

Revision Guide for Chapter 11

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I can show my understanding of effects, ideas and relationships by describing and explaining cases involving:

<p>momentum as the product of mass \times velocity</p> <p>force defined as rate of change of momentum</p> <p>conservation of momentum when objects interact; Newton's third law as a consequence</p> <p>Revision Notes: momentum, Newton's laws of motion</p> <p>Summary Diagrams: Conservation of momentum, Two craft collide, More collisions, Momentum, invariance, symmetry, Jets and rockets</p>	
<p>work done (<i>as force \times distance moved in the direction of the force</i>: including cases where the force does not act in the direction of the resulting motion)</p> <p>changes of gravitational potential energy to kinetic energy and vice versa when objects move in a gravitational field</p> <p>motion in a uniform gravitational field</p> <p>Revision Notes: work, gravitational field, potential energy, kinetic energy, projectile</p> <p>Summary Diagrams: Graph showing g against h</p>	
<p>the gravitational field and gravitational potential due to a point mass</p> <p>Revision Notes: gravitational field, gravitational potential</p> <p>Summary Diagrams: Field and potential</p>	
<p>motion in a horizontal circle and in a circular gravitational orbit about a central mass</p> <p>Revision Notes: circular motion, Kepler's laws</p> <p>Summary Diagrams: Centripetal acceleration, Geometry rules the Universe, A geostationary satellite</p>	

I can use the following words and phrases accurately when describing effects and observations:

<p>force, momentum</p> <p>Revision Notes: momentum</p>	
<p>kinetic energy, potential energy</p> <p>Revision Notes: kinetic energy, potential energy</p>	
<p>gravitational field, gravitational potential, equipotential surface</p> <p>Revision Notes: gravitational field, gravitational potential</p> <p>Summary Diagrams: Relationship between g and V_g</p>	

I can sketch, plot and interpret:

graphs showing the variation of a gravitational field with distance, and know that the area under the graph shows the change in gravitational potential Revision Notes: gravitational field Summary Diagrams: Field and potential , Graph showing g against h	
graphs showing the variation of gravitational potential with distance, and know that the tangent to the curve gives the gravitational field strength Revision Notes: gravitational potential Summary Diagrams: Field and potential	
diagrams illustrating gravitational fields and the corresponding equipotential surfaces Summary Diagrams: Relationship between g and V_g , Field and potential	

I can make calculations and estimates involving:

kinetic energy $\frac{1}{2}mv^2$, gravitational potential energy change mgh energy transfers and exchanges using the idea: work done $\Delta E = F\Delta s \cos\theta$ (no work is done when F and s are perpendicular) Revision Notes: kinetic energy , potential energy , gravitational potential , projectile	
momentum $p = mv$ and $F = \Delta(mv) / \Delta t$ Revision Notes: momentum , Newton's laws of motion	
circular and orbital motion: $a = v^2 / r$; $F = mv^2 / r$ Revision Notes: satellite motion , Kepler's laws Summary Diagrams: Centripetal acceleration , Speeds and accelerations in the Solar System , Acceleration of the Moon , A geostationary satellite	
gravitational fields: for the radial components $F_{\text{grav}} = -\frac{GmM}{r^2}, \quad g = \frac{F_{\text{grav}}}{m} = -\frac{GM}{r^2}$ gravitational potential energy $-\frac{GmM}{r}$ gravitational potential $V_{\text{grav}} = \frac{E_{\text{grav}}}{m} = -\frac{GM}{r}$ Revision Notes: gravitational field , gravitational potential Summary Diagrams: Field and potential , Apollo returns from the Moon	

Revision Notes

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Momentum

Momentum is *mass* × *velocity*. Momentum is a **vector quantity**. The SI unit of momentum is kg m s^{-1} .

Newton's second law defines the magnitude of the force as the magnitude of the rate of change of momentum, with the force in the direction of the change of momentum

$$F = \frac{\Delta(mv)}{\Delta t}.$$

If the mass is constant this can be expressed as 'force = mass × acceleration' because acceleration is rate of change of velocity.

The change of momentum of an object acted on by a force is:

$$\Delta(mv) = F\Delta t.$$

The product $F\Delta t$ is called the impulse of the force.

The thrust on a rocket of the jet of gases that it ejects is equal to the rate at which the jet carries away momentum. This is given by the *mass ejected per second* × *the velocity of the jet*.

When two objects interact, for example in a collision, one object loses an amount of momentum and the other object gains an equal amount. The total momentum of the two objects is the same after the interaction as before. This is the principle of **conservation of momentum**.

Since the time of interaction Δt is the same for both objects, the forces acting on the objects are equal and opposite. This is **Newton's third law**. It is a consequence of the conservation of momentum.

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Newton's laws of motion

Newton's first law of motion states that an object remains at rest or moves with constant velocity unless acted on by a resultant force.

Newton's first law defines what a force is, namely any physical effect that is capable of changing the motion of an object. If an object is at rest or in uniform motion, either no force acts on it or forces do act on it and the resultant force is zero.

Newton's second law of motion states that the magnitude of the rate of change of momentum of an object is equal to the magnitude of the resultant force on the object, with the change of momentum in the direction of the force.

That is, $F = dp / dt$, where $p = mv$ is the momentum of an object acted on by a resultant force F .

For an object of constant mass m , acted on by a force F

$$F = m \frac{dv}{dt} = ma$$

The SI unit of force is the newton (N). 1 N is the force that gives a 1 kg mass an acceleration of 1 m s^{-2} .

Newton's third law of motion states that when two objects interact, there is an equal and opposite force (of the same kind of force) on each object. So e.g. for a book at rest on a table, the "Newton pair" of forces on the book are its weight and the gravitational force the book exerts upon the planet Earth (i.e. not the reaction force of the table on the book, which is not a gravitational force).

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Work

Work is the change in energy when a force moves in the direction of that force.

The work W done by a force of magnitude F that moves its point of application by a distance s in the direction of the force is given by $W = F s$.

Work is a primary means of measuring amounts of energy transferred from one thing to another.

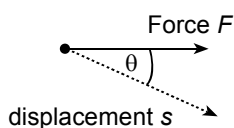
The SI unit of energy and of work is the joule (J). Work and energy are **scalar quantities**.

Work is done whenever an object moves under the action of a force with a component of the force along the displacement. If there is no movement of the object, no work is done. However, an outstretched hand holding a book does need to be supplied with energy to keep the arm muscles taut. No work is done on the book provided it is stationary. But, energy has to be supplied to repeatedly contract the muscle fibres so as to keep the muscles taut.

No work is done on an object by a force when the displacement of the object is at right angles to the direction of the force.

If an object is moved by a force F a distance s along a line that is at angle θ to the direction of the force, the work done by the force is given by $W = F s \cos\theta$.

Work done



$$\text{Work done} = F s \cos \theta$$

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Gravitational field

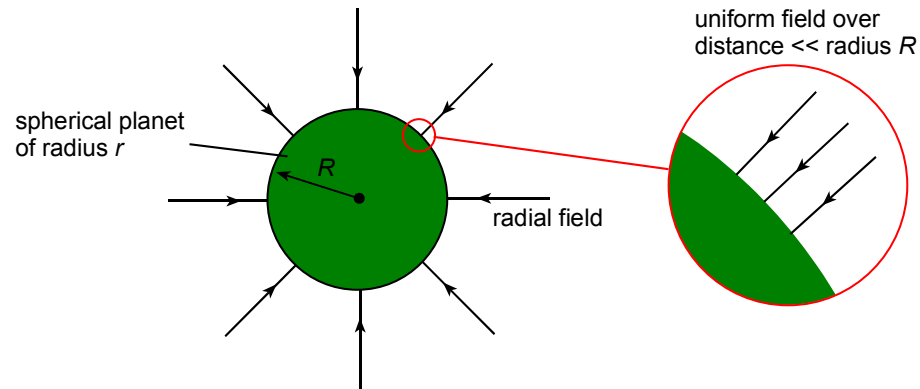
The strength g of a gravitational field at a point is the gravitational force per unit mass acting on a small mass at that point. Gravitational field strength is a **vector quantity** in the direction of the gravitational force.

The SI unit of gravitational field strength is N kg^{-1} or equivalently m s^{-2} .

The magnitude of the gravitational force F on a mass m at a point in a gravitational field (i.e. the object's weight) is given by $F = m g$, where g is the magnitude of the gravitational field strength at that point.

Close to the surface of the Earth, the gravitational field is almost uniform. The lines of force are parallel and at right angles to the Earth's surface.

A uniform gravitational field



On a large scale, the gravitational field is radial.

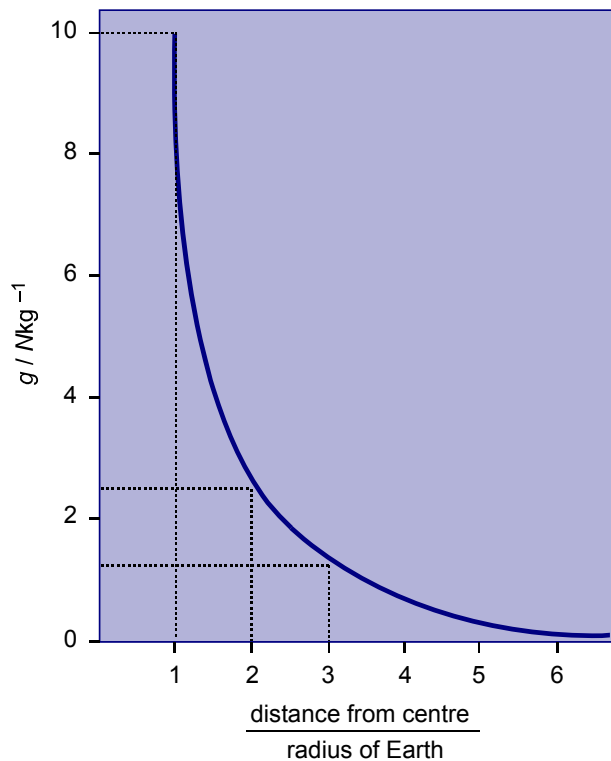
Newton's law of gravitation states that the radial component of the force of gravitational attraction F of a mass M on another mass m obeys an inverse square law:

$$F = -\frac{GMm}{r^2}$$

where r is the distance from the centre of M to m and the minus sign indicates that the radial component of the force acts towards the mass M . The measured value of the Universal Gravitational Constant G is $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$.

The radial component of the gravitational field strength due to M is $g = F / m = -GM / r^2$ at distance r from the centre of the mass M .

Variation of g with distance from the centre of the Earth



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Potential energy

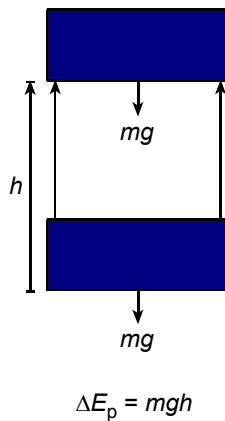
The potential energy of a system is the energy associated with the position of objects relative to one another, for example a mass raised above the Earth.

The SI unit of potential energy is the joule. Potential energy can be thought of as stored in a field, for example a gravitational field.

The potential energy is measured by the capacity to do work if positions of objects change. For example, if the potential energy of two magnets at rest repelling each other was 100 J, then releasing the magnets would enable them to use the potential energy in the field to do work.

If the height of an object above the Earth changes, its potential energy changes. Since the force of gravity on an object of mass m due to the Earth is equal to mg , the energy transferred when an object of mass m is raised a height h above the Earth = force \times distance moved along the line of action of the force = mgh , where g is the gravitational field strength. Hence the change of potential energy = mgh (provided the field g does not vary appreciably with height h).

Gain of potential energy



Potential energy can be negative as well as positive. The potential energy of a 1 kg mass and the Earth, if the mass is at the surface of the Earth, is -64 MJ. The negative sign means that the object is lower in energy at the surface than it is further away from the Earth. In this case the 'zero' of potential has been set at a very great distance ('infinity') from the Earth.

Change of potential energy = $m g h$, for a small change of height h of an object of mass m .

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Kinetic energy

The kinetic energy of a moving object is the energy it carries due to its motion.

For an object of mass m moving at speed v , the kinetic energy $E_k = \frac{1}{2} m v^2$, provided its speed is much less than the speed of light.

If an object of mass m initially at rest is acted on by a constant force of magnitude F for a time t , the object accelerates to speed v where $F t = m v$.

Since the distance moved by the object, $s = \frac{1}{2} v t$, then the work done on the object is equal to

$$F s = \frac{m v}{t} \frac{v t}{2} = \frac{1}{2} m v^2.$$

The work done is equal to the gain of kinetic energy. Hence the kinetic energy at speed v is $E_k = \frac{1}{2} m v^2$. Note that the relationship is an approximation for speeds which are small compared with the speed of light. At all speeds, from the theory of relativity, the kinetic energy of a free particle may be defined as $E_k = E_{total} - E_{rest}$, where E_{rest} , the rest energy, is given by $E_{rest} = m c^2$, and E_{total} , the total energy, is given by $\gamma m c^2$. Here γ is the relativistic factor

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}} \quad (\text{see chapter 12}).$$

Relationships

$$E_k = \frac{1}{2} m v^2$$

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Projectile

A projectile is any object in motion close to the Earth and acted on only by the Earth's gravity.

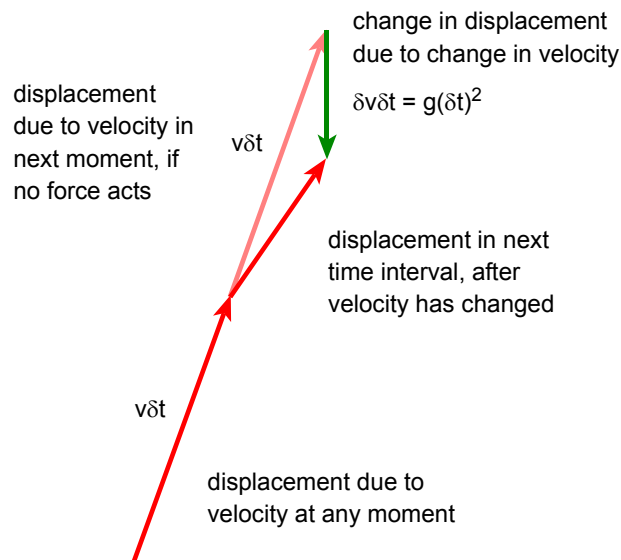
At any point on its flight path (assuming air drag to be negligible):

1. its horizontal component of acceleration is zero,
2. its vertical component of acceleration is equal to g , the gravitational field strength at that point.

Any projectile travels equal horizontal distances in equal times because its horizontal component of acceleration is zero. Its vertical motion is unaffected by its horizontal motion. The combination of constant horizontal velocity and constant downward acceleration leads to a parabolic path.

The path can be calculated in two ways: (1) calculating the path step by step or (2) using the kinematic equations for constant acceleration.

Step by step calculation



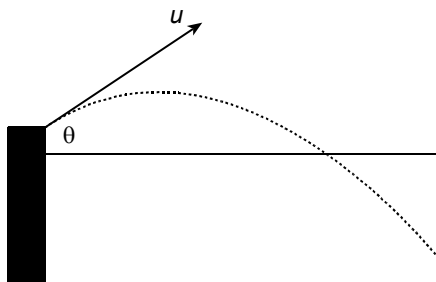
Suppose the magnitude of the velocity at a given moment is v . The displacement in a short time δt will be $v\delta t$, in the direction of the velocity. Draw such a vector displacement. If there were no gravitational force, the velocity would continue unchanged. The next displacement would again be $v\delta t$, continuing on from the last. Copy the first displacement vector and add it to the tip of the first vector. However, the velocity **does** change. If the change in velocity is δv , the change in displacement due to this change in velocity will be $\delta v\delta t$. Its direction, if the acceleration is due to gravity, will be downwards. Since $\delta v = g\delta t$, the downward change in displacement $\delta v\delta t = g\delta t\delta t = g(\delta t)^2$. Draw in such a vector, and find the resultant of it and the displacement had there been no force. This is the new segment of path, in the next time interval. Repeat the process, starting now with the new segment of path.

The change in displacement due to the change in velocity is the same in every time interval, if the intervals are equal. This fact results in the curve taking the shape of a parabola.

Using kinematic equations

If a projectile is launched horizontally at speed u , then at time t after launch, its horizontal distance from the launch point $x = ut$ and its vertical distance $y = -\frac{1}{2}gt^2$ (where the minus sign signifies that the upwards direction has been taken to be positive).

Projection at a non-zero angle above the horizontal



If the projectile is launched with an initial speed u at angle θ above the horizontal, its initial horizontal component of velocity = $u \cos\theta$. Because its horizontal component of acceleration is zero, then its horizontal component of velocity is constant. Hence the horizontal component of its displacement from launch, at time t after being launched, $x = ut \cos\theta$ and its initial vertical component of velocity = $u \sin\theta$. Hence:

Its vertical component of velocity at time t after being launched, $u_y = u \sin\theta - gt$.

Its vertical component of displacement from launch, $y = ut \sin\theta - \frac{1}{2}gt^2$.

The co-ordinates (x, y) relative to the launch point are therefore $x = ut \cos\theta$, $y = ut \sin\theta - \frac{1}{2}gt^2$.

The highest point is reached at time t_0 when $u_y = u \sin\theta - gt_0 = 0$, hence $t_0 = u \sin\theta / g$.

The shape of the path is parabolic.

The total energy of a projectile is constant because the change of kinetic energy is equal and opposite to the change of potential energy. A projectile directed upwards at a non-zero angle to the vertical loses kinetic energy which is stored as potential energy in the gravitational field, until it reaches its highest point. As it descends, it takes potential energy from the field, gaining kinetic energy. At any point, the kinetic energy plus potential energy equals the initial kinetic energy plus initial potential energy.

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Gravitational potential

The gravitational potential at a point is the potential energy per unit mass of a small object placed at that point. This is the work done per unit mass to move a small object from infinity to that point.

The gravitational potential energy E_P of a point mass m is given by $E_P = m V_G$, where V_G is the gravitational potential at that point.

The SI unit of gravitational potential is J kg^{-1} . Gravitational potential is a **scalar quantity**.

An **equipotential** is a surface of constant potential. No change of potential energy occurs when an object is moved along an equipotential. The lines of force are therefore always perpendicular to the equipotential surfaces.

The gravitational field strength at a point in a gravitational field is the negative of the potential gradient at that point. In symbols $g = -dV_G / dx$.

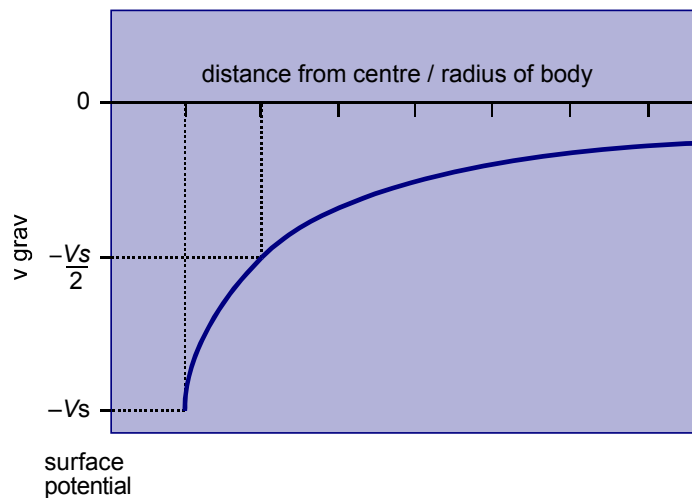
In an inverse square gravitational field due to a mass M , the radial component of the field strength is:

$$g = -\frac{GM}{r^2}.$$

and the gravitational potential is:

$$V_G = -\frac{GM}{r}$$

Variation of gravitational potential with distance from the centre of a spherical body



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Circular motion

An object moving in a horizontal circle at constant speed changes its direction of motion continuously. Its velocity is not constant because its direction of motion is not constant. The resultant force is directed towards the centre of the circle. It is called the **centripetal force**.

For an object moving at constant speed v along a circular path of radius r , the acceleration towards the centre is:

$$a = \frac{v^2}{r}$$

and the centripetal force F acting on it is:

$$F = ma = \frac{mv^2}{r}$$

where m is the mass of the object.

The centripetal force does no work on the moving mass because the force is always at right angles to the direction of motion. The energy of the motion is therefore constant.

The time T taken to move once round the circular path is

$$T = 2\pi r / v$$

For a proof that $a = \frac{v^2}{r}$ see Summary Diagrams: [Centripetal acceleration](#)

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Kepler's laws

Kepler's laws have to do with the motion of planets in the solar system, and were explained by Newton using his theory of gravitation based on an inverse square law for the gravitational attraction between a planet and the Sun.

Kepler's three laws are as follows:

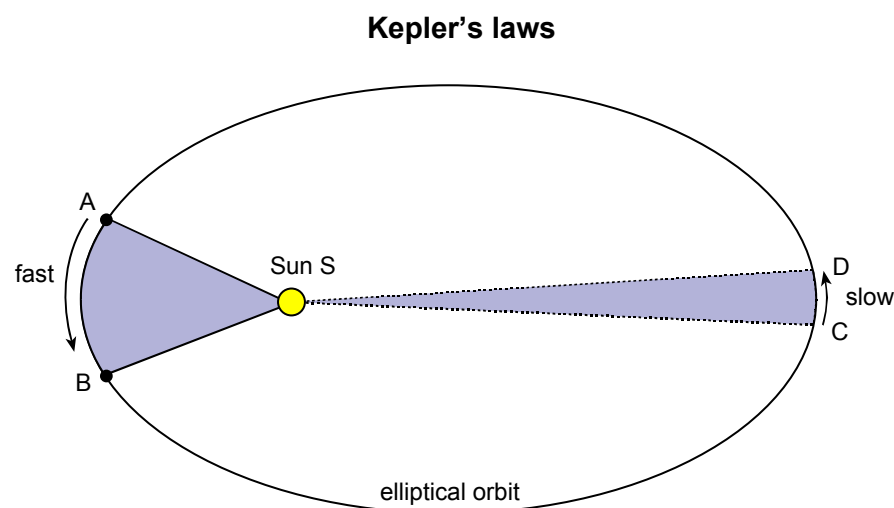
Kepler's first law: each planet moves round the Sun in an elliptical orbit in which the Sun is at one of the focal points of the ellipse.

Kepler's second law: the line from the Sun to a planet sweeps out equal areas in equal times as the planet moves round its orbit.

Kepler's third law: the square of the time period of a planet is in proportion to the cube of its mean radius of orbit. It can be more simply expressed for a circular orbit by saying that the centripetal acceleration v^2 / r is equal to the gravitational field GM / r^2 , so that the product $v^2 r$ is constant, equal to GM .

A planet continues to move round the Sun because of the gravitational force of attraction between it and the Sun. The force on the planet always acts towards the Sun, slowing the planet down as it moves on its orbit away from the Sun and speeding it up as it approaches the Sun. Most planets apart from Mercury and Pluto move on orbits that are almost circular. Kepler's laws also apply to the motion of comets which move round the Sun on highly elliptical orbits.

The first law describes the shape of a planetary orbit in general terms. The Sun is on the major axis at one of the two focal points of the elliptical orbit.



Time from A to B = time from C to D if area ABS = area CDS

The second law arose from Kepler's observation that the least distance from Mars to the Sun is 0.9 times the greatest distance. He also observed that at its greatest distance from the Sun, the line joining Mars and the Sun turned at a rate 0.81 times slower than at its least distance. Consider the line joining the Sun and a planet. Let it turn through a small angle $\delta \theta$ in time δt

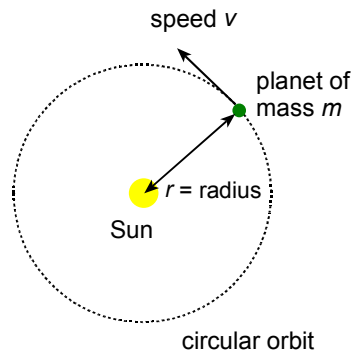
as the planet moves along its orbit. The planet moves a distance $r \delta \theta$ in time δt . Going to the limit as δt tends to zero, the speed $v = r d\theta / dt$.

The angular momentum of the planet is $m v r$, and is thus equal to $m r^2 d\theta / dt$.

But $r^2 d\theta / dt$ is the area swept out by the line joining the Sun and planet.

Kepler's second law is therefore equivalent to the statement that the angular momentum of a planet is the same at all points on its orbit.

Proof of Kepler's third law for the case of a circular orbit



A planet moving round a circular orbit of radius r in time T has speed

$$v = \frac{2\pi r}{T}.$$

The force of gravitational attraction on the planet due to the Sun is

$$F = \frac{GMm}{r^2}$$

where M is the mass of the Sun and m is the mass of the planet.

The centripetal acceleration of the planet on its orbit round the Sun is

$$a = \frac{v^2}{r}.$$

This must be equal to the gravitational field

$$g = GM / r^2$$

Thus:

$$v^2 = \frac{GM}{r}$$

which is independent of the mass m of the planet.

Substituting

$$v = \frac{2\pi r}{T}$$

therefore gives

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r},$$

which can be rearranged to give

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

in agreement with Kepler's third law.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn
Time period in years, T (years)	0.24	0.61	1.0	1.9	11.9	29.6
Mean radius of orbit, r / mean distance of Earth from Sun	0.39	0.72	1.0	1.5	5.2	9.5

Relationships

Kepler's second law:

$$r^2 \frac{d\theta}{dt} = \text{constant}$$

Kepler's third law:

$$r^3 = \frac{GM}{4\pi^2} T^2$$

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Satellite motion

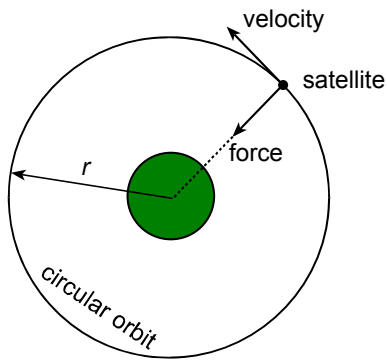
A satellite is any object in orbit about a larger astronomical object.

The planets are satellites of the Sun, the Moon is a satellite of the Earth. Artificial satellites in orbit about the Earth are used for communications, meteorology and surveillance.

The time period of a satellite is the time taken to complete one orbit. The time period of an artificial satellite depends on its distance from the Earth.

A **geostationary satellite** is a satellite in an equatorial orbit at such an altitude and direction that it remains at the same point above the equator because its time period is exactly 24 hours.

A satellite orbiting the Earth is kept in its orbit by the force of gravity between it and the Earth. In general, a satellite orbit is an ellipse. For a circular orbit (a special case of an ellipse), the velocity of the satellite is always perpendicular to the force of gravity on the satellite, and is given by equating the force of gravitational attraction GMm/r^2 to the centripetal force $m v^2 / r$ where M is the mass of the Earth, m is the mass of the satellite, r is the radius of orbit and v is the speed of the satellite.



Hence $v^2 = GM/r$ gives the speed of a satellite in a circular orbit.

The time period $T = 2\pi r / v$ for a circular orbit.

These equations can be used to show that:

$T = 24$ hours for $r = 42\,300$ km. Thus the height of a geostationary orbit must be $35\,900$ km because the Earth's radius is 6400 km. Geostationary satellites are used for communications because satellite transmitters and receivers once pointed towards the satellite remain pointing to it without having to track it.

$r = 8000$ km for $T = 2$ hours, thus a satellite in a polar orbit with a time period of 2 hours needs to be at a height of 1600 km above the Earth. In such an orbit, the satellite makes 12 orbits every 24 hours, crossing the equator 30° further west on each successive transit in the same direction. Such satellites are used for meteorological and surveillance purposes.

The total energy of a satellite, $E = E_K + E_P$, where E_K is its kinetic energy ($= \frac{1}{2}mv^2$) and E_P is its potential energy ($= -GMm/r$).

For a circular orbit, $v^2 = GM/r$ so $E_K = \frac{1}{2}mGM/r = -\frac{1}{2}E_P$.

Thus the kinetic energy has magnitude equal to half the potential energy, and the total energy $E = -\frac{1}{2}E_P + E_P = \frac{1}{2}E_P = -GMm/2r$.

Relationships

For circular orbits:

$$v^2 = GM/r.$$

$$T = 2\pi r / v.$$

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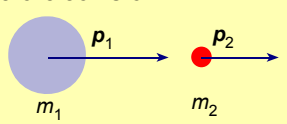
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Conservation of momentum

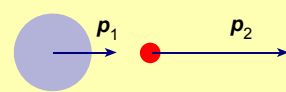
Conservation of momentum $p = mv$

Before collision:



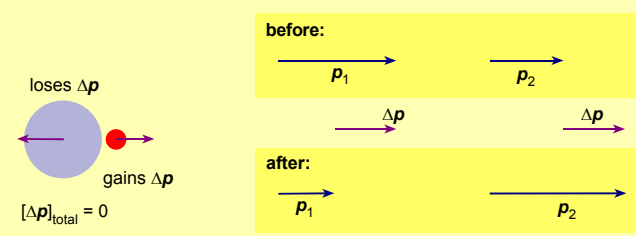
[total momentum p]_{before} = $[m_1v_1 + m_2v_2]_{\text{before}}$

After collision:



[total momentum p]_{after} = $[m_1v_1 + m_2v_2]_{\text{after}}$

During collision: momentum Δp goes from one mass to the other



loses Δp

gains Δp

$[\Delta p]_{\text{total}} = 0$

before:

p_1 p_2

Δp Δp

after:

p_1 p_2

Momentum conserved

$[p_1]_{\text{after}} = [p_1]_{\text{before}} - \Delta p$

$[p_2]_{\text{after}} = [p_2]_{\text{before}} + \Delta p$

therefore:

$[p_1 + p_2]_{\text{after}} = [p_1 + p_2]_{\text{before}}$

Changes of velocity:

$m_1\Delta v_1 = -\Delta p$ therefore: $-\frac{\Delta v_2}{\Delta v_1} = \frac{m_1}{m_2}$

$m_2\Delta v_2 = +\Delta p$

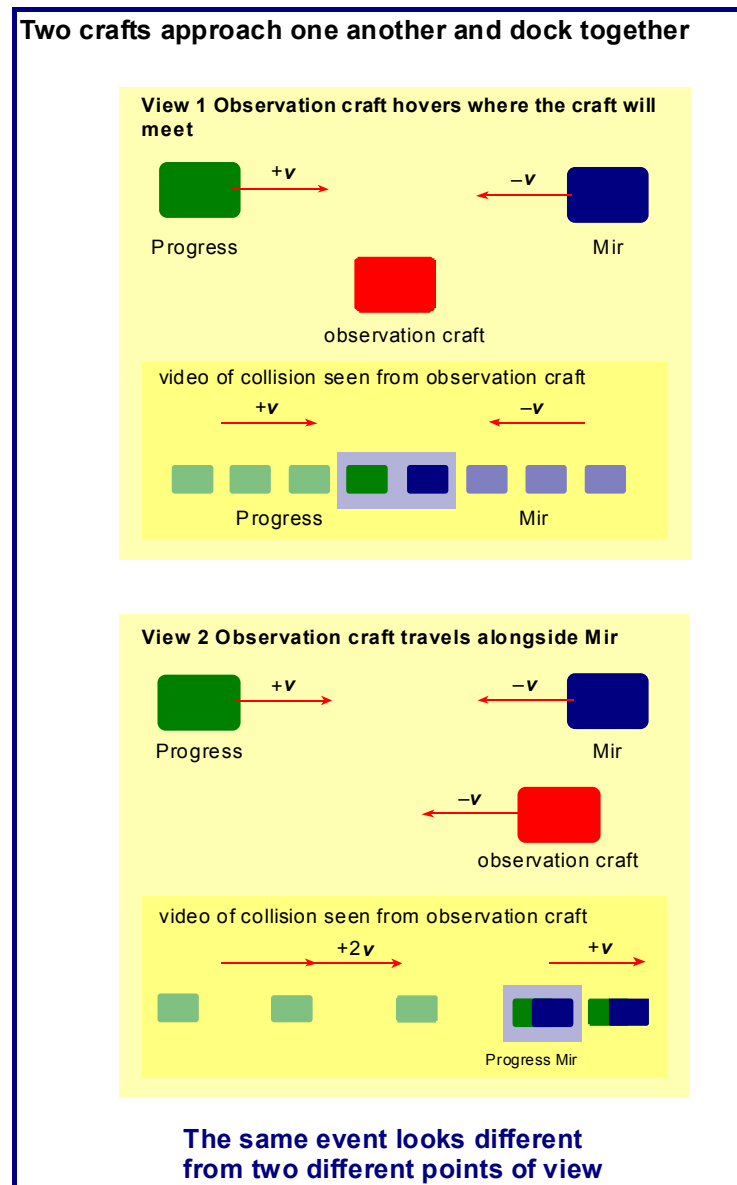
changes of momentum are equal and opposite
changes of velocity are in inverse proportion to mass

Momentum just goes from one object to the other. The total momentum is constant

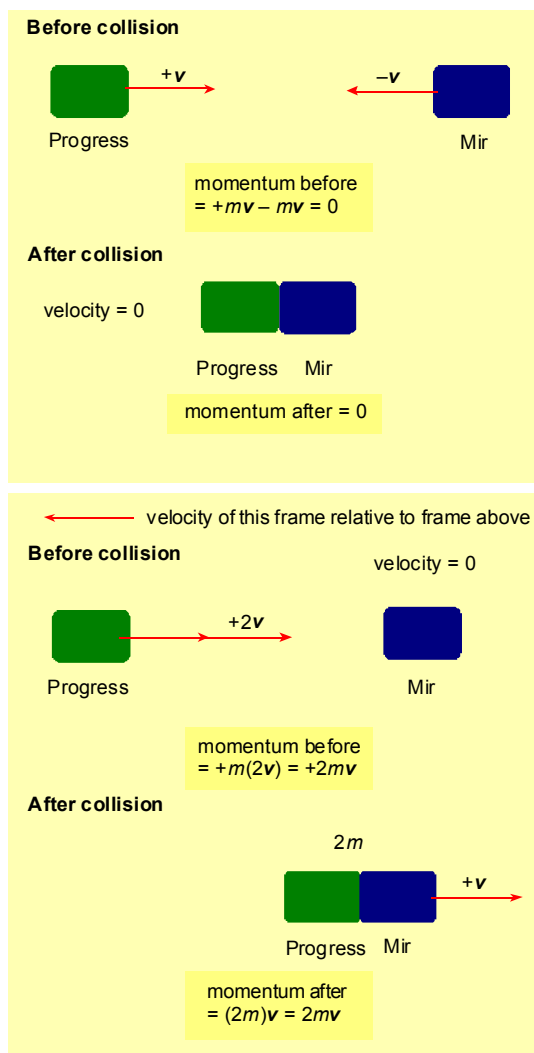
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Two craft collide

Two equally massive spacecraft dock together and join. The collision is seen from two different moving points of view. Momentum is conserved from both points of view



One event, two points of view












Momentum is different in the two views of the same event, but in each case: momentum after = momentum before

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More collisions

Here are six collisions. Notice that the total momentum before is always equal to the total momentum after.

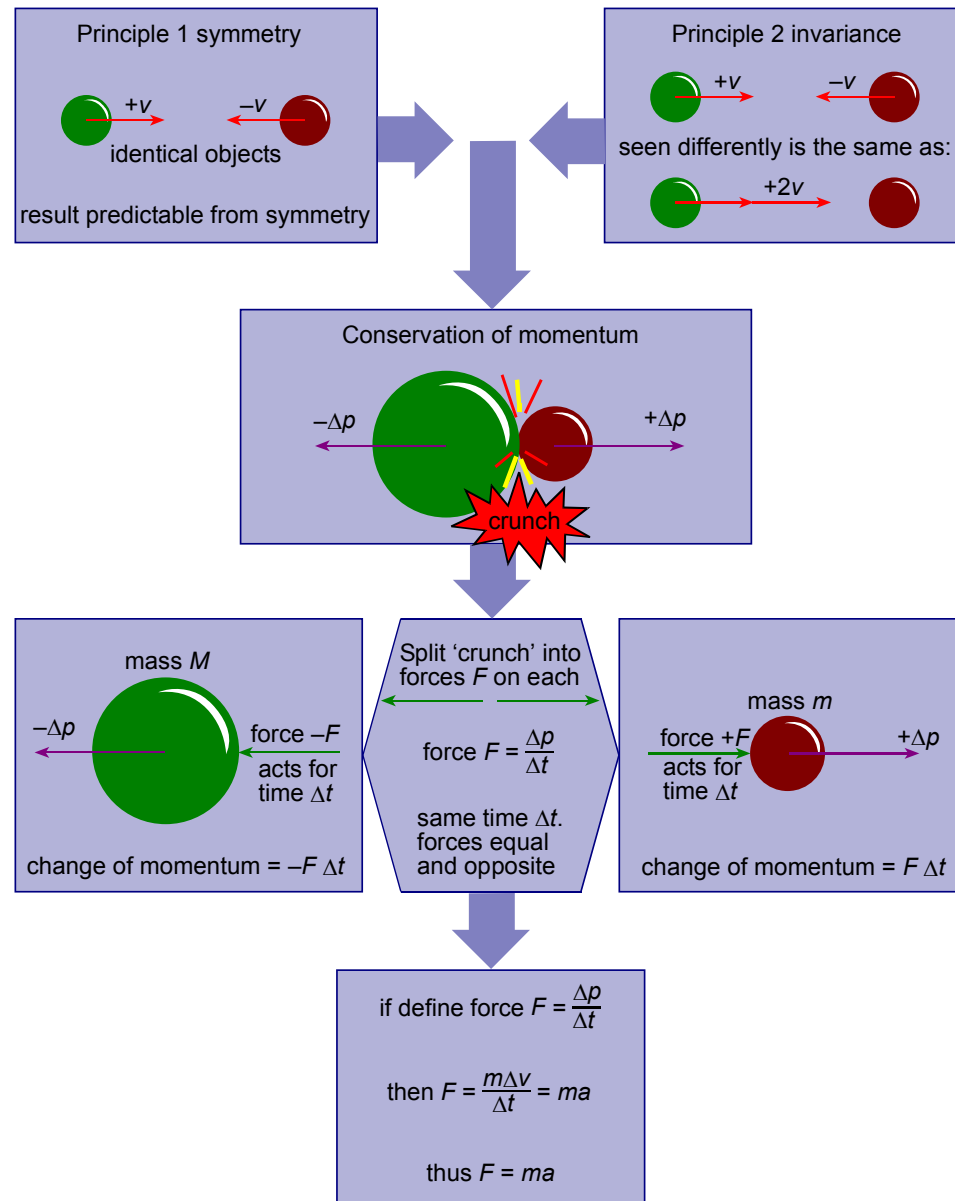
equal masses, inelastic collision		total momentum
before 		before 0
during 		
after 		after 0
equal masses, elastic collision		total momentum
before 		before →
during 		
after 		after →
equal masses, elastic collision		total momentum
before 		before 0
during 		
after 		after 0

unequal masses, inelastic collision		total momentum
before	before	
<p>velocity v velocity zero</p>	→	
during		
after	<p>velocity a little less than v</p>	after →
unequal masses, elastic collision		total momentum
before	before	
<p>velocity v velocity zero</p>	→	
during		
after	<p>velocity a little less than v velocity much less than v</p>	after →
unequal masses, elastic collision		total momentum
before	before	
<p>velocity v velocity zero</p>	→	
during		
after	<p>velocity much less than v velocity up to $2v$</p>	after →

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Momentum, invariance, symmetry

Thinking about momentum and forces



From symmetry and invariance (looking differently can't change events):

1. momentum is conserved
2. define mass from change of velocity in collision
3. define force as rate of change of momentum, giving $F = ma$
4. forces on interacting objects act in equal and opposite pairs

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Jets and rockets

Jets and rockets

momentum carried by gas plus momentum change of rocket = 0

rocket velocity V increases by ΔV in time t

rocket mass M

momentum carried away by jet: $p = v m$ in time t

change of momentum of rocket: $p = M \Delta V$ in time t

mass m ejected in time t

gas velocity v

equal and opposite

for jet: $v m = -p$

for rocket: $p = M \Delta V$

$$M \Delta V = -v m$$

$$\Delta V = \frac{-v m}{M}$$

$$\text{thrust} = \frac{p}{t} = \frac{M \Delta V}{t} = \frac{-v m}{t}$$

Rocket thrust = $-v \frac{m}{t}$

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Graphs showing g against h

Gravitational field and gravitational potential energy

Uniform gravitational field

Field picture

force on mass in gravitational field = mg

field = force/mass = g

Potential energy picture

potential energy change E

mass m lift by height h

potential energy change = force distance = $mg h$

gravitational potential difference = potential change/mass = $mg h/m = g h$

Field is slope of potential hill

potential energy per kg

field g = slope

gravitational potential difference

displacement upwards h

Potential difference is area under graph

field strength g

area $g h$ = gravitational potential difference

displacement upwards h

The field is the rate of change of potential with displacement

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Field and potential

Gravitational field and gravitational potential

radius r

gravitational potential V_{grav}

at radius r : $V = -\frac{GM}{r}$

at radius $r + r$: $V = -\frac{GM}{r + r}$

Difference in potential V between r and $r + r$ is:

$$V = -\frac{GM}{r} - \left(-\frac{GM}{r+r}\right)$$

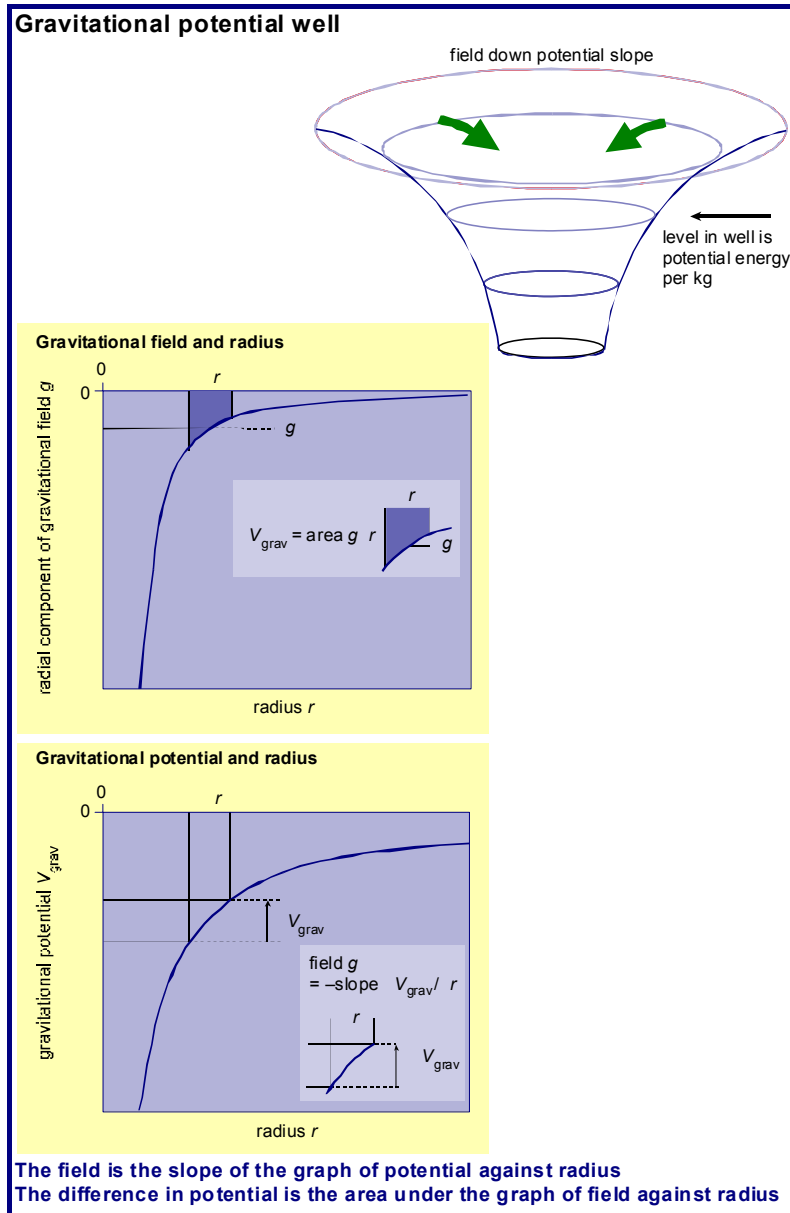
$$V = \frac{-GM(r+r) + GMr}{r(r+r)}$$

if r is small: $V = \frac{GM}{r^2}$

field $g = -\frac{V}{r}$

Thus: radial component of field $g = -\frac{GM}{r^2}$

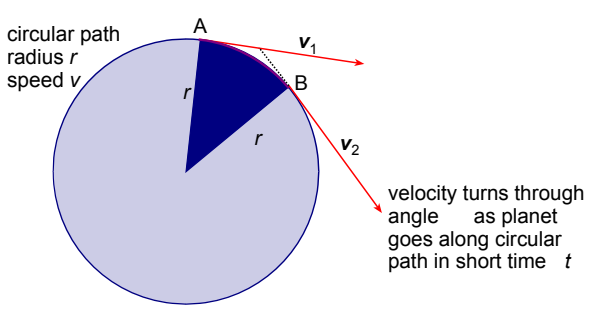
Radial component of field $= -\frac{dV}{dr}$. Since $V = -\frac{GM}{r}$, then radial component of field $g = -\frac{GM}{r^2}$



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Centripetal acceleration

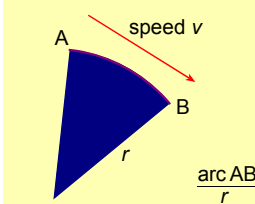
Circular motion



circular path
radius r
speed v

velocity turns through angle as planet goes along circular path in short time t

radius turns through

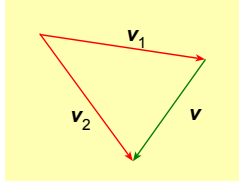


speed v

arc AB = distance in time t at speed v
arc AB = $v t$

$\frac{v t}{r}$

velocity turns through



change of velocity v towards centre of circle

$\frac{v}{v}$

$\frac{v t}{r} \longleftrightarrow \frac{v}{v}$

$\frac{v t}{r}$ $\frac{v}{v}$

multiply by v : $v \frac{v t}{r} = v^2$

divide by t : $\frac{v^2}{r} \frac{v}{t} = \text{acceleration}$

The acceleration towards the centre of a circular orbit = $\frac{v^2}{r}$

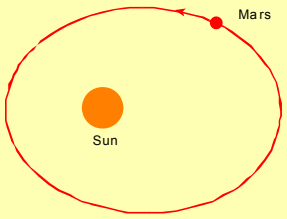
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Geometry rules the Universe

Kepler: Geometry rules the Universe

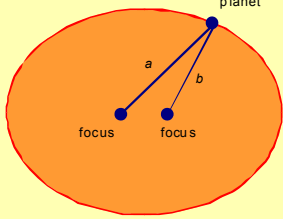
Law 1: a planet moves in an ellipse with the Sun at one focus

Astronomy



Orbit of Mars is an ellipse with Sun at a focus

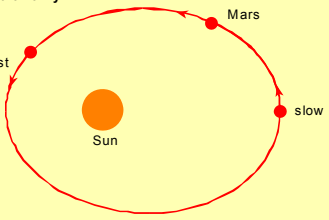
Geometry



Ellipse: curve such that sum of a and b is constant

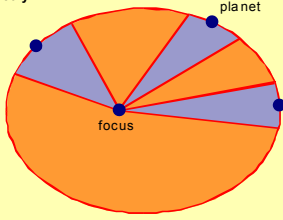
Law 2: the line from the Sun to a planet sweeps out equal areas in equal times

Astronomy



Speed of planet faster near Sun, slower away from Sun

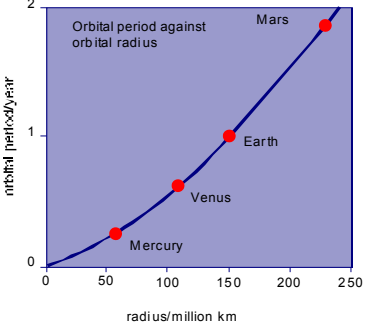
Geometry



Areas swept out in same time are equal

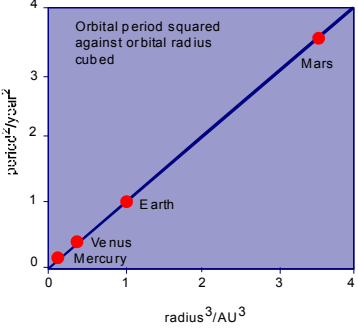
Law 3: square of orbital time is proportional to cube of orbital radius

Orbital period against orbital radius



Planet	radius/million km	orbital period/year
Mercury	~58	~0.088
Venus	~108	~0.225
Earth	~150	~1.0
Mars	~228	~1.88

Orbital period squared against orbital radius cubed



Planet	radius ³ /AU ³	period ² /year ²
Mercury	~0.05	~0.008
Venus	~0.1	~0.045
Earth	1.0	1.0
Mars	~3.4	~3.5

Kepler formulated these three laws governing the motion of planets around the sun

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A geostationary satellite

A geostationary satellite

m = mass of satellite
 R = radius of satellite orbit
 v = speed in orbit
 G = gravitational constant
 $= 6.67 \times 10^{-11} \text{ N kg}^{-2} \text{ m}^2$
 M = mass of Earth
 $= 5.98 \times 10^{24} \text{ kg}$
 T = time of orbit
 $= 24 \text{ hours} = 86400 \text{ s}$

orbit radius
 $R = 42000 \text{ km}$

gravitational force

satellite orbit turns at same rate as Earth turns

Calculating the radius of orbit

force producing acceleration to centre $\frac{mv^2}{R}$ equal gravitational force on satellite $\frac{GMm}{R^2}$

forces are equal: $\frac{mv^2}{R} = \frac{GMm}{R^2}$

divide by m : $\frac{v^2}{R} = \frac{GM}{R^2}$

multiply by R : $v^2 = \frac{GM}{R}$ equal speed in orbit depends on time of orbit and radius $v = \frac{2\pi R}{T}$

$v^2 = \frac{4\pi^2 R^2}{T^2}$

equate expressions for v^2 : $\frac{GM}{R} = \frac{4\pi^2 R^2}{T^2}$

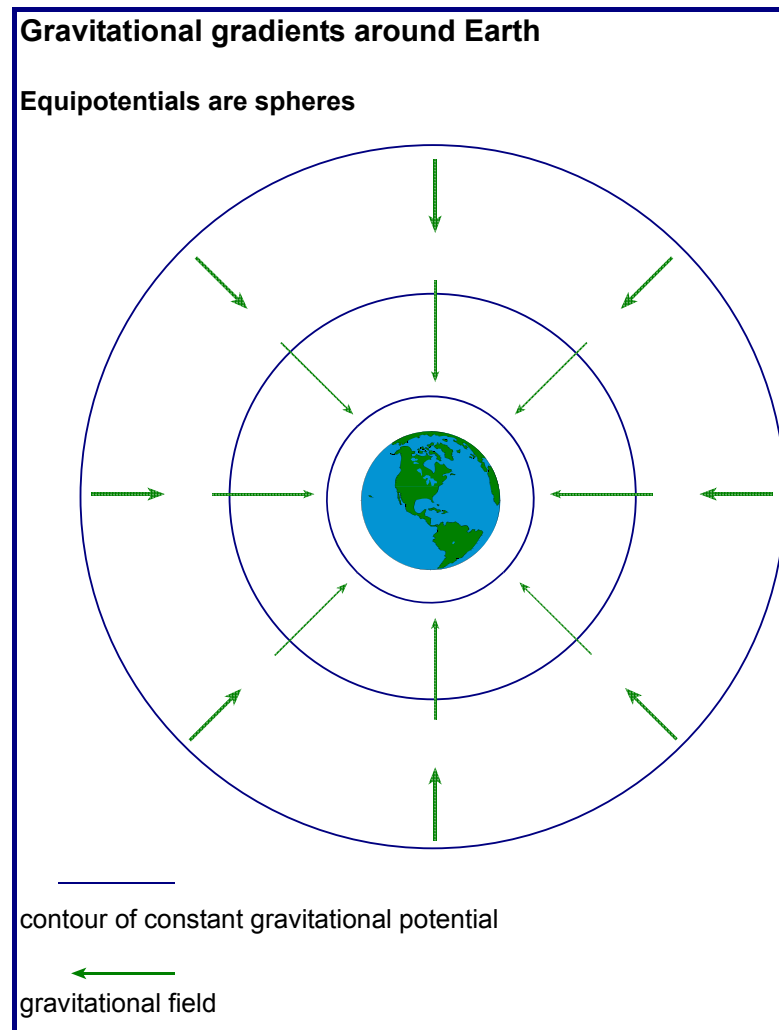
rearrange to calculate R : $\frac{GMT^2}{4\pi^2} = R^3$ **Kepler's third law deduced**

insert values of G , M and T : $R = 4.2 \times 10^4 \text{ km}$
 $R = 6.6$ radius of Earth (6400 km)

Kepler's third law, and the orbit radius of a geostationary satellite, can be deduced from first principles

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Relationship between g and V_g



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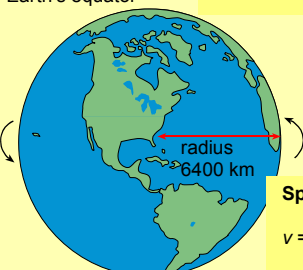
Speeds and accelerations in the Solar System

Speeds and accelerations in the Solar System

What speed and acceleration? *why don't you notice them?*

Time
1 day = 24 hours

Travelling round once a day on Earth's equator



radius 6400 km

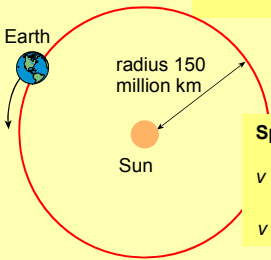
Distance
circumference = $2 \times 6400 \text{ km}$
= 40200 km

Speed
 $v = \frac{40200 \text{ km}}{24 \text{ hour}}$
 $v = 1700 \text{ km per hour}$

Acceleration
 $a = \frac{v^2}{r}$
 $a = \frac{(1700 \text{ km per hour})^2}{6400 \text{ km}}$
 $a = 440 \text{ km h}^{-1} \text{ per hour}$
 $a = 0.034 \text{ m s}^{-2}$

Time
1 year = 365 days
= 8800 hours

Travelling round the Sun once a year on Earth's orbit



radius 150 million km

Sun

Distance
circumference = $2 \times 1.5 \times 10^8 \text{ km}$
= $9.4 \times 10^8 \text{ km}$

Speed
 $v = \frac{9.4 \times 10^8 \text{ km}}{8800 \text{ hour}}$
 $v = 110000 \text{ km per hour}$

Acceleration
 $a = \frac{v^2}{r}$
 $a = \frac{(110000 \text{ km per hour})^2}{9.4 \times 10^8 \text{ km}}$
 $a = 80 \text{ km h}^{-1} \text{ per hour}$
 $a = 0.006 \text{ m s}^{-2}$

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Acceleration of the Moon

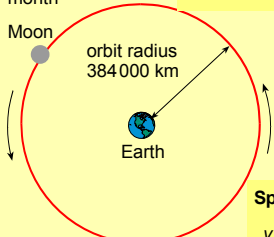
The acceleration of the Moon and the inverse square law

Acceleration of the Moon

Time
 1 Moon month = 27.3 days
 = 27.3 × 24 × 3600 s
 = 2.35 × 10⁶ s

Distance
 circumference = 2 × 3.84 × 10⁸ m
 = 24.1 × 10⁸ m

Travelling around the Earth once a month



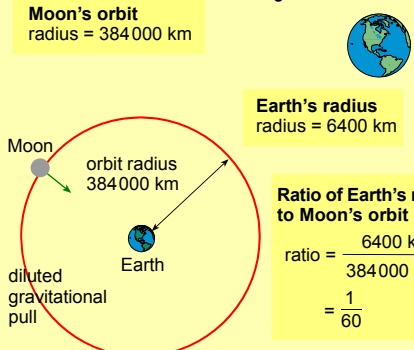
Speed
 $v = \frac{24.1 \times 10^8 \text{ m}}{2.35 \times 10^6 \text{ s}}$
 $v = 1020 \text{ m s}^{-1}$

Acceleration
 $a = \frac{v^2}{r}$
 $a = \frac{(1020 \text{ m s}^{-1})^2}{3.84 \times 10^8 \text{ m}}$
 $a = 0.0027 \text{ m s}^{-2}$ ← acceleration found from motion

Diluting Earth's gravity

$g = 9.8 \text{ m s}^{-2}$ at surface

Moon's orbit
 radius = 384,000 km



Earth's radius
 radius = 6,400 km

Ratio of Earth's radius to Moon's orbit
 $\text{ratio} = \frac{6400 \text{ km}}{384000 \text{ km}}$
 $= \frac{1}{60}$

Gravity diluted by inverse square law

Earth's surface:
 $g = 9.8 \text{ m s}^{-2}$

At Moon's orbit:
 acceleration = $\frac{9.8 \text{ m s}^{-2}}{60^2}$
 $a = 0.0027 \text{ m s}^{-2}$ ← same acceleration found from inverse square law

The acceleration of the Moon is simply diluted Earth gravity. The acceleration measures the gravitational field

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Apollo returns from the Moon

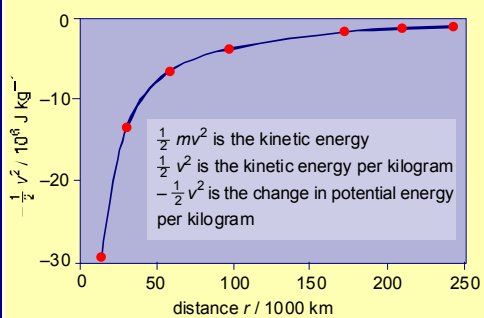
Apollo 11 comes back from the Moon

Pairs of observations of speed and distance

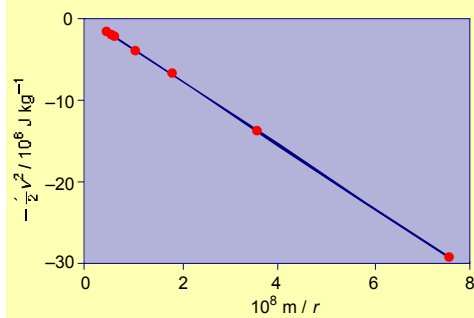
Apollo 11 is coasting home 'downhill' with rockets turned off. Distances r taken from centre of Earth

distance / 10^6 m	speed / m s^{-1}	$-\frac{1}{2}v^2 / 10^6 \text{ J kg}^{-1}$	$10^8 \text{ m} / r$
241.6	1521	-1.16	0.414
209.7	1676	-1.40	0.477
170.9	1915	-1.83	0.585
96.8	2690	-3.62	1.033
56.4	3626	-6.57	1.774
28.4	5201	-13.53	3.518
13.3	7673	-29.44	7.513

Variation of gravitational potential with distance



Variation of gravitational potential with $1/r$



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