Revision Guide for Chapter 8

Contents

Revision Checklist

Back to list of Contents

I can show my understanding of effects, ideas and relationships by describing and explaining:

that a vector has both size and direction, while a scalar just has size Revision Notes: scalar quantity, vectors, vector components	
how to use vectors to represent displacement and velocity Revision Notes: displacement, velocity and speed Summary Diagrams: Average and instantaneous speed	
that vectors can be broken down into components acting in different directions, and that the effect of the original vector isn't changed by doing this Revision Notes: vectors, vector components Summary Diagrams: Components of a vector by projection, Components of a vector from angles	
that vectors can be added to find a resultant vector Revision Notes: vector addition	
how graphs of speed–time and distance–time can describe the movement of an object Revision Notes: distance–time graph, speed–time graph	
that speed is distance / time: $v = \frac{s}{t}$ velocity is displacement / time: $\boldsymbol{v} = \frac{\boldsymbol{s}}{t}$ Revision Notes: displacement, velocity and speed Summary Diagrams: Average and instantaneous speed	
the meaning of average and instantaneous speed Revision Notes: velocity and speed Summary Diagrams: Average and instantaneous speed	
the meaning of the area under a speed–time graph Revision Notes: speed–time graph Summary Diagrams: Average and instantaneous speed	

I can use the following words and phrases accurately when describing the motion of objects:

vector, scalar displacement and velocity as vectors distance and speed as scalars	

component of a vector average speed, instantaneous speed Revision Notes: scalar quantity, vectors, vector components, displacement, velocity and speed Summary Diagrams: Components of a vector by projection, Components of a vector from angles, Average and instantaneous speed	
gradient, slope, tangent (with respect to graphs) Revision Notes: distance–time graph, speed–time graph	

I can interpret:

vector diagrams showing components and the addition of vectors Revision Notes: vector addition	
graphs of speed–time and distance–time for the movement of an object Revision Notes: distance–time graph, speed–time graph	

I can calculate:

the components of a vector acting at right angles to each other by (i) making scale drawings **and** (ii) using geometry or trigonometry Revision Notes: vector components Summary Diagrams: Components of a vector by projection, Components of a vector from angles	
the resultant vector produced by adding two vectors the resultant vector produced by subtracting one vector from another Revision Notes: vector addition	
the unknown quantity given any two in the equation $v = \frac{s}{t}$ Revision Notes: displacement, velocity and speed	
speed from the gradient (slope) of a distance–time graph distance from the area under a speed–time graph Revision Notes: distance–time graph, speed–time graph	

I can show my ability to make better measurements by:

measuring speed, distance and time with known uncertainty Revision Notes: accuracy and precision, systematic error, uncertainty	

Revision Notes

Back to list of Contents

Scalar quantity

A scalar quantity is any physical quantity that can be represented by a single numerical value.

Vector quantities have components, which specify their direction as well as their magnitude.

Examples of scalar quantities include distance, speed, mass, energy, work, power, temperature, charge and potential. A scalar quantity is completely specified by its magnitude at a given position or time.

The magnitude of a vector is a scalar quantity.

Many scalar quantities are the product or the quotient of two other scalar quantities. For example, power = energy transferred divided by time taken, which is a quotient of two scalar quantities.

A vector multiplied or divided by a scalar quantity is a vector. Examples: $\boldsymbol{v} = \boldsymbol{a}t$, $\boldsymbol{v} = \boldsymbol{s} / t$

Some scalar quantities are the product of two vector quantities. For example, the work W done by a force of magnitude F which moves its point of application through a distance s is given by the equation $W = F s \cos \theta$, where θ is the angle between the force vector and the displacement vector.

Back to Revision Checklist

Vectors

Vector quantities are used to represent physical quantities that have both magnitude and direction. Displacements, velocities, accelerations and forces are examples of vector quantities.

Vectors are often represented by the length and direction of an arrow. The length of the arrow is proportional to the magnitude of the vector. For example, the displacement from the Channel Tunnel exit to Paris is 230 km nearly due South. It can be represented on a map by an arrow 230 mm long pointing (say) downwards.

Notice that the magnitude of a vector is unchanged by rotating the frame of reference (the distance to Paris stays the same if you turn the map around).

A vector quantity is a package of similar quantities bundled together. It is made of components. The components of the sum of two vectors are equal to the sums of the components of the vectors. A two-dimensional vector has two components. A vector in three dimensions needs three components.

A bold type symbol e.g. $\boldsymbol{s}$ is used to refer to the whole vector quantity.

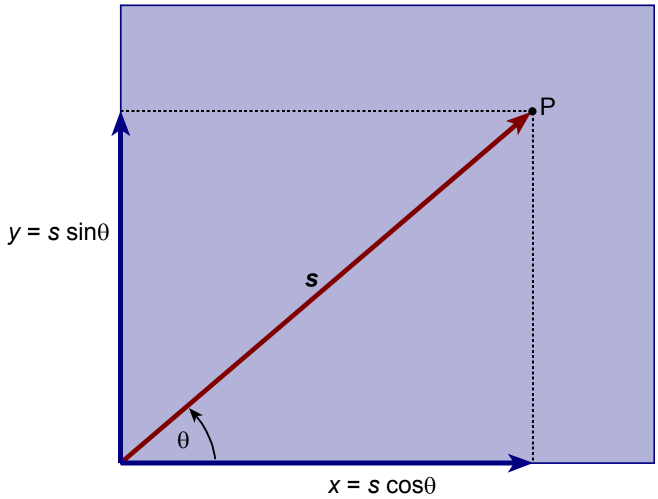
Back to Revision Checklist

Vector components

The components of a vector are projections of the vector on to any two perpendicular axes.

The process of expressing a vector of known magnitude and direction as perpendicular components is called **resolving** the vector.

Vector components



Components x and y of the vector $\boldsymbol{s}$ are $x = s \cos \theta$ and $y = s \sin \theta$, where θ is the angle between the vector $\boldsymbol{s}$ and the x-axis. By Pythagoras' theorem the magnitude s of the vector $\boldsymbol{s}$ is given by

$s = \sqrt{(x^2 + y^2)}$.

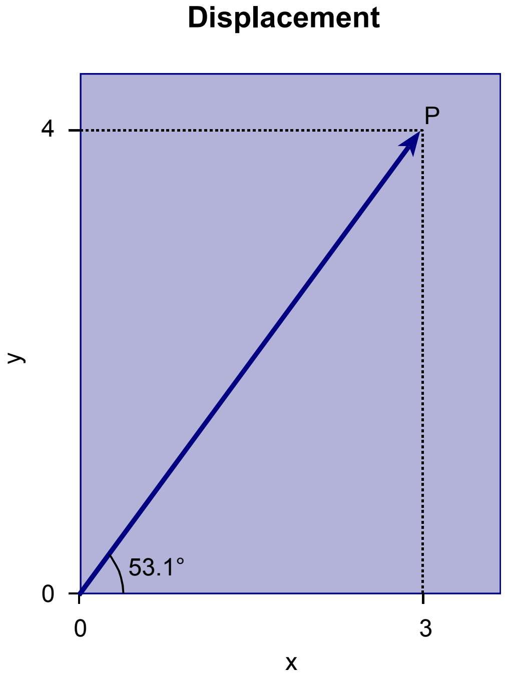
Back to Revision Checklist

Displacement

Displacement is distance moved in a given direction. Displacement is a **vector quantity**.

The displacement of an object moved from one point to another can be represented as a vector arrow pointing from the starting point to the finishing point. Displacement can be expressed as components in two perpendicular directions.

Distance is a scalar quantity, indicating the length along a path, without regard to the shape or direction of the path. Thus an object going once around a complete circle radius r travels a distance $2\pi r$, but has zero net displacement.



For example, using *x* and *y* coordinates, a displacement (3 m, 4 m) from the origin O of the coordinates would be represented by a vector arrow from the origin to point P which has coordinates x = 3 m, y = 4 m.

1. The distance from O to P = $\sqrt{3^2 + 4^2}$ = 5 m.
2. The direction from O to P is 53.1° above the *x*-axis, the angle whose tangent is 4/3.

Back to Revision Checklist

Velocity and speed

The velocity of an object is its rate of change of displacement. The unit of velocity is the metre per second (m s^{-1}). Velocity is a **vector quantity**.

Thus the vector velocity $\boldsymbol{v} = \boldsymbol{s} / t$, where $\boldsymbol{s}$ is the vector displacement and t is the time taken. The direction of the velocity is the direction of the displacement.

The **speed** of an object is the distance it travels per unit time. Speed, like distance, is a scalar quantity. The speed of an object is the magnitude of its velocity.

If an object travels in a straight line, its velocity changes only if its speed changes. But an object travelling along a curved path changes its velocity because the direction of motion changes.

The **average speed** of an object over a distance *s*, $v_{av} = s / t$, where t is the time taken.

The **instantaneous speed** of an object is the rate of change of distance moved at a given instant. This is usually written as $v = \mathrm{d}s / \mathrm{d}t$.

Back to Revision Checklist

Vector addition

Vector addition is the addition of two vectors. It can be done simply by adding the components of the two vectors to obtain the components of the resultant vector. It can be achieved without breaking the vectors into components by adding the two vectors ‘tip-to-tail’ using a scale drawing.

Two vectors in the **same** direction add to give a resultant vector in the same direction and of magnitude equal to the sum of the individual magnitudes.

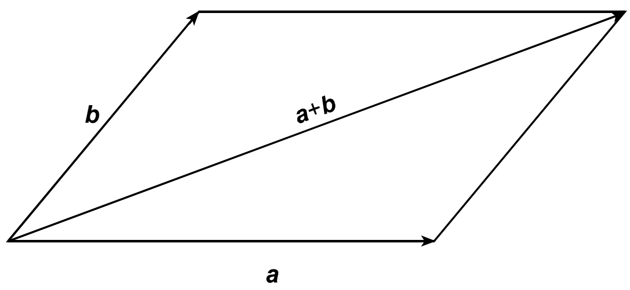
Two vectors in **opposite** directions add to give a resultant vector in the direction of the larger vector and of magnitude equal to the difference of the individual magnitudes.

Adding vectors in different directions

The **parallelogram rule** for adding two vectors is an accurate geometrical method of finding the resultant of two vectors. The two vectors are drawn so they form adjacent sides of a parallelogram. The resultant is the diagonal of the parallelogram from the start of one vector to the end of the other vector.

Vectors add 'tip-to-tail' so the resultant vector is from the start of one vector to the end of the other vector.

The parallelogram rule



Adding components

Two vectors may be added by adding their components. For example, if the components of $\boldsymbol{A}$ and $\boldsymbol{B}$ with respect to the x-, y-axes are:

$\boldsymbol{A} = (A_x, A_y)$

$\boldsymbol{B} = (B_x, B_y)$

then

$\boldsymbol{A} + \boldsymbol{B} = (A_x + B_x, A_y + B_y)$.

The components of the vector sum are the sums of the components of the added vectors.

Back to Revision Checklist

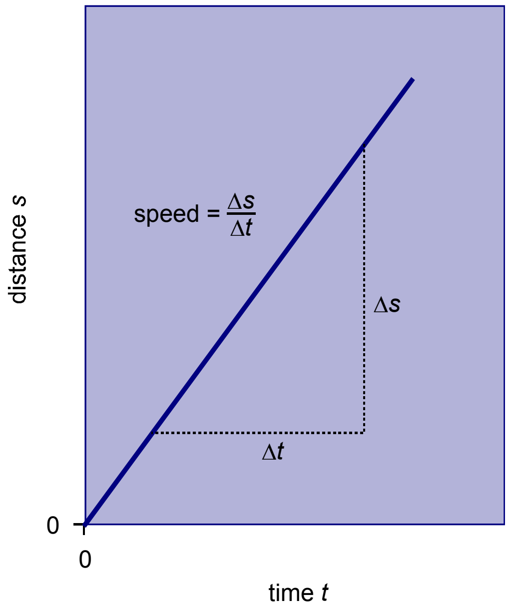
Distance–time graph

A distance–time graph is a graph of distance on the y-axis against time on the x-axis. The progress of an object from a fixed point O can be shown as a graph of distance from O on the y-axis against time on the x-axis.

The gradient at any point of a distance–time graph is equal to the speed at that point.

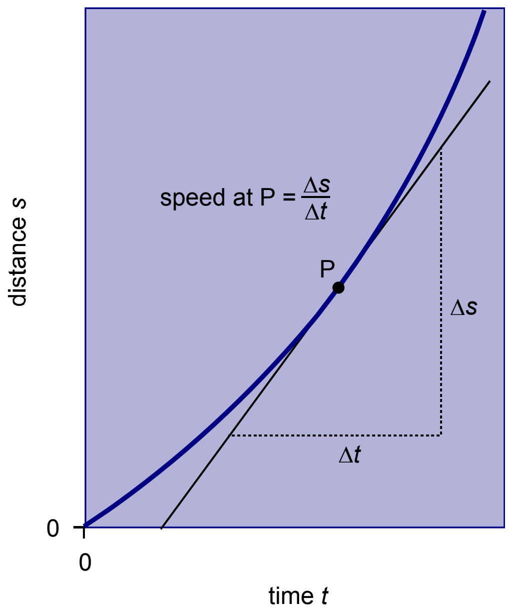
The speed of the object is the rate of change of distance with time. If the line on the graph representing the progress of the object is straight and not flat, the object must be moving at constant speed. The gradient of the line represents the speed.

Distance-time graphs



constant speed

To find the speed at a point on a distance–time graph where the line is curved, the tangent to the curve is drawn at the relevant point. The gradient of the tangent is equal to the gradient of the curve at that point.



increasing speed

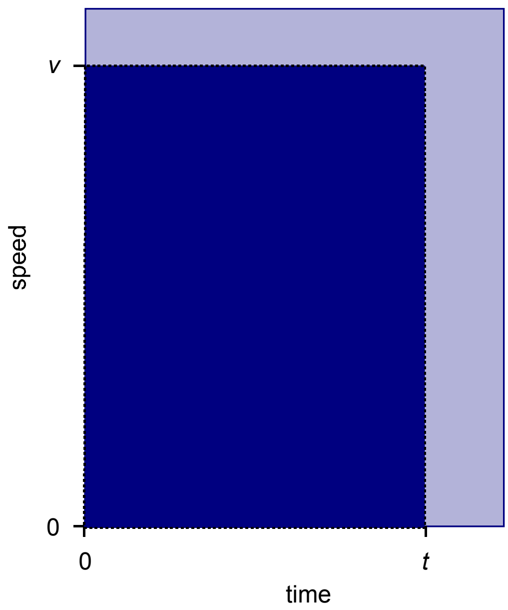
Back to Revision Checklist

Speed–time graph

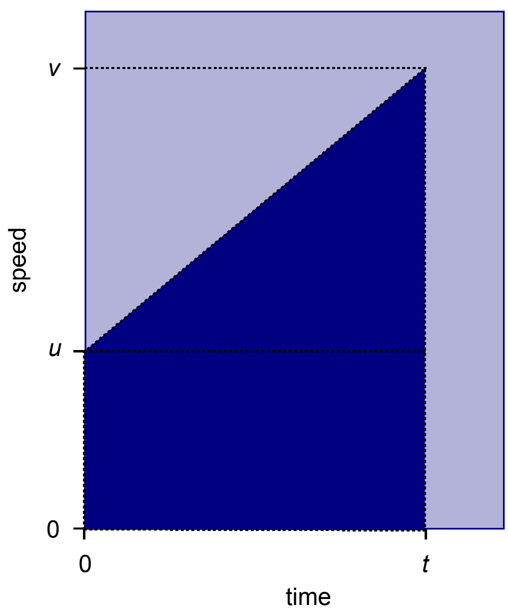
A speed–time graph is a graph of speed on the y-axis against time on the x-axis.

A speed–time graph is a useful way of showing how the motion of an object changes. In addition, the acceleration and distance travelled can be worked out from the graph.

Speed–time graphs



area in dark blue represents distance $s = vt$



area in dark blue represents distance $s = \frac{1}{2}(u+v)t$

In general, at any given time, the acceleration of an object $a = \mathrm{d}v / \mathrm{d}t$ where v is its speed at that time. The gradient of a speed–time graph at any given point is equal to the acceleration at that point. The gradient can be determined by measuring the gradient of the tangent to the curve at that point.

The area between the graph and the *x*-axis between any two points in time gives the distance travelled in that time interval.

For constant speed *v*, the distance moved in time *t*, $s = vt$ which is represented on the graph by a rectangular area *vt*.

Back to Revision Checklist

Accuracy and precision

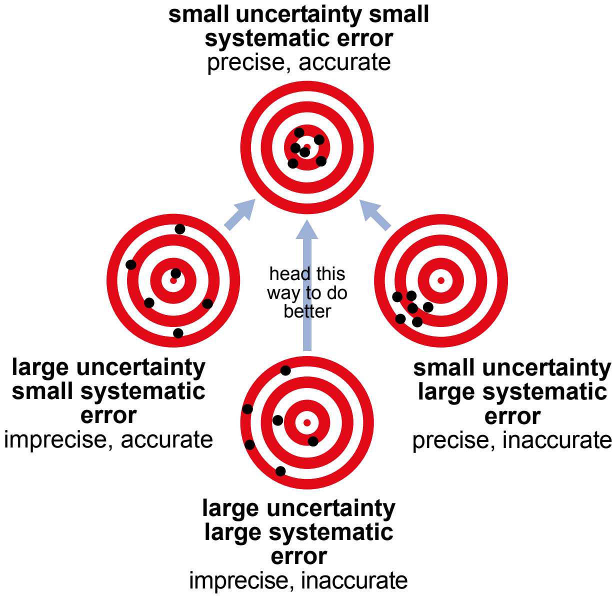
A measurement is accurate if it is close to the true value. A measurement is precise if values cluster closely, with small uncertainty.

A watch with an accuracy of 0.1% could be up to five minutes astray within a few days of being set. A space probe with a trajectory accurate to 0.01 % could be more than 30 km off target at the Moon.

Think of the true value as like the bullseye on a target, and measurements as like arrows or darts aimed at the bullseye.

Uncertainty and systematic error

Think of measurements as shots on a target. Imagine the ‘true value’ is at the centre of the target



An accurate set of measurements is like a set of hits that centre on the bullseye. In the diagram above at the top, the hits also cluster close together. The uncertainty is small. This is a measurement that gives the true result rather precisely.

On the left, the accuracy is still good (the hits centre on the bullseye) but they are more scattered. The uncertainty is higher. This is a measurement where the average still gives the true result, but that result is not known very precisely.

On the right, the hits are all away from the bullseye, so the accuracy is poor. But they cluster close together, so the uncertainty is low. This is a measurement that has a systematic error, giving a result different from the true result, but where other variations are small.

Finally, at the bottom, the accuracy is poor (systematic error) and the uncertainty is large.

A statement of the result of a measurement needs to contain two distinct estimates:

1. The best available estimate of the value being measured.
2. The best available estimate of the range within which the true value lies.

Note that both are statements of belief based on evidence, not of fact.

For example, a few years ago discussion of the 'age-scale' of the Universe put it at 14 plus or minus 2 thousand million years. Earlier estimates gave considerably smaller values but with larger ranges of uncertainty. The current (2008) estimate is 13.7 ± 0.2 Gy. This new value lies within the range of uncertainty for the previous value, so physicists think the estimate has been improved in precision but has not fundamentally changed.

Fundamental physical constants such as the charge of the electron have been measured to an astonishing small uncertainty. For example, the charge of the electron is $1.602\,173\,335 \times 10^{-19}$ C to an uncertainty of $0.000\,000\,005 \times 10^{-19}$ C, better than nine significant figures.

There are several different reasons why a recorded result may differ from the true value:

1. **Constant systematic bias**, such as a zero error in an instrument, or an effect which has not been allowed for.

   Constant systematic errors are very difficult to deal with, because their effects are only observable if they can be removed. To remove systematic error is simply to do a better experiment. A clock running slow or fast is an example of systematic instrument error. The effect of temperature on the resistance of a strain gauge is an example of systematic experimental error.

2. **Varying systematic bias**, or drift, in which the behaviour of an instrument changes with time, or an outside influence changes.

   Drift in the sensitivity of an instrument, such as an oscilloscope, is quite common in electronic instrumentation. It can be detected if measured values show a systematic variation with time. Another example: the measured values of the speed of light in a pipe buried in the ground varied regularly twice a day. The cause was traced to the tide coming in on the nearby sea-shore, and compressing the ground, shortening the pipe a little.

3. **Limited resolution of an instrument**. For example the reading of a digital voltmeter may change from say 1.25 V to 1.26 V with no intermediate values. The true potential difference lies in the 0.01 V range 1.25 V to 1.26 V.

   All instruments have limited resolution: the smallest change in input which can be detected. Even if all of a set of repeated readings are the same, the true value is not exactly equal to the recorded value. It lies somewhere between the two nearest values which can be distinguished.

4. **Accidental momentary effects**, such as a 'spike' in an electrical supply, or something hitting the apparatus, which produce isolated wrong values, or 'outliers'.

   Accidental momentary errors, caused by some untoward event, are very common. They can often be traced by identifying results that are very different from others, or which depart from a general trend. The only remedy is to repeat them, discarding them if further measurements strongly suggest that they are wrong. Such values should never be included in any average of measurements, or be used when fitting a line or curve.

5. **Human errors**, such as misreading an instrument, which produce isolated false recorded values.

Human errors in reading or recording data do occur, such as placing a decimal point wrongly, or using the wrong scale of an instrument. They can often be identified by noticing the kinds of mistake it is easy to make. They should be removed from the data, replacing them by repeated check observations.

6. **Random fluctuations**, for example noise in a signal, or the combined effect of many unconnected minor sources of variation, which alter the measured value unpredictably from moment to moment.

   Truly random variations in measurements are rather rare, though a number of unconnected small influences on the experiment may have a net effect similar to random variation. But because there are well worked out mathematical methods for dealing with random variations, much emphasis is often given to them in discussion of the estimation of the uncertainty of a measurement. These methods can usually safely be used when inspection of the data suggests that variations around an average or a fitted line or curve are small and unsystematic. It is important to look at visual plots of the variations in data before deciding how to estimate uncertainties.

Back to Revision Checklist

Systematic error

Systematic error is any error that biases a measurement away from the true value.

All measurements are prone to systematic error. A systematic error is any biasing effect, in the environment, methods of observation or instruments used, which introduces error into an experiment. For example, the length of a pendulum will be in error if slight movement of the support, which effectively lengthens the string, is not prevented, or allowed for.

Incorrect zeroing of an instrument leading to a **zero error** is an example of systematic error in instrumentation. It is important to check the zero reading during an experiment as well as at the start.

Systematic errors can change during an experiment. In this case, measurements show trends with time rather than varying randomly about a mean. The instrument is said to show **drift** (e.g. if it warms up while being used).

Systematic errors can be reduced by checking instruments against known standards. They can also be detected by measuring already known quantities.

The problem with a systematic error is that you may not know how big it is, or even that it exists. The history of physics is littered with examples of undetected systematic errors. The only way to deal with a systematic error is to identify its cause and either calculate it and remove it, or do a better measurement which eliminates or reduces it.

Back to Revision Checklist

Uncertainty

The uncertainty of an experimental result is the range of values within which the true value may reasonably be believed to lie. To estimate the uncertainty, the following steps are needed.

1. Removing from the data **outlying** values which are reasonably suspected of being in serious error, for example because of human error in recording them correctly, or because of an unusual external influence, such as a sudden change of supply voltage. Such values should not be included in any later averaging of results or attempts to fit a line or curve to relationships between measurements.
2. Estimating the possible magnitude of any **systematic error**. An example of a constant systematic error is the increase in the effective length of a pendulum because the string's

support is able to move a little as the pendulum swings. The sign of the error is known (in effect increasing the length) and it may be possible to set an upper limit on its magnitude by observation. Analysis of such systematic errors points the way to improving the experiment.

3. Assessing the **resolution** of each instrument involved, that is, the smallest change it can detect. Measurements from it cannot be known to less than the range of values it does not distinguish.
4. Assessing the magnitude of other small, possibly random, unknown effects on each measured quantity, which may include human factors such as varying speed of reaction. Evidence of this may come from the spread of values of the measurement conducted under what are as far as possible identical conditions. The purpose of repeating measurements is to decide how far it appears to be possible to hold conditions identical.
5. Determining the combined effect of possible **uncertainty** in the result due to the limited resolution of instruments (3 above) and uncontrollable variation (4 above).

To improve a measurement, it is essential to identify the largest source of uncertainty. This tells you where to invest effort to reduce the uncertainty of the result.

Having eliminated accidental errors, and allowed for systematic errors, the range of values within which the true result may be believed to lie can be estimated from (a) consideration of the resolution of the instruments involved and (b) evidence from repeated measurements of the variability of measured values.

Most experiments involve measurements of more than one physical quantity, which are combined to obtain the final result. For example, the length L and time of swing T of a simple pendulum may be used to determine the local acceleration of free fall, g , using

$$T = 2\pi\sqrt{\frac{L}{g}}$$

so that

$$g = \frac{4\pi^2 L}{T^2}.$$

The range in which the value of each quantity may lie needs to be estimated. To do so, first consider the resolution of the instrument involved – say ruler and stopwatch. The uncertainty of a single measurement cannot be better than the resolution of the instrument. But it may be worse. Repeated measurements under supposedly the same conditions may show small and perhaps random variations.

If you have repeated measurements, 'plot and look', to see how the values vary. A simple estimate of the variation is the $\text{spread} = \pm \frac{1}{2}\,\text{range}$.

A simple way to see the effect of uncertainties in each measured quantity on the final result is to recalculate the final result, but adding or subtracting from the values of variables the maximum possible variation of each about its central value. This is pessimistic because it is unlikely that 'worst case' values all occur together. However, pessimism may well be the best policy: physicists have historically tended to underestimate uncertainties rather than overestimate them. The range within which the value of a quantity may reasonably be believed to lie may be reduced somewhat by making many equivalent measurements, and averaging them. If there are N independent but equivalent measurements, with range R, then the range of their average is likely to be approximately R divided by the factor $\sqrt{N}$. These benefits are not automatic, because in collecting many measurements conditions may vary.

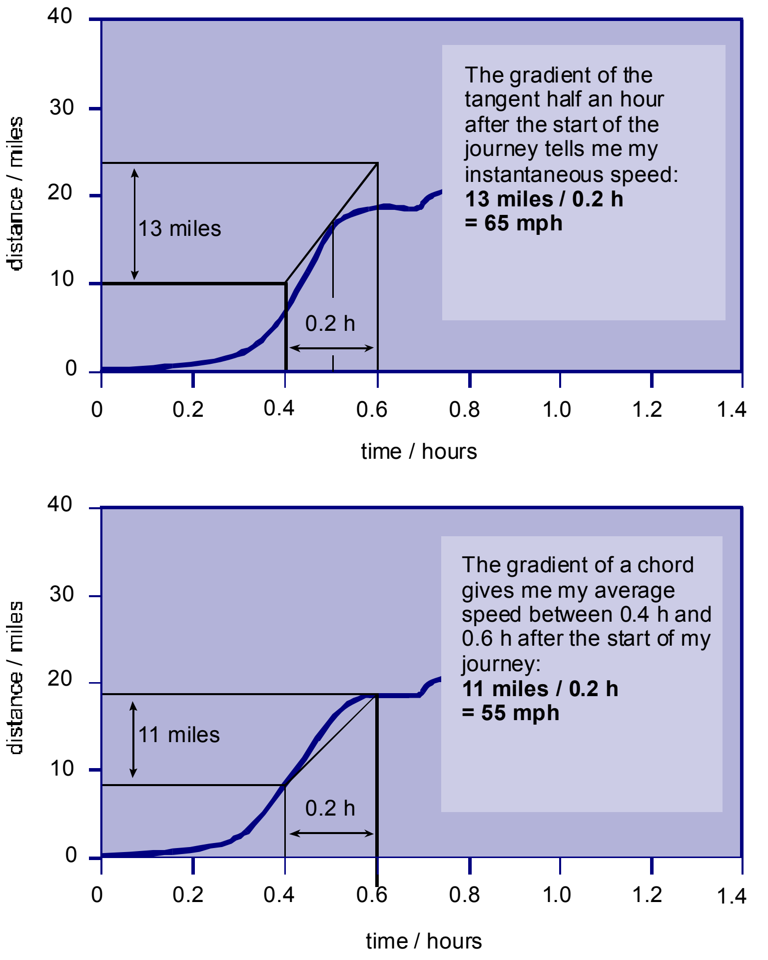
Back to Revision Checklist

Summary Diagrams

Back to list of Contents

Average and instantaneous speed

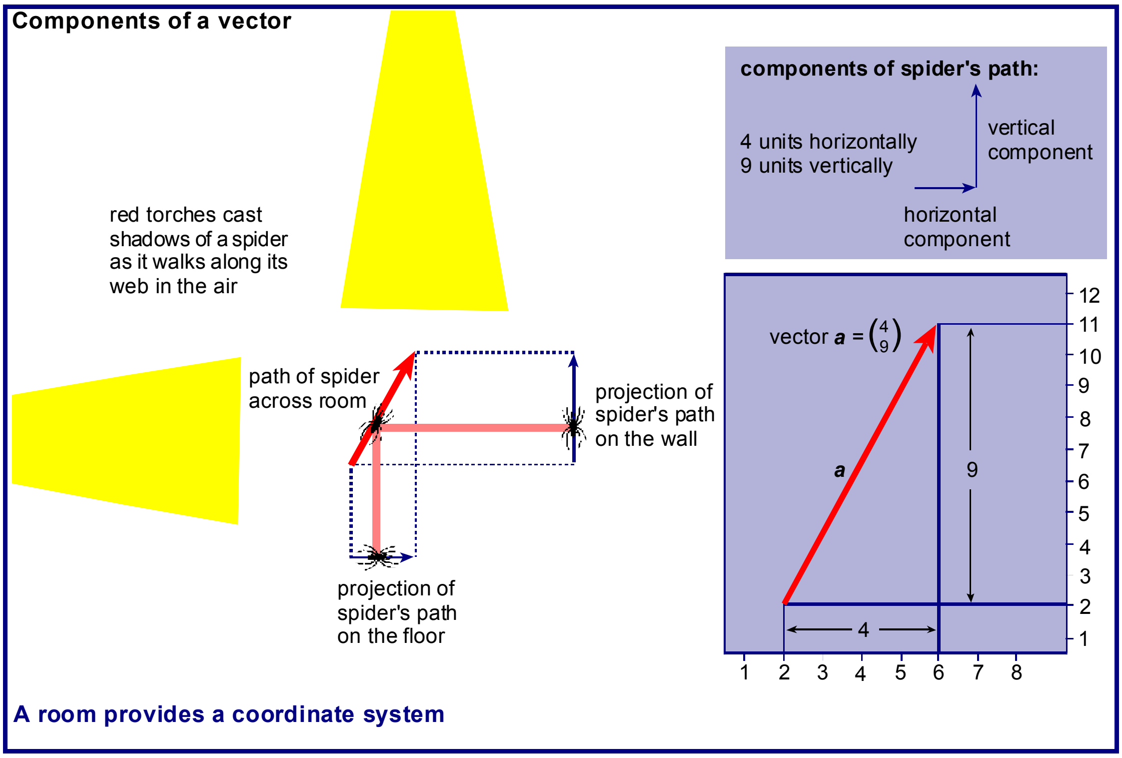
Average and instantaneous speed



Average and instantaneous speed can both be calculated from the gradient of a straight line

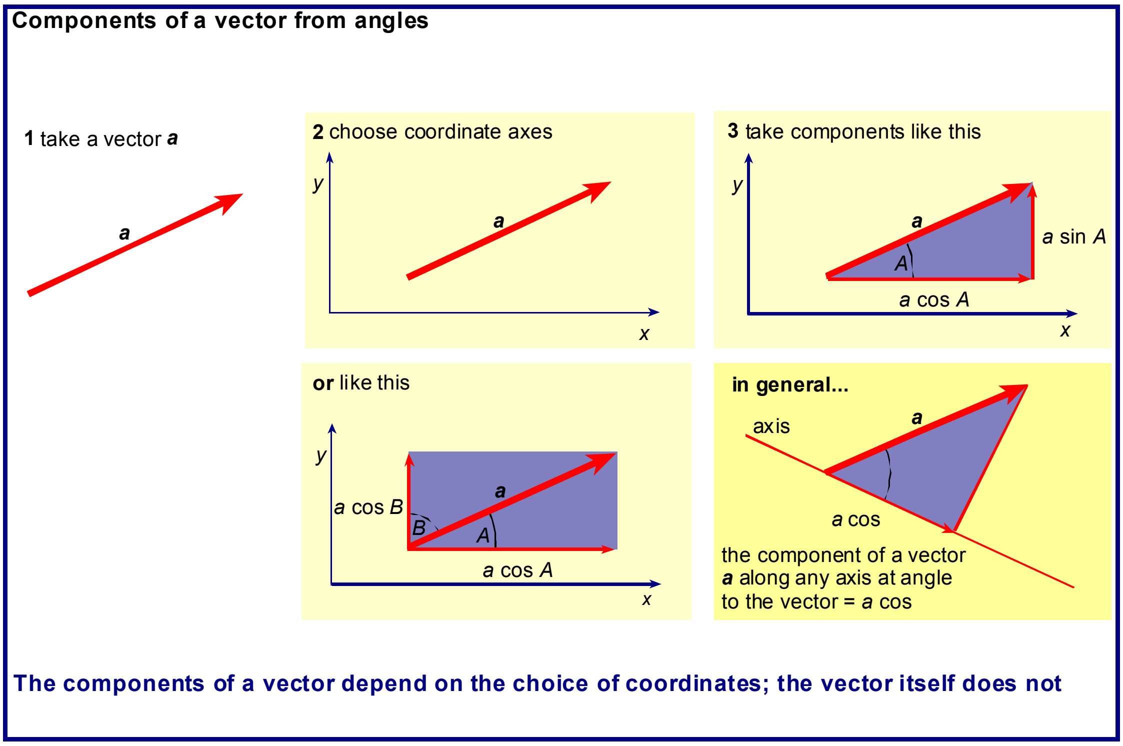
Back to Revision Checklist

Components of a vector by projection



Back to Revision Checklist

Components of a vector from angles



Back to Revision Checklist

Back to list of Contents