

Revision Guide for Chapter 9

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I can show my understanding of effects, ideas and relationships by describing and explaining:

<p>the meaning of relative velocity</p> <p>Revision Notes: relative velocity</p> <p>Summary Diagrams: Relative velocity</p>	
<p>motion under constant acceleration and force</p> <p>Revision Notes: acceleration, mass, Newton's laws of motion</p> <p>Summary Diagrams: Stepping through uniform acceleration, Logic of motion 1, Logic of motion 2, Graphs for constant acceleration, Graphs for realistic motion, Computing uniform acceleration</p>	
<p>the parabolic trajectory of a projectile</p> <p>that the horizontal and vertical components of the velocity of a projectile are independent</p> <p>that a force changes only the component of velocity in the direction of the force</p> <p>Revision Notes: projectile</p> <p>Summary Diagrams: A parabola from steps</p>	
<p>that work done = force × displacement in the direction of the force</p> <p>power as the rate of transfer of energy (energy transferred per second)</p> <p>Revision Notes: work, kinetic energy, potential energy, power</p> <p>Summary Diagrams: Calculating kinetic energy, Kinetic and potential energy, Flow of energy to a train, Power, force and velocity</p>	

I can use the following words and phrases accurately when describing the motion of objects:

<p>relative velocity</p> <p>Revision Notes: relative velocity</p> <p>Summary Diagrams: Relative velocity</p>	
<p>acceleration, force, mass</p> <p>Revision Notes: acceleration, mass, Newton's laws of motion</p> <p>Summary Diagrams: Stepping through uniform acceleration</p>	
<p>gravitational force, weight, acceleration of free fall</p> <p>Revision Notes: weight, mass, free fall</p>	
<p>kinetic energy, potential energy, work done, power</p> <p>Revision Notes: kinetic energy, potential energy, work, power</p> <p>Summary Diagrams: Calculating kinetic energy, Kinetic and potential energy, Flow of energy to a train, Power, force and velocity</p>	

I can interpret:

vector diagrams showing relative velocities Revision Notes: relative velocity Summary Diagrams: Relative velocity	
graphs of speed–time and distance–time for accelerated motion, including the area under a speed–time graph and the slope of a distance–time and speed–time graph Revision Notes: acceleration Summary Diagrams: Graphs for constant acceleration , Graphs for realistic motion	

I can calculate:

the resultant vector produced by subtracting one vector from another Revision Notes: relative velocity Summary Diagrams: Relative velocity	
speed from the gradient (slope) of a distance–time graph distance from the area under a speed–time graph Summary Diagrams: Graphs for constant acceleration , Graphs for realistic motion	
the unknown variable, when given other relevant data, using the kinematic equations: $v = u + at;$ $v^2 = u^2 + 2as;$ $s = ut + \frac{1}{2}at^2;$ $s = \frac{(u+v)}{2}t$ Summary Diagrams: Logic of motion 1 , Logic of motion 2	
the unknown quantity, given other relevant data, using the equation $F = m a$ Revision Notes: Newton's laws of motion	
the path of a moving body acted upon by the force of gravity when the body is (a) moving vertically and (b) moving both vertically and horizontally, including use of the kinematic equations and of step by step changes of velocity and displacement in short time intervals Revision Notes: free fall , projectile Summary Diagrams: Stepping through uniform acceleration , Computing uniform acceleration , A parabola from steps	
the kinetic energy of a moving body using $KE = \frac{1}{2} m v^2$ Revision Notes: kinetic energy Summary Diagrams: Calculating kinetic energy	
the change in potential energy when a massive body changes height in a gravitational field using $\Delta E_p = m g h$ Revision Notes: potential energy	

Summary Diagrams: Kinetic and potential energy	
the work done (energy transferred) $\Delta E = F \Delta s$ force, energy and power: $\text{power} = \Delta E / t = F v$ Revision Notes: work , power Summary Diagrams: Power, force and velocity , Flow of energy to a train , Kinetic and potential energy	

I can show my ability to make better measurements by:

measuring force, acceleration, velocity, kinetic and potential energy, work, power with known uncertainty Revision Notes: accuracy and precision , systematic error , uncertainty	
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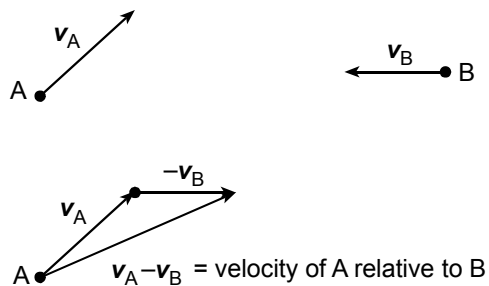
Relative velocity

If two objects A and B move at velocities \mathbf{v}_A and \mathbf{v}_B in a given frame of reference, the relative velocity of A with respect to B, $\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$.

Relative velocity is a **vector quantity**.

A velocity vector may be represented as an arrow having a length in proportion to the speed in the appropriate direction. The relative velocity of an object A to an object B can therefore be represented as the velocity vector $-\mathbf{v}_B$ added on to the end of the velocity vector \mathbf{v}_A , giving a resultant velocity vector $\mathbf{v}_A - \mathbf{v}_B$ as shown below.

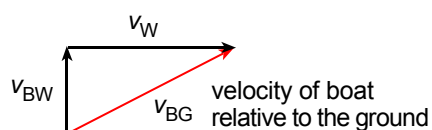
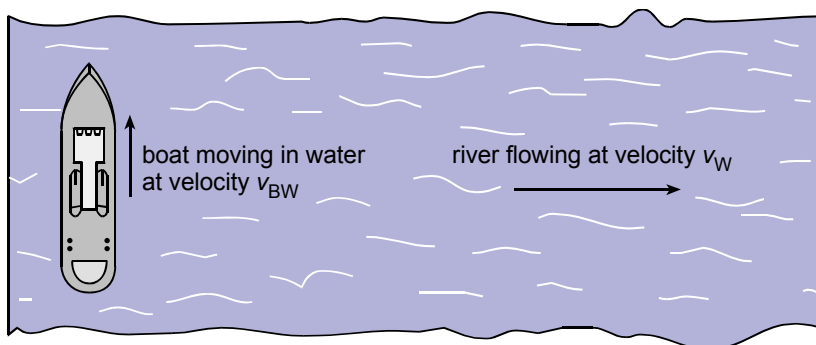
Relative velocity



Using components

Given the velocity components of two objects, the differences of the velocity components give the components of the relative velocity.

Velocity in a flowing river



The velocity of a boat crossing a river is affected by the velocity of the water. The boat velocity relative to the ground, $\mathbf{v}_{BG} = \mathbf{v}_{BW} + \mathbf{v}_W$, where \mathbf{v}_{BW} is the velocity of the boat relative to the water and \mathbf{v}_W is the velocity of the water relative to the ground. This is because the

velocity of the water is added on to the velocity of the boat relative to the water to give the velocity of the boat relative to the ground. For example, if the boat is travelling at a speed of 5 m s^{-1} relative to the water in a direction opposite to the flow of the water which is moving at 2 m s^{-1} , the boat velocity relative to the ground is 3 m s^{-1} ($= 5 - 2 \text{ m s}^{-1}$) in the opposite direction to the flow.

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Acceleration

Acceleration is the rate of change of velocity. The SI unit of acceleration is the metre per second per second (m s^{-2}). Acceleration is a **vector quantity**.

If in time Δt the vector velocity \mathbf{v} changes by the vector amount $\Delta \mathbf{v}$, then the vector acceleration \mathbf{a} is given by:

$$\mathbf{a} = \Delta \mathbf{v} / \Delta t$$

The acceleration is in the direction of the change of velocity.

A decelerating object, whose speed decreases, has a change in velocity opposite in direction to the velocity.

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Newton's Laws of Motion

Newton's laws of motion describe the motion of objects acted on by forces, doing so to a very good approximation as long as the speeds are small compared with the speed of light.

Newton's first law of motion states that an object remains at rest or moves with constant velocity unless acted on by a resultant force.

Newton's first law defines what a force is, namely any physical effect that is capable of changing the motion of an object. For example, an object released from rest in mid-air accelerates because the force of gravity acts on it. If an object speeds up, slows down or changes direction, it must have a resultant force acting on it. If an object is at rest or in uniform motion, either no force acts on it or forces do act on it and their resultant force is zero.

Newton's second law of motion states that the rate of change of momentum of an object is proportional to the resultant force on the object.

Newton's second law is expressed mathematically as $\mathbf{F} = d\mathbf{p} / dt$, where $\mathbf{p} = m\mathbf{v}$ is the momentum of an object acted on by a resultant force \mathbf{F} and d / dt means 'rate of change'. The SI unit of force, the newton (N), is the force that gives a 1 kg mass an acceleration of 1 m s^{-2} . In this case, the momentum increases at a rate of 1 kg m s^{-1} every second.

For an object of constant mass m , acted on by a constant force \mathbf{F} only

$$\mathbf{F} = m \frac{d\mathbf{v}}{dt}$$

since its momentum $\mathbf{p} = m \mathbf{v}$ and $d\mathbf{p} = m d\mathbf{v}$.

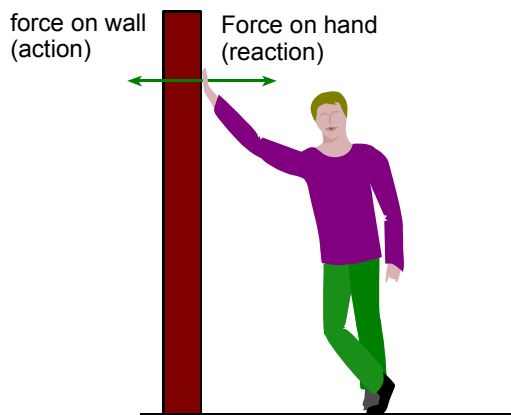
Hence in this case $\mathbf{F} = m\mathbf{a}$, where $\mathbf{a} = d\mathbf{v} / dt$ is the acceleration of the object.

Newton's third law of motion states that when two objects interact, there is an equal and opposite force on each. The forces are of the same kind. This is a consequence of the conservation of momentum.

Newton's third law applies to any form of interaction between two objects.

1. In a collision between two objects or an explosion where two objects fly apart, the momentum of each object changes. One object gains momentum at the expense of the other object, and the total change of momentum is zero. The total momentum is therefore conserved, in accordance with the principle of conservation of momentum. It follows from the definition of force as rate of change of momentum that the forces must be thought of as equal and opposite.
2. If two objects are in contact with each other at rest or with no relative motion, the two objects exert equal and opposite forces on each other. For example, someone at rest leaning on a wall exerts a force on the wall and is acted on by an equal and opposite force from the wall.

Equal and opposite forces



Relationships

Newton's second law: force = mass × acceleration.

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Mass

A massive object changes its velocity less than does a less massive object, when acted on by the same force for the same time.

If two bodies interact, their velocities change in inverse proportion to their masses

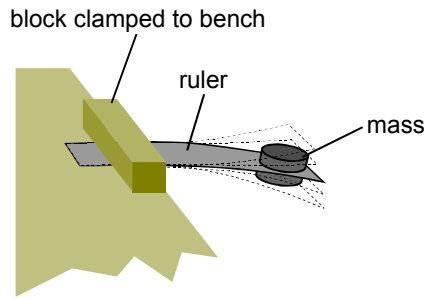
$$\frac{\Delta v_1}{\Delta v_2} = \frac{m_2}{m_1}$$

The larger mass suffers the smaller change in velocity. This relationship only defines ratios of masses. To establish a unit, a mass has to be assigned to some particular object. Mass is measured in kilograms. The kilogram is defined as the mass of a certain quantity of platinum iridium alloy kept in an international standards laboratory in Paris.

Mass is a **scalar quantity**.

Masses of objects can be compared by comparing their changes of velocity under the same forces or in the same interaction. One way to do this is to compare the frequencies of oscillation of masses fixed to the same springy support.

Oscillations of a loaded rule



Mass is also the source of the gravitational field and is acted on by a gravitational field. Masses are conveniently compared by comparing the gravitational forces on them in the same gravitational field. This is what a balance does. A standard mass can be used to calibrate a spring balance.

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Weight

The weight of an object is the gravitational force acting on it. Weight is measured in newtons (N).

The strength of a gravitational field, g , at a point in a gravitational field is the force per unit mass acting on a small mass at that point.

Gravitational field strength is a **vector quantity** in the direction of the gravitational force on a mass. The SI unit of gravitational field strength is the newton per kilogram (N kg^{-1}) or (equivalently) m s^{-2} .

The force F on a point mass m at a point in a gravitational field is given by $F = m g$, where g is the gravitational field strength at that point. Thus the weight = mg .

If an object is supported at rest, both the object and its support will be compressed. This compression can be used in a spring balance to weigh the object.

An object that is in free fall is sometimes said to be 'weightless' even though the force of gravity still acts on it. Such an object will appear to weigh nothing if put on a spring balance falling freely with it, not because the Earth is not exerting a force on it, but because both object and balance are falling together.

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Free fall

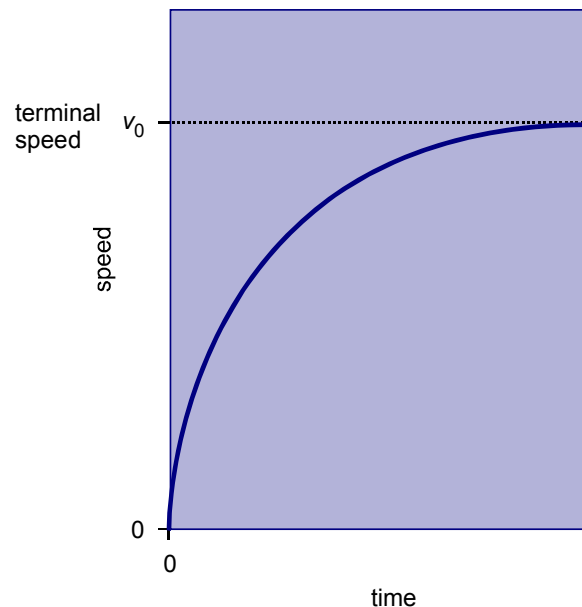
Objects acted on by gravity alone are said to be in free fall.

For an object of mass m , its weight = $m g$, where m is the mass of the object and g is the gravitational field strength at the object.

The acceleration of the object, a , is the force of gravity divided by the mass = $m g / m = g$. Hence the acceleration of a freely falling object is equal to g , independent of its mass. The acceleration is constant if the gravitational force is constant, and no other forces act.

If the object is acted on by air resistance as it falls, its acceleration gradually decreases to zero and its velocity reaches a maximum value known as its terminal velocity. The object is thus not in free fall as it is 'partially supported' by air resistance. The air-resistance force F_R , increases with velocity so the resultant force ($= m \mathbf{g} - F_R$) and hence the acceleration decreases. At the **terminal velocity**, the force due to air resistance is equal and opposite to the weight of the object so the resultant force and hence the acceleration is zero.

Terminal speed



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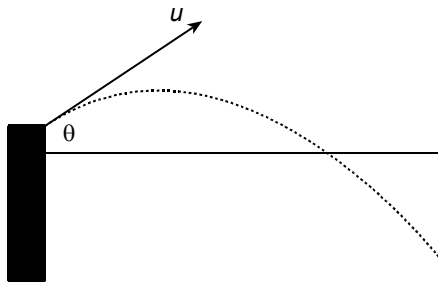
Projectile

At any point on the path of a projectile, provided that air resistance is negligible:

1. the horizontal component of acceleration is zero
2. the downward vertical component of acceleration is equal to \mathbf{g} , the gravitational field strength at that point.

A projectile travels equal horizontal distances in equal times because its horizontal component of acceleration is zero. Its horizontal motion is unaffected by its vertical motion. The combination of constant horizontal velocity and constant downward acceleration leads to a parabolic path.

See [A parabola from steps](#) for a graphical calculation of the parabolic path of a projectile.

Using kinematic equations**Projection at a non-zero angle above the horizontal**

The projectile is launched with an initial speed u at angle θ above the horizontal at time $t = 0$.

Its constant horizontal component of velocity $u_x = u \cos\theta$. Thus the horizontal component of its displacement at time t is

$$x = u_x t$$

Its initial vertical component of velocity $u_y = u \sin\theta$. Thus its vertical component of velocity at time t is $u_y - gt$. The vertical component of displacement is

$$y = u_y t - \frac{1}{2} gt^2.$$

The highest point is reached at time t_0 when the vertical component of velocity $u_y - g t_0 = 0$, hence $t_0 = u_y / g$.

The shape of the path is parabolic.

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Work

The work W done by a force F that moves its point of application by a distance s in the direction of the force is given by $W = F s$.

Work measures the amount of energy transferred from one thing to another.

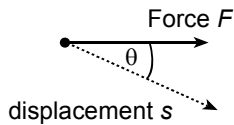
The SI unit of energy and of work is the joule (J). Work and energy are **scalar quantities**.

Work is done whenever an object moves under the action of a force with a component of the force along the displacement. If there is no movement of the object, no work is done. However, an outstretched hand holding a book does need to be supplied with energy to keep the arm muscles taut. No work is done on the book provided it is stationary. But, energy has to be supplied to repeatedly contract the muscle fibres so as to keep the muscles taut.

No work is done on an object by a force when the displacement of the object is at right angles to the direction of the force.

If an object is moved by a force F a distance s along a line that is at angle θ to the direction of the force, the work done by the force is given by $W = F s \cos\theta$.

Work done



Work done = $Fs \cos \theta$

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Kinetic energy

The kinetic energy of a moving object is the energy it carries due to its motion. An object of mass m moving at speed v has kinetic energy $E_k = \frac{1}{2}mv^2$.

To show this, consider an object of mass m initially at rest acted on by a constant force F for a time t . The object accelerates to speed v where $Ft = mv$.

Since the average speed is $v/2$, the distance moved by the object, $s = \frac{1}{2}vt$, and the work done on the object is equal to

$$Fs = \frac{mv}{t} \frac{vt}{2} = \frac{1}{2}mv^2.$$

The work done is equal to the gain of kinetic energy. Hence the kinetic energy at speed v is $E_k = \frac{1}{2}mv^2$

See also [Calculating kinetic energy](#)

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Potential energy

The gravitational potential energy of a system is the energy associated with the position of objects relative to one another, for example a mass raised above the Earth.

The SI unit of potential energy is the joule.

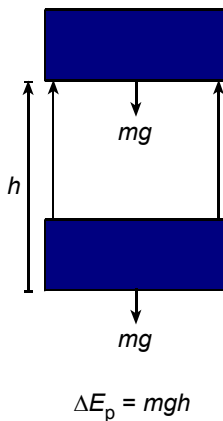
Potential energy can be thought of as stored in a field, in this example the gravitational field.

The potential energy is measured by the capacity to do work if the positions of objects change.

If the height of an object above the Earth changes, the potential energy changes. The energy transferred when it is raised is equal to the force \times distance moved along the line of action of the force.

Since the force of gravity on an object of mass m is equal to mg , then if the object is raised by a height h , the change of potential energy = mgh .

Gain of potential energy



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Power

The power of a device is the rate at which it transfers energy. The SI unit of power is the watt (W). 1 watt is 1 joule per second. Power is a scalar quantity.

If energy ΔE is transferred in time Δt the average power $P = \Delta E / \Delta t$.

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Accuracy and precision

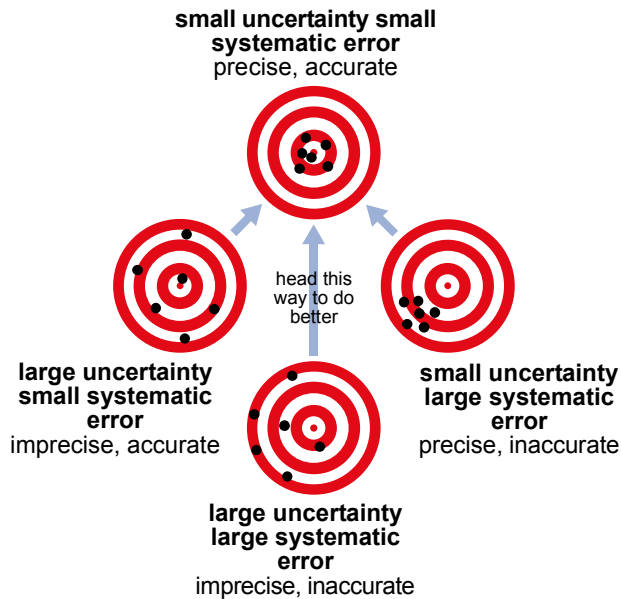
A measurement is accurate if it is close to the true value. A measurement is precise if values cluster closely, with small uncertainty.

A watch with an accuracy of 0.1% could be up to five minutes astray within a few days of being set. A space probe with a trajectory accurate to 0.01 % could be more than 30 km off target at the Moon.

Think of the true value as like the bullseye on a target, and measurements as like arrows or darts aimed at the bullseye.

Uncertainty and systematic error

Think of measurements as shots on a target. Imagine the 'true value' is at the centre of the target



An accurate set of measurements is like a set of hits that centre on the bullseye. In the diagram above at the top, the hits also cluster close together. The uncertainty is small. This is a measurement that gives the true result rather precisely.

On the left, the accuracy is still good (the hits centre on the bullseye) but they are more scattered. The uncertainty is higher. This is a measurement where the average still gives the true result, but that result is not known very precisely.

On the right, the hits are all away from the bullseye, so the accuracy is poor. But they cluster close together, so the uncertainty is low. This is a measurement that has a systematic error, giving a result different from the true result, but where other variations are small.

Finally, at the bottom, the accuracy is poor (systematic error) and the uncertainty is large.

A statement of the result of a measurement needs to contain two distinct estimates:

1. The best available estimate of the value being measured.
2. The best available estimate of the range within which the true value lies.

Note that both are statements of belief based on evidence, not of fact.

For example, a few years ago discussion of the 'age-scale' of the Universe put it at 14 plus or minus 2 thousand million years. Earlier estimates gave considerably smaller values but with larger ranges of uncertainty. The current (2008) estimate is 13.7 ± 0.2 Gy. This new value lies within the range of uncertainty for the previous value, so physicists think the estimate has been improved in precision but has not fundamentally changed.

Fundamental physical constants such as the charge of the electron have been measured to an astonishing small uncertainty. For example, the charge of the electron is $1.602\,173\,335 \times 10^{-19}$ C to an uncertainty of $0.000\,000\,005 \times 10^{-19}$ C, better than nine significant figures.

There are several different reasons why a recorded result may differ from the true value:

1. **Constant systematic bias**, such as a zero error in an instrument, or an effect which has not been allowed for.

Constant systematic errors are very difficult to deal with, because their effects are only observable if they can be removed. To remove systematic error is simply to do a better experiment. A clock running slow or fast is an example of systematic instrument error. The effect of temperature on the resistance of a strain gauge is an example of systematic experimental error.

2. **Varying systematic bias**, or drift, in which the behaviour of an instrument changes with time, or an outside influence changes.

Drift in the sensitivity of an instrument, such as an oscilloscope, is quite common in electronic instrumentation. It can be detected if measured values show a systematic variation with time. Another example: the measured values of the speed of light in a pipe buried in the ground varied regularly twice a day. The cause was traced to the tide coming in on the nearby sea-shore, and compressing the ground, shortening the pipe a little.

3. **Limited resolution of an instrument**. For example the reading of a digital voltmeter may change from say 1.25 V to 1.26 V with no intermediate values. The true potential difference lies in the 0.01 V range 1.25 V to 1.26 V.

All instruments have limited resolution: the smallest change in input which can be detected. Even if all of a set of repeated readings are the same, the true value is not exactly equal to the recorded value. It lies somewhere between the two nearest values which can be distinguished.

4. **Accidental momentary effects**, such as a 'spike' in an electrical supply, or something hitting the apparatus, which produce isolated wrong values, or 'outliers'.

Accidental momentary errors, caused by some untoward event, are very common. They can often be traced by identifying results that are very different from others, or which depart from a general trend. The only remedy is to repeat them, discarding them if further measurements strongly suggest that they are wrong. Such values should never be included in any average of measurements, or be used when fitting a line or curve.

5. **Human errors**, such as misreading an instrument, which produce isolated false recorded values.

Human errors in reading or recording data do occur, such as placing a decimal point wrongly, or using the wrong scale of an instrument. They can often be identified by noticing the kinds of mistake it is easy to make. They should be removed from the data, replacing them by repeated check observations.

6. **Random fluctuations**, for example noise in a signal, or the combined effect of many unconnected minor sources of variation, which alter the measured value unpredictably from moment to moment.

Truly random variations in measurements are rather rare, though a number of unconnected small influences on the experiment may have a net effect similar to random variation. But because there are well worked out mathematical methods for dealing with random variations, much emphasis is often given to them in discussion of the estimation of the uncertainty of a measurement. These methods can usually safely be used when inspection of the data suggests that variations around an average or a fitted line or curve are small and unsystematic. It is important to look at visual plots of the variations in data before deciding how to estimate uncertainties.

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Systematic error

Systematic error is any error that biases a measurement away from the true value.

All measurements are prone to systematic error. A systematic error is any biasing effect, in the environment, methods of observation or instruments used, which introduces error into an

experiment. For example, the length of a pendulum will be in error if slight movement of the support, which effectively lengthens the string, is not prevented, or allowed for.

Incorrect zeroing of an instrument leading to a **zero error** is an example of systematic error in instrumentation. It is important to check the zero reading during an experiment as well as at the start.

Systematic errors can change during an experiment. In this case, measurements show trends with time rather than varying randomly about a mean. The instrument is said to show **drift** (e.g. if it warms up while being used).

Systematic errors can be reduced by checking instruments against known standards. They can also be detected by measuring already known quantities.

The problem with a systematic error is that you may not know how big it is, or even that it exists. The history of physics is littered with examples of undetected systematic errors. The only way to deal with a systematic error is to identify its cause and either calculate it and remove it, or do a better measurement which eliminates or reduces it.

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Uncertainty

The uncertainty of an experimental result is the range of values within which the true value may reasonably be believed to lie. To estimate the uncertainty, the following steps are needed.

1. Removing from the data **outlying** values which are reasonably suspected of being in serious error, for example because of human error in recording them correctly, or because of an unusual external influence, such as a sudden change of supply voltage. Such values should not be included in any later averaging of results or attempts to fit a line or curve to relationships between measurements.
2. Estimating the possible magnitude of any **systematic error**. An example of a constant systematic error is the increase in the effective length of a pendulum because the string's support is able to move a little as the pendulum swings. The sign of the error is known (in effect increasing the length) and it may be possible to set an upper limit on its magnitude by observation. Analysis of such systematic errors points the way to improving the experiment.
3. Assessing the **resolution** of each instrument involved, that is, the smallest change it can detect. Measurements from it cannot be known to less than the range of values it does not distinguish.
4. Assessing the magnitude of other small, possibly random, unknown effects on each measured quantity, which may include human factors such as varying speed of reaction. Evidence of this may come from the spread of values of the measurement conducted under what are as far as possible identical conditions. The purpose of repeating measurements is to decide how far it appears to be possible to hold conditions identical.
5. Determining the combined effect of possible **uncertainty** in the result due to the limited resolution of instruments (3 above) and uncontrollable variation (4 above).

To improve a measurement, it is essential to identify the largest source of uncertainty. This tells you where to invest effort to reduce the uncertainty of the result.

Having eliminated accidental errors, and allowed for systematic errors, the range of values within which the true result may be believed to lie can be estimated from (a) consideration of the resolution of the instruments involved and (b) evidence from repeated measurements of the variability of measured values.

Most experiments involve measurements of more than one physical quantity, which are combined to obtain the final result. For example, the length L and time of swing T of a simple pendulum may be used to determine the local acceleration of free fall, g , using

$$T=2\pi\sqrt{\frac{L}{g}}$$

so that

$$g=\frac{4\pi^2L}{T^2}.$$

The range in which the value of each quantity may lie needs to be estimated. To do so, first consider the resolution of the instrument involved – say ruler and stopwatch. The uncertainty of a single measurement cannot be better than the resolution of the instrument. But it may be worse. Repeated measurements under supposedly the same conditions may show small and perhaps random variations.

If you have repeated measurements, ‘plot and look’, to see how the values vary. A simple estimate of the variation is the spread = $\pm \frac{1}{2}$ range .

A simple way to see the effect of uncertainties in each measured quantity on the final result is to recalculate the final result, but adding or subtracting from the values of variables the maximum possible variation of each about its central value. This is pessimistic because it is unlikely that ‘worst case’ values all occur together. However, pessimism may well be the best policy: physicists have historically tended to underestimate uncertainties rather than overestimate them. The range within which the value of a quantity may reasonably be believed to lie may be reduced somewhat by making many equivalent measurements, and averaging them. If there are N independent but equivalent measurements, with range R , then the range of their average is likely to be approximately R divided by the factor \sqrt{N} . These benefits are not automatic, because in collecting many measurements conditions may vary.

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Relative velocity

An air miss

your velocity, reversed

225 m s⁻¹

your plane's velocity

your velocity, reversed

225 m s⁻¹

the other aircraft seems to approach you along this track.

212 m s⁻¹

other plane's velocity

relative velocity

other aircraft

225 m s⁻¹

your velocity, reversed

You imagine yourself 'at rest', so your plane has been 'stopped' by adding an equal velocity in the opposite direction to give it zero velocity.

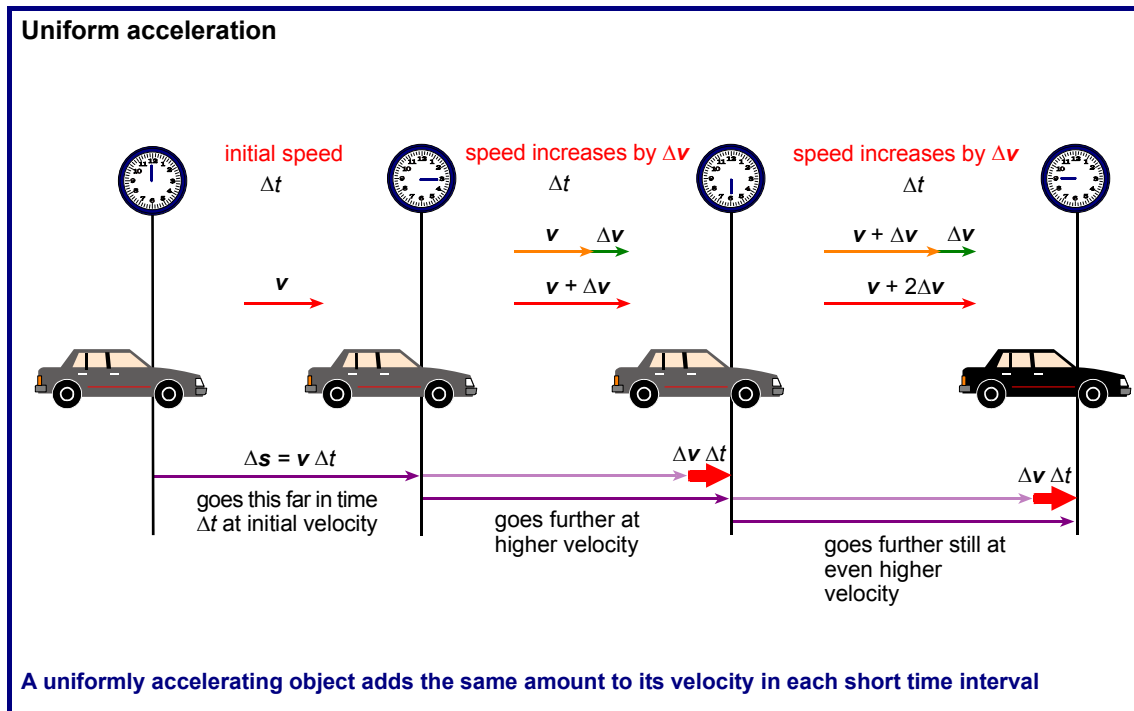
A velocity equal and opposite to your plane's velocity has been added to the velocity of the other aircraft. Together with its velocity, they combine to give the two aircraft's **relative velocity**.

Rules:

1. Add a velocity opposite to that of one plane to the velocities of both
2. Find the resultant relative velocity, adding vectors tip to tail
3. See if the direction of the relative velocity hits your plane. If so, take avoiding action!

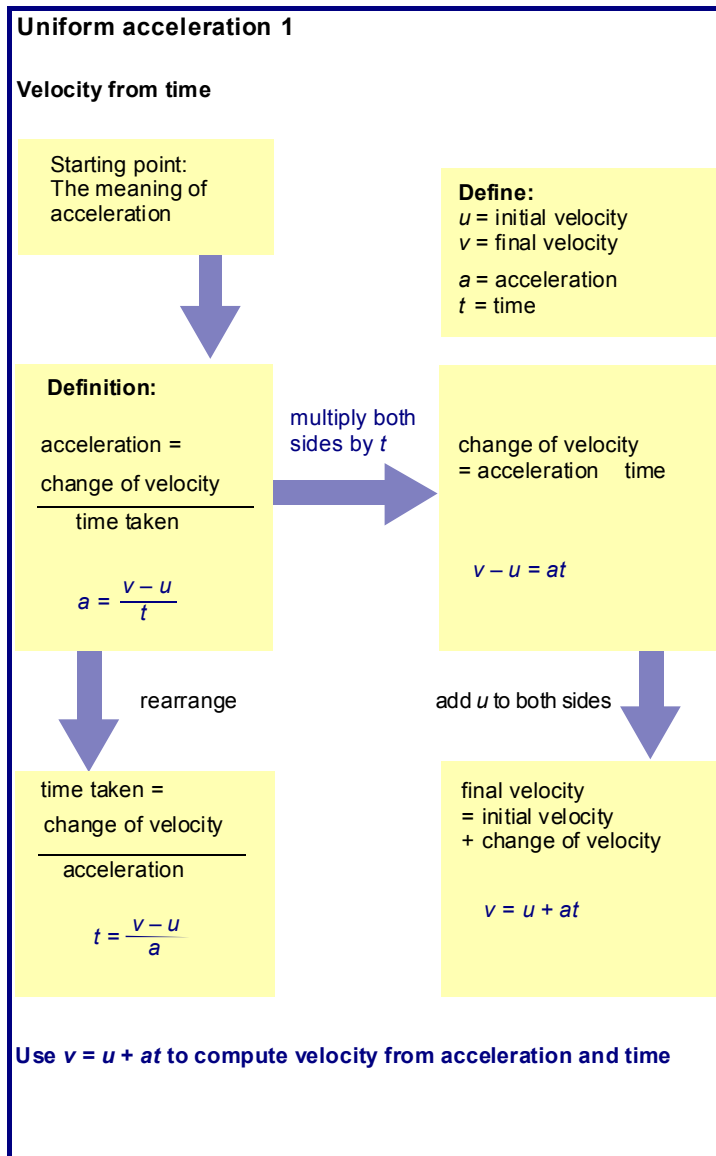
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Stepping through uniform acceleration



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Logic of motion 1



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Logic of motion 2

Uniform acceleration 2
Distance from time; velocity from distance

Starting point: average velocity

average velocity = $\frac{\text{initial plus final velocity}}{2}$
 average velocity = $\frac{u + v}{2}$
 distance gone = average velocity \times time

$s = \frac{u + v}{2} t$

From the meaning of acceleration:

$t = \frac{v - u}{a}$

Replace t :

$s = \frac{u + v}{2} \frac{v - u}{a}$

since:
 $(v + u)(v - u) = v^2 - u^2$

then:
 $s = \frac{v^2 - u^2}{2a}$

rearrange:
 $v^2 = u^2 + 2as$

Use $s = ut + \frac{1}{2}at^2$ to compute distance gone from acceleration and time

From the meaning of acceleration:

$v = u + at$

Replace v :

$s = \frac{u + u + at}{2} t$

since:
 $\frac{u + u + at}{2} = u + \frac{1}{2}at$

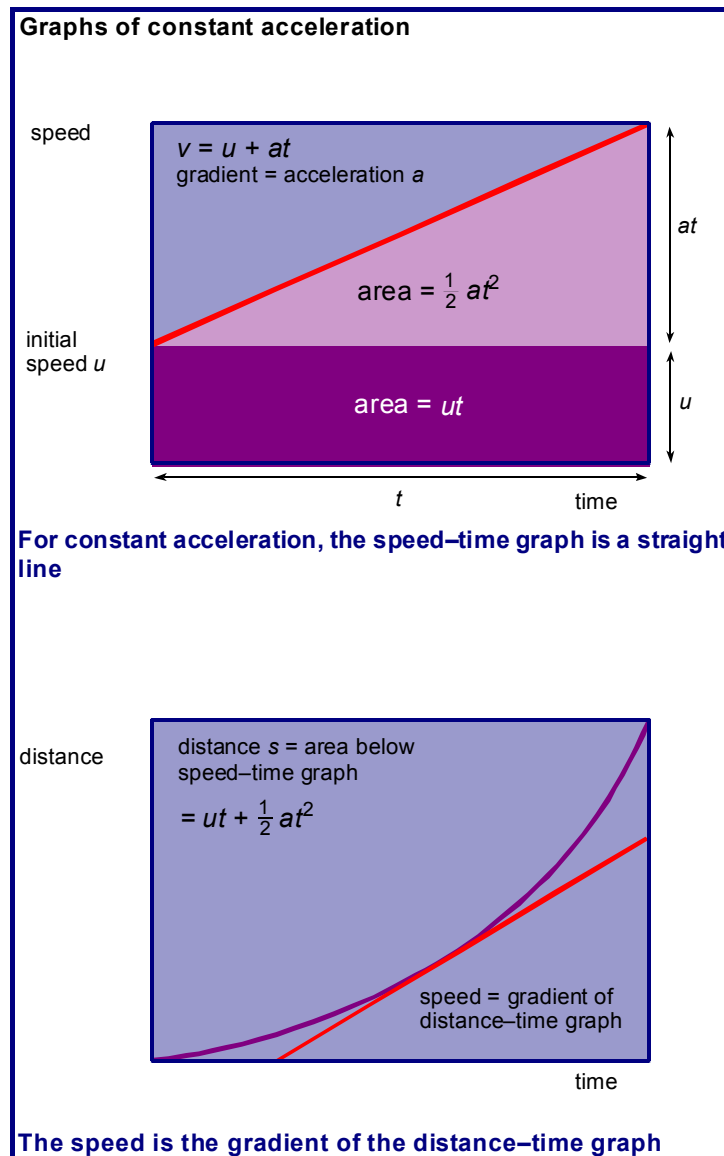
then:
 $s = (u + \frac{1}{2}at)t$

rearrange:
 $s = ut + \frac{1}{2}at^2$

Use $v^2 = u^2 + 2as$ to compute final velocity from acceleration and distance

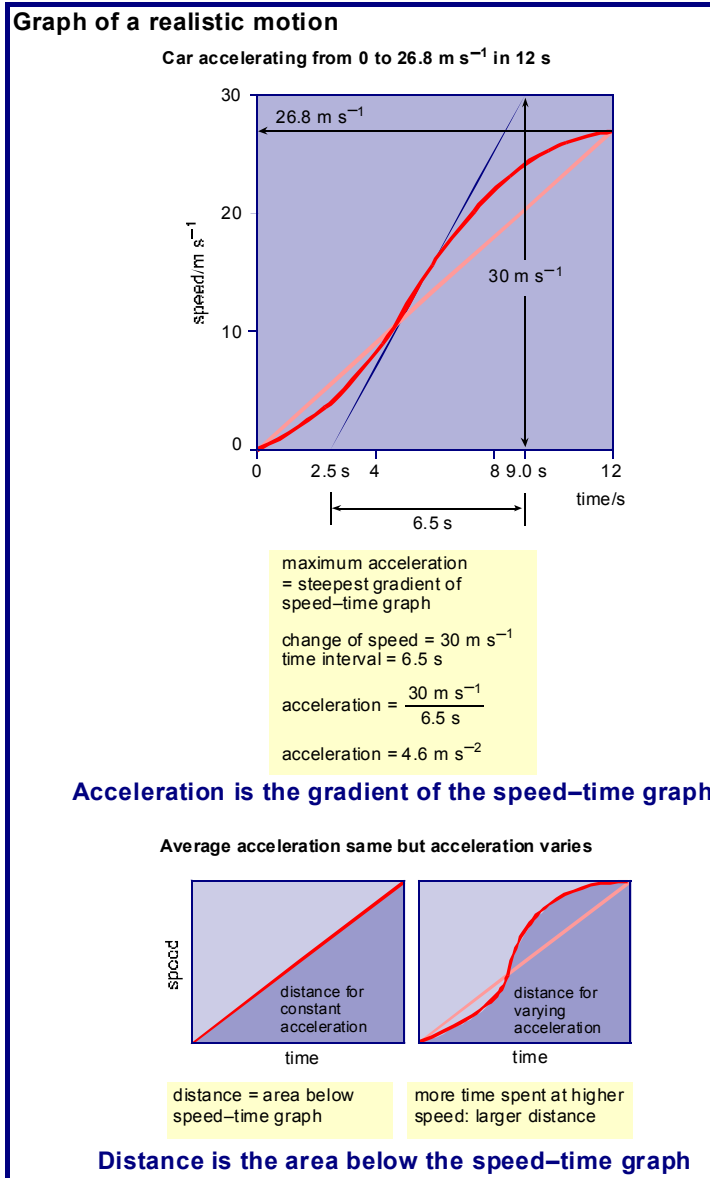
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Graphs for constant acceleration



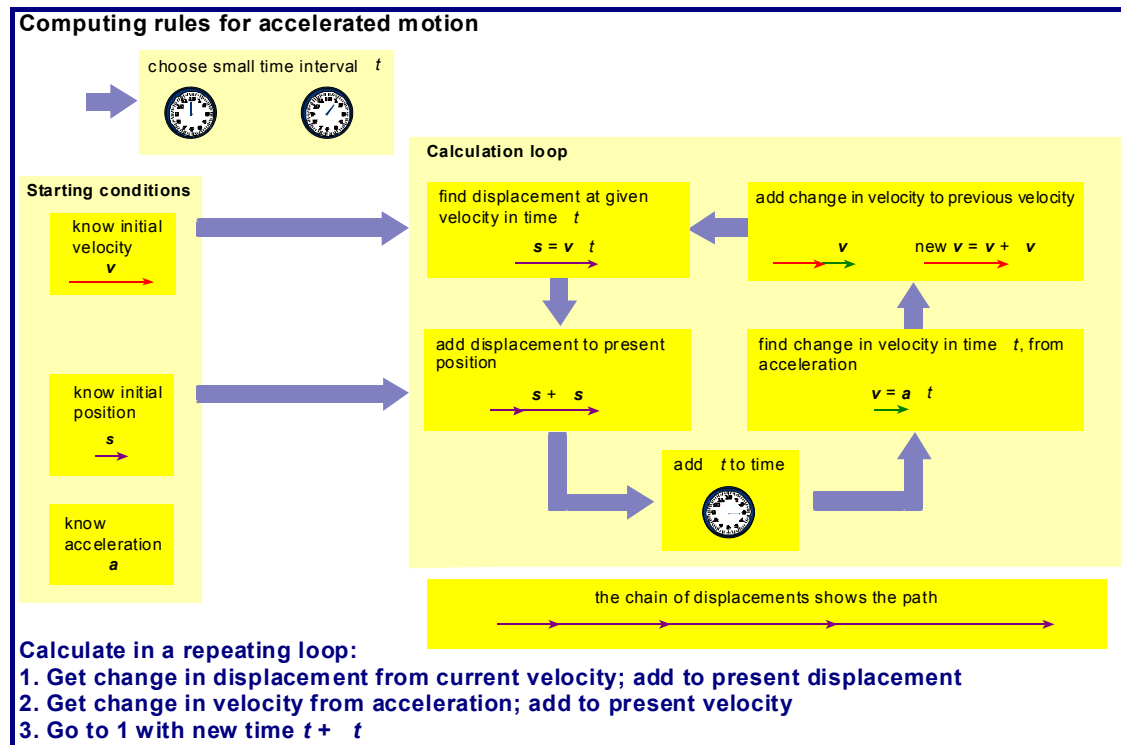
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Graphs for realistic motion



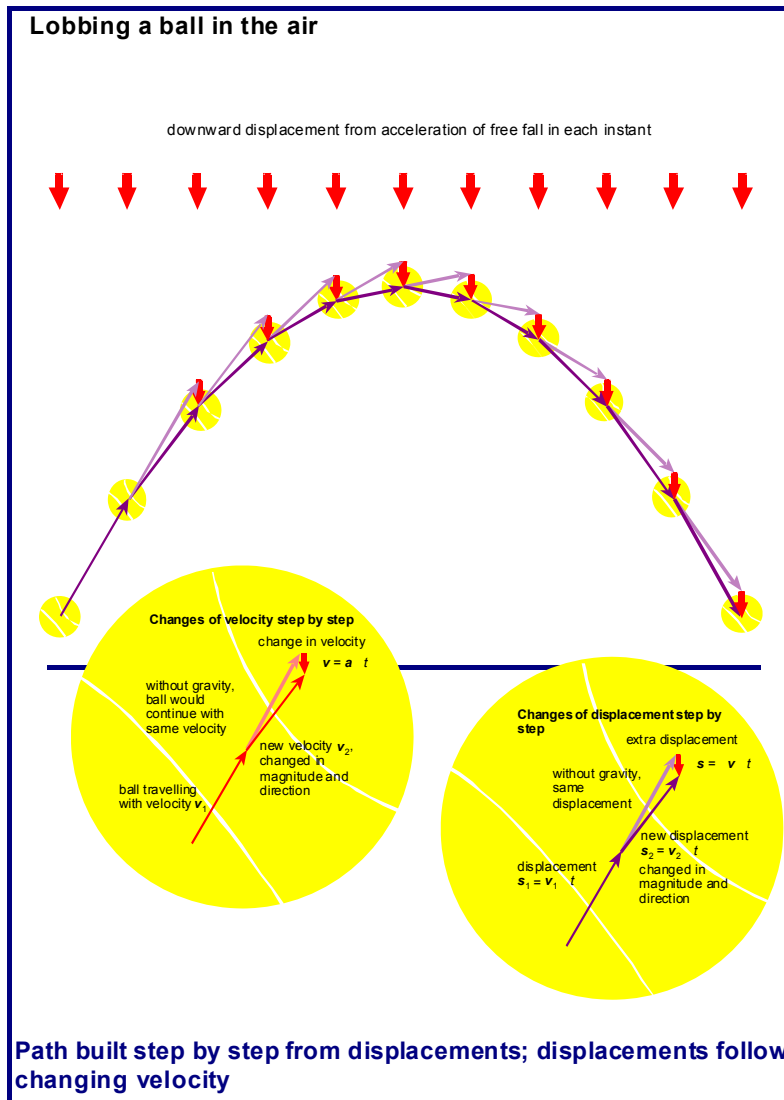
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Computing uniform acceleration



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A parabola from steps



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Calculating kinetic energy

Kinetic energy = $\frac{1}{2} mv^2$

Force × time

$F = ma$
 $v = at$

Force × time = mass × acceleration × time
acceleration × time = velocity

Force × time = mv

mv is called momentum

Force × displacement

distance = average velocity × time
average velocity = $v/2$

Force × distance
= force × average velocity × time
= force × time × $v/2$

force × displacement = $mv \times v/2$

Force × displacement = $\frac{1}{2} mv^2$

$\frac{1}{2} mv^2$ is called kinetic energy

Momentum mv says
how big a force is needed to stop in a given time

Kinetic energy $\frac{1}{2} mv^2$ says
how big a force is needed to stop in a given distance

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Kinetic and potential energy

Force × distance under gravity

mass m initial velocity = 0
 height h
 acceleration g
 force of gravity mg
 velocity v
 force × distance = mgh
 Falls with constant acceleration:
 average velocity = $\frac{1}{2}v$
 $v = gt$, so time $t = v/g$
 distance = average velocity × time
 $h = \frac{1}{2}v \times (v/g)$
 $gh = \frac{1}{2}v^2$
 $gh = \frac{1}{2}v^2$
 $mgh = \frac{1}{2}mv^2$

Potential energy **Kinetic energy**

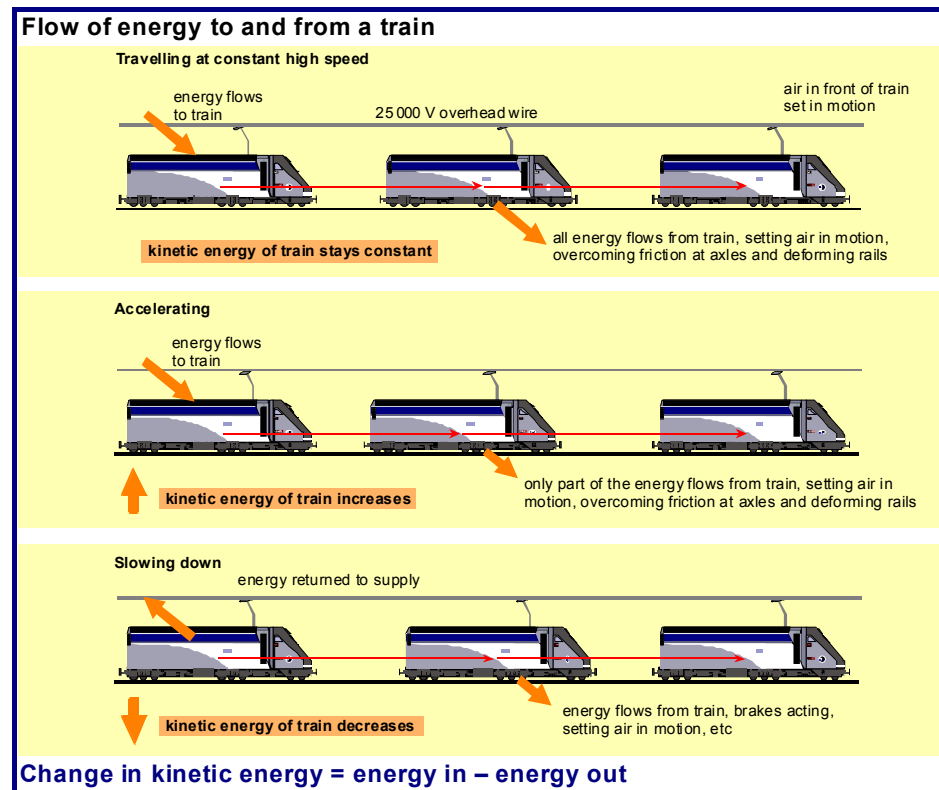
mgh work = force × distance = energy transfer $\frac{1}{2}mv^2$

Energy comes from the gravitational field: decrease in potential energy = mgh
 Energy now carried by motion of ball: increase in kinetic energy = $\frac{1}{2}mv^2$
 Decrease in potential energy = increase in kinetic energy

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Flow of energy to a train

Going at a steady speed means that as much energy flows in every second as flows out. If more energy flows in than out then the train speeds up. If more energy flows out than in then the train slows down.



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Power, force and velocity

Power = force × velocity

Travelling at constant high speed
 energy flows to train
 25000 V overhead wire
 air in front of train set in motion
 effective drag force F
 equal energy flows from train, setting air in motion, overcoming friction at axles and deforming rails
 velocity v
 time t
 distance travelled $s = vt$

Eurostar has two locomotives each 5.6 MW
 speed 140 km h^{-1}
 $= 39 \text{ m s}^{-1}$

Power used
 Energy from train to surroundings = drag force × distance moved
 $E = Fs$
 Power used by train = rate of dissipation of energy
 $P = \frac{E}{t}$
 $P = \frac{Fs}{t}$ with $s = vt$
 $P = Fv$
Power used by train = drag force × velocity

power in	power out
two locomotives each 5.6 MW	maximum speed 140 km per hour $v = 39 \text{ m s}^{-1}$
$P = 2 \times 5.6 \text{ MW} = 11.2 \text{ MW}$	power in = power out $P = Fv$ $11.2 \times 10^6 \text{ W} = F \times 39 \text{ m s}^{-1}$ $F = 290 \text{ kN}$

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