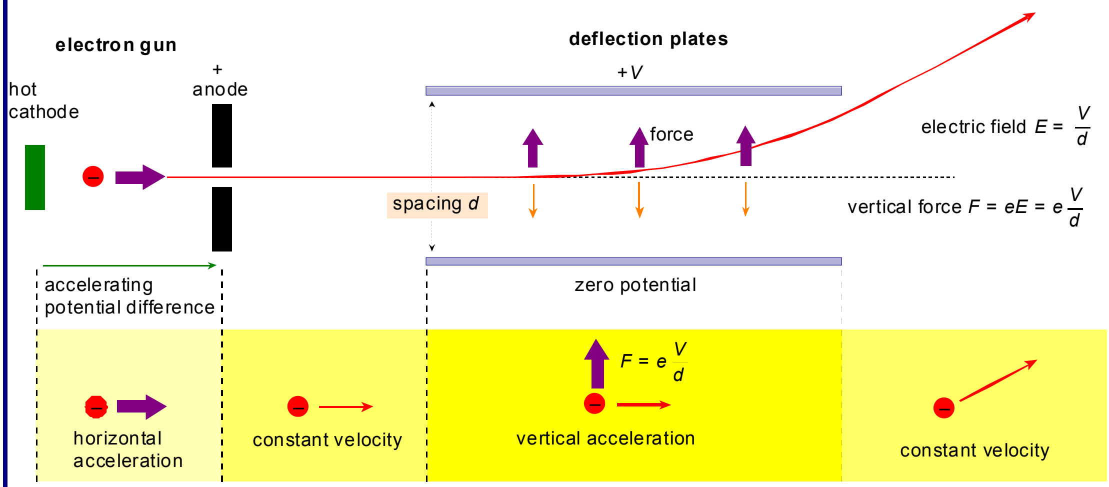
How an electric field deflects an electron beam

Deflections of electron beam by an electric field



A uniform electric field deflects a charged particle along a parabolic path

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Force, field, energy and potential

Relationships between force, field, energy and potential

Electricity

Interaction of two charges

Force
(radial component)
unit N

$F = \frac{q_1 q_2}{4 \quad {}_0 r^2}$

Potential energy
unit J

$\text{potential} = \frac{q_1 q_2}{4 \quad {}_0 r}$

divide by charge

multiply by charge

Behaviour of isolated charge

Field
(radial component)
unit N C^{-1}

$E = \frac{q}{4 \quad {}_0 r^2}$

Potential
unit V = J C^{-1}

$V_{electric} = \frac{q}{4 \quad {}_0 r}$

area under graph of force or field against *r*

slope of graph of potential energy or potential against *r*

Relationships between force, field, energy and potential

Gravity

Interaction of two masses

Force
(radial component)
unit N

$F = -G\frac{m_1 m_2}{r^2}$

Potential energy
unit J

$\text{potential} = -G\frac{m_1 m_2}{r}$

divide by mass

multiply by mass

Behaviour of isolated mass

Field
(radial component)
unit N kg^{-1}

$g = -G\frac{m}{r^2}$

Potential
(radial component)
unit J kg^{-1}

$V_{grav} = -G\frac{m}{r}$

area under graph of force or field against *r*

slope of graph of potential energy or potential against *r*

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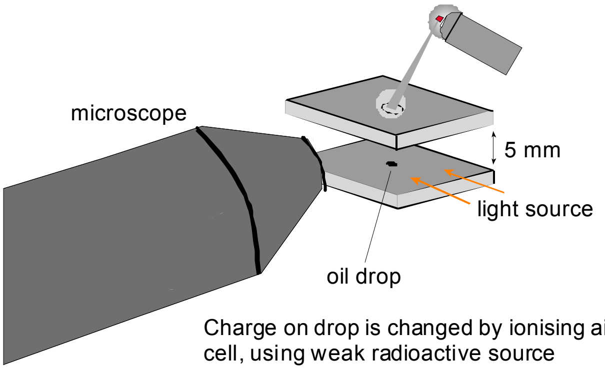
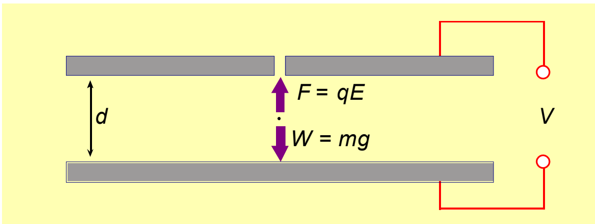
Millikan's experiment

Discreteness of charge



Charge on drop is changed by ionising air in cell, using weak radioactive source



When drop is held stationary:

electric force F = gravitational force W

$F = qE$ $\quad$ $W = mg$

Uniform electric field $E = \frac{V}{d}$

$qE = mg$

$V = \frac{mgd}{q}$

If charges q are discrete multiples n of electron charge e, then $q = ne$

$V = \frac{mgd}{ne}$

Vn = constant

Changes can be detected that are due to single electrons leading to a calculated value of e.

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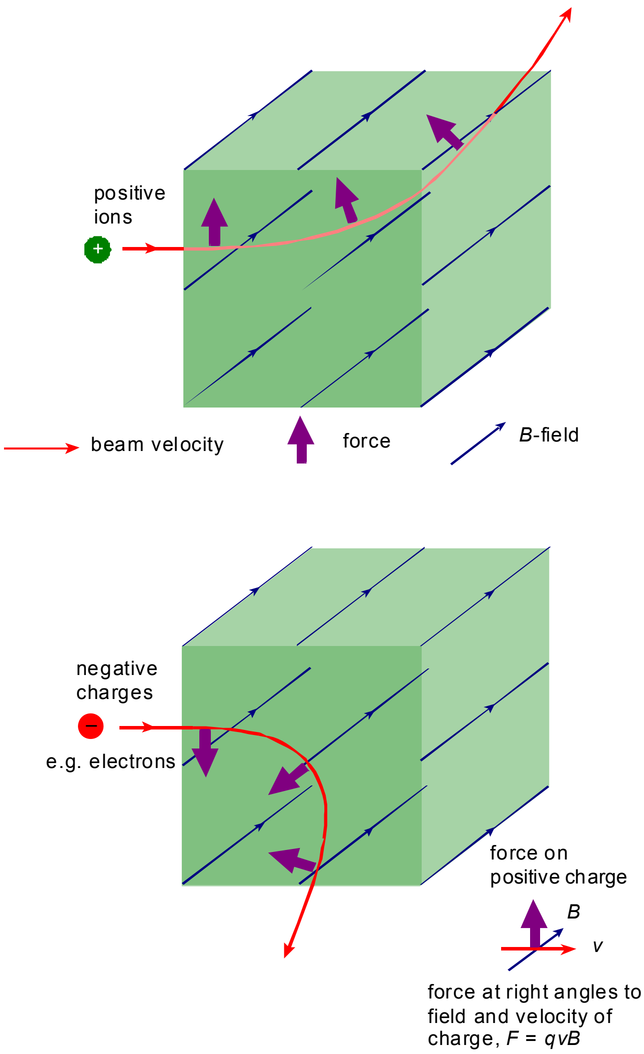
How a magnetic field deflects an electron beam

Magnetic deflection



Magnetic fields deflect moving charged particles in circular paths

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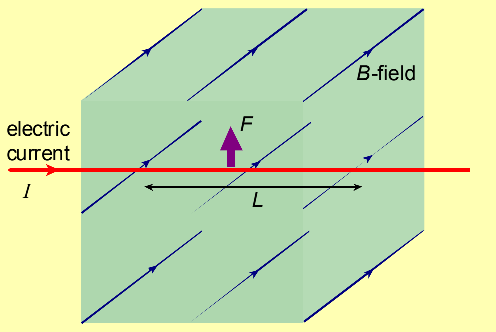
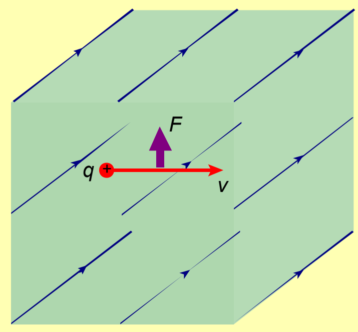
Force on a current: force on a moving charge

Force on current: force on moving charge

Electric motor



Moving charge



Force on current I in length L

$F = ILB$

$$\text{current} = \frac{\text{charge flow}}{\text{time}}$$

$$I = \frac{q}{t}$$

$$IL = q\frac{L}{t} = qv$$

Force on charge q at velocity v

$F = qvB$

The force which drives electric motors is the same as the force that deflects moving charged particles

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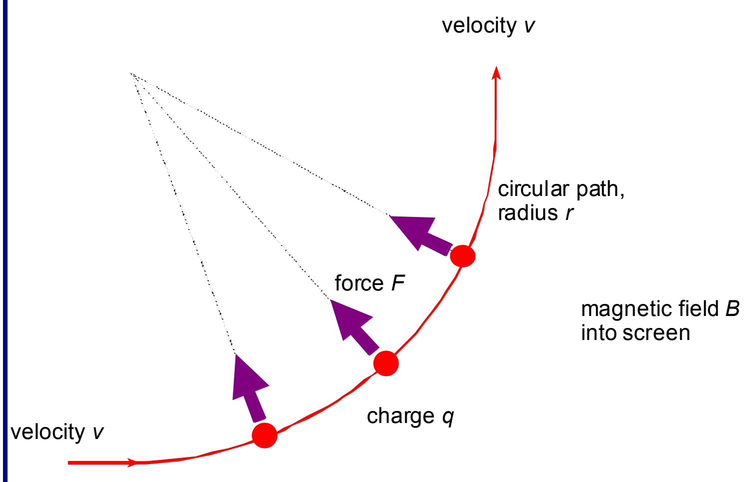
Measuring the momentum of moving charged particles

Measuring the momentum of moving charged particles



motion in circle

$$\frac{mv^2}{r} = \text{force } F$$

magnetic force

force $F = qvB$

$$\frac{mv^2}{r} = qvB$$

$p = mv = qrB$

at relativistic speed
$p = qrB$
is still true but
$p = \ mv$

Momentum of the particle is proportional to the radius of curvature of its path

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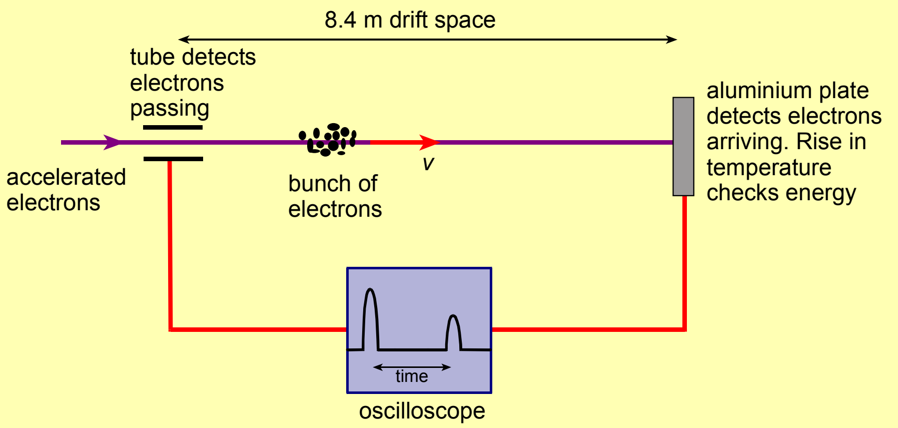
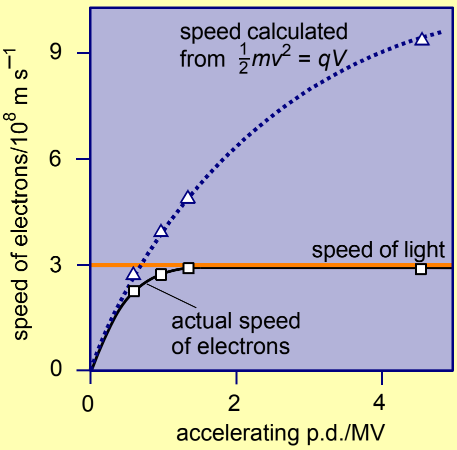
The ultimate speed – Bertozzi's demonstration

The ultimate speed: Bertozzi's demonstration



The results:



The difference made by relativity

As particles are accelerated

- speed v reaches a limit, c
- kinetic energy E_K increases without limit
- momentum p increases without limit

At all speeds

$E_K = qV$

At low speeds

$p \approx mv$

$E_K \approx \frac{1}{2}mv^2$

At high speeds

$v \approx c$

$E_K \approx pc$

Powerful accelerators can't increase the speed of particles above *c*, but they go on increasing their energy and momentum

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Relativistic momentum $p = \gamma mv$

Einstein redefines momentum

Problem:

time *t* depends on relative motion because of time dilation (chapter 12)

Newton's definition of momentum

$p = mv$

$p = m \frac{x}{t}$

Einstein's solution:

Replace *t* by , the change in wristwatch time , which does not depend on relative motion

from time dilation:

$= \frac{1}{\sqrt{1 - v^2/c^2}}$

t =

substitute for

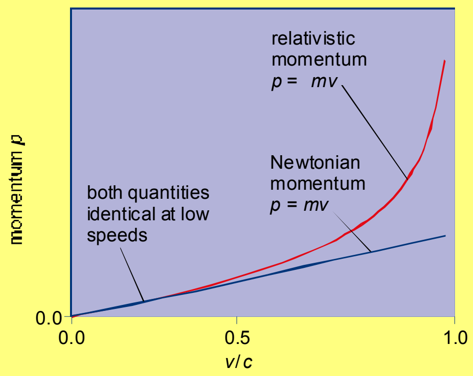
$\frac{x}{t} = v$

Einstein's new definition of momentum

$p = m \ \ x$

$p = m \frac{x}{t}$

$p = mv$



relativistic momentum

$p = \ mv$

Relativistic momentum *p* = *mv* increases faster than Newtonian momentum *mv* as *v* increases towards *c*

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Relativistic energy $E_{total} = \gamma mc^2$

Einstein rethinks energy

relativistic idea:

Space and time are related. Treat variables x and ct similarly.
Being at rest means moving in wristwatch time .

relativistic momentum

for component of movement in space

$p = m \frac{x}{}$

so invent

relativistic momentum for 'movement in wristwatch time'

Just write ct in place of x

from time dilation $\frac{x}{} =$

multiply p_0 by c, getting a quantity E having units of energy (momentum × speed)

relativistic momentum

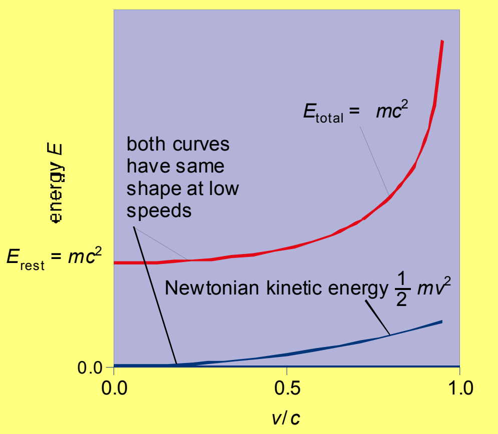
for component of movement in time

$p_0 = m \frac{ct}{}$

$p_0 = mc \frac{t}{}$

$p_0 = mc$

$p_0 c = E = mc^2$



interpret energy $E = mc^2$

1 particle at rest: $v = 0$ and = 1 → $E_{rest} = mc^2$ → particle has rest energy

2 particle moving at speed v energy mc^2 greater than rest energy → $E_{kinetic} = mc^2 - mc^2 = (-1)mc^2$

3 at low speeds $v \ll c$ kinetic energy has same value as for Newton → $E_{kinetic} = (-1)mc^2 \approx \frac{1}{2}mv^2$ (see graph)

$E_{total} = mc^2$

interpret E as total energy = kinetic energy + rest energy

relativistic energy

$E_{total} = mc^2$

$E_{rest} = mc^2$

$= \frac{E_{total}}{E_{rest}}$

Total energy = mc^2. Rest energy = mc^2. Total energy = kinetic energy + rest energy.

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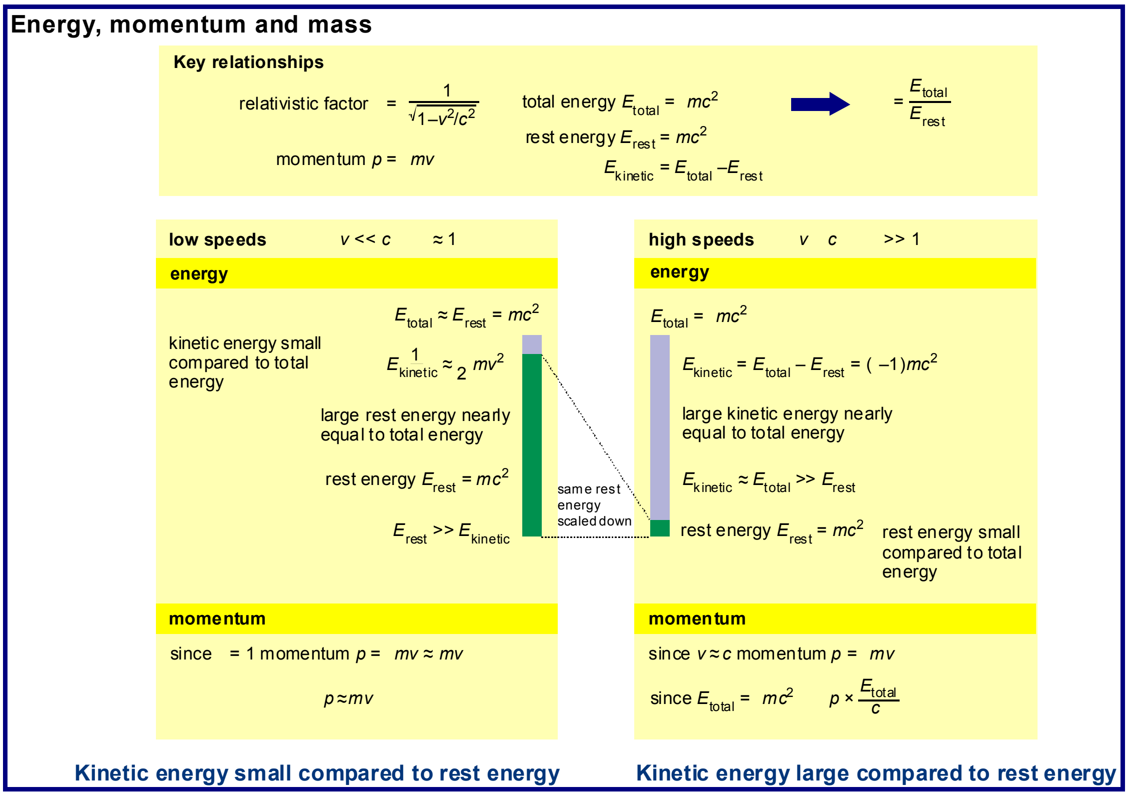
Energy, momentum and mass



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