Revision Guide for Chapter 6

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I can show my understanding of effects, ideas and relationships by describing and explaining:

how standing waves are formed by sets of wave travelling in opposite directions *e.g. by drawing diagrams to show what happens*

Revision Notes: <u>amplitude, frequency, wavelength and wave speed</u>, <u>travelling and standing</u> <u>waves</u>

Summary Diagrams: Standing waves, Standing waves on a guitar, Standing waves in pipes

how waves passing through two slits combine and interfere (superpose) to produce a wave / no-wave pattern

Revision Notes: interference, path difference, double-slit interference, superposition

Summary Diagrams: <u>Two-slit interference</u>

what happens when waves pass through a single narrow gap (diffraction)

Revision Notes: diffraction, phase and phasors

Summary Diagrams: Diffraction

how a diffraction grating works in producing a spectrum

Revision Notes: gratings and spectra, phase and phasors

Summary Diagrams: <u>A transmission grating</u>

I can use the following words and phrases accurately when describing effects and observations:

wave, standing wave, frequency, wavelength, amplitude, phase, phasor

Revision Notes: <u>amplitude, frequency, wavelength and wave speed</u>, <u>travelling and standing</u> <u>waves</u>

Summary Diagrams: Phase and angle, Adding oscillations using phasors

path difference, interference, diffraction, superposition, coherence

Revision Notes: interference, path difference, double-slit interference, diffraction, superposition, coherence

Summary Diagrams: <u>Two-slit interference</u>, <u>Coherence</u>

I can sketch and interpret diagrams:

illustrating the propagation of waves

Revision Notes: <u>amplitude, frequency, wavelength and wave speed</u>, <u>travelling and standing</u> <u>waves</u>

showing how waves propagate in two dimensions using Huygens' wavelets

Revision Notes: <u>Huygen's wavelets</u>

showing how waves that have travelled to a point by different paths combine to produce the wave amplitude at that point

i.e. by adding together the different phases of the waves, using phasors

Revision Notes: interference, double-slit interference, diffraction, gratings and spectra

Summary Diagrams: Two-slit interference, A transmission grating

I can calculate:

wavelengths, wave speeds and frequencies by using (and remembering) the formula $v = f \lambda$	
Revision Notes: amplitude, frequency, wavelength and wave speed	
wavelengths of standing waves e.g. in a string or a column of air	
Revision Notes: travelling and standing waves	
Summary Diagrams: Standing waves, Standing waves on a guitar, Standing waves in pipes	
path differences for waves passing through double slits and diffraction gratings	
Revision Notes: interference, double-slit interference, diffraction, gratings and spectra	
Summary Diagrams: Two-slit interference, A transmission grating	
the unknown quantity when given other relevant data in using the formula $n \lambda = d \sin \theta$	
Revision Notes: gratings and spectra	
Summary Diagrams: <u>A transmission grating</u>	

I can show my ability to make better measurements by:

measuring the wavelength of light

Revision Notes: accuracy and precision, systematic error, uncertainty

I can show an appreciation of the growth and use of scientific knowledge by:

commenting on how and why ideas about the nature of light have changed

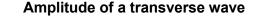
Revision Notes

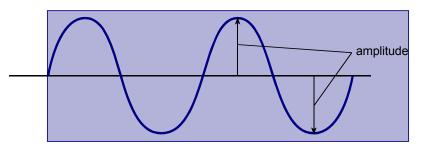
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Amplitude, frequency, wavelength and wave speed

Waves are characterised by several related parameters: amplitude (how big they are); frequency (how rapidly they oscillate); wavelength (the distance over which they repeat) and their speed of travel.

The amplitude of a wave at a point is the maximum displacement from some equilibrium value at that point.





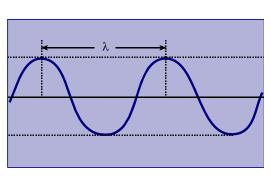
The period T of an oscillation is the time taken for one complete oscillation.

The frequency f of an oscillation is the number of complete cycles of oscillation each second.

The SI unit of frequency is the hertz (Hz), equal to one complete cycle per second.

The wavelength λ of a wave is the distance along the direction of propagation between adjacent points where the motion at a given moment is identical, for example from one wave crest to the next.

The SI unit of wavelength is the metre.



Wavelength

Relationships

Frequency f and period T

$$f = \frac{1}{T}$$
$$T = \frac{1}{f}$$

Frequency *f* , wavelength λ and wave speed *v*

$$v = f\lambda$$

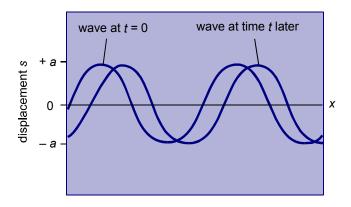
Displacement *s* at any one point in a wave, where ϕ is the phase.

 $s = A \sin(2\pi f t + \phi)$

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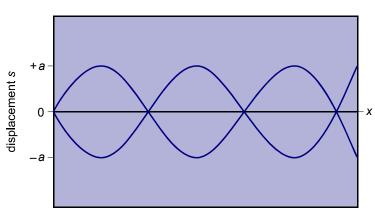
Travelling and standing waves

Travelling waves propagate through space or through a substance.



A travelling wave

Stationary or **standing** waves are produced when travelling waves of the same frequency and amplitude pass through one another in opposing directions.



A standing wave

The resultant wave has the same frequency of oscillation at all points. The wave does not travel. Its amplitude varies with position. Positions of minimum amplitude are called displacement nodes and positions of maximum amplitude are called displacement antinodes.

Nodes and antinodes alternate in space. The nodes (and the antinodes) are half a wavelength apart.

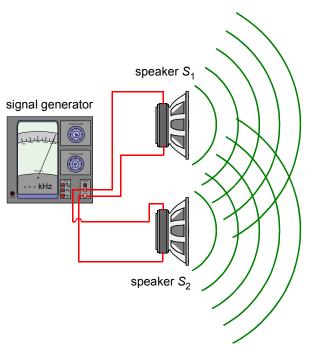
Standing waves are an example of **wave superposition**. The waves on a guitar string or in an organ pipe are standing waves.

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Interference

When waves overlap, the resultant displacement will be equal to the sum of the individual displacements at that point and at that instant (if the waves superpose linearly).

Interference is produced if waves from two coherent sources overlap or if waves from a single source are divided and then reunited.



Interference of sound

Interference using sound waves can be produced by two loudspeakers connected together to an oscillator. If you move about where the waves overlap you will detect points of reinforcement (louder) and of cancellation (quieter).

Another way to produce interference is to divide the waves from one source and then recombine them. The diagram below shows this being done for microwaves, sending part of the wave along one path and part along another. The receiver gives a minimum response when the paths differ by half a wavelength.

Division of amplitude metal reflector microwave transmitter hardboard plate as a semi-reflective mirror microwave receiver

Other examples of interference include the 'blooming' of camera lenses, and the colours of oil films and soap bubbles.

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Path difference

The path difference between two waves will determine what happens when they superpose.

If the path difference between two wavefronts is a whole number of wavelengths, the waves reinforce.

If the path difference is a whole number of wavelengths plus or minus one half of a wavelength, the waves cancel.

The importance of a path difference is that it introduces a time delay, so that the phases of the waves differ. It is the difference in phase that generates the superposition effects.

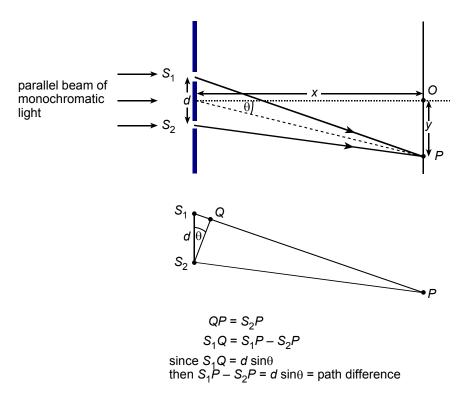
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Double-slit interference

The double-slit experiment with light requires light from a narrow source to be observed after passing through two closely spaced slits. A pattern of alternate bright and dark fringes is observed.

In the diagram below the path difference from the two slits to a point P on the screen is equal to $d \sin \theta$, where d is the spacing between the slit centres and θ is the angle between the initial direction of the beam and the line from the centre of the slits to the point P.

Double slit interference



Bright fringes: the waves arriving at P **reinforce** if the path difference is a whole number of wavelengths, i.e. $d \sin \theta = n \lambda$, where λ is the wavelength of the light used and *n* is an integer.

Dark fringes: the waves arriving at P **cancel** if the path difference is a whole number of wavelengths plus a half wavelength, i.e. $d \sin \theta = (n + \frac{1}{2}) \lambda$, where λ is the wavelength of the light used and *n* is an integer.

The angle $\sin \theta = y / L$ where y is the distance OP to the fringe and L is the distance from the fringe to the centre of the slits. However, y is very small so L does not differ appreciably from X, the distance from the centre of the fringe pattern to the slits. Hence, for a bright fringe:

$$\sin \theta = \frac{y}{X} = \frac{n\lambda}{d}$$

which gives

 $\frac{y}{X} = \frac{n\lambda}{d}$

Adjacent fringes have values of *n* equal to *n* and *n*+1. Thus the spacing between pairs of $\frac{1}{\sqrt{n}}$

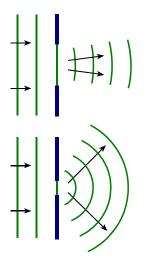
adjacent bright (or dark) fringes = $\frac{\lambda X}{d}$. Or: <u>fringe width</u> <u>slit to screen distance</u> = <u>wavelength</u> <u>slit separation</u>

Diffraction

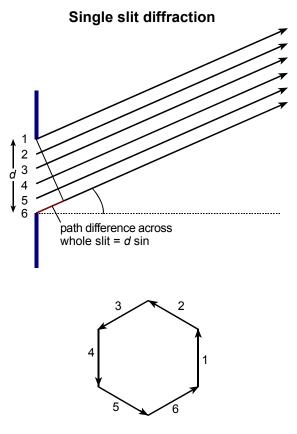
Diffraction is the spreading of waves after passing through a gap or past the edge of an obstacle.

The spreading increases if the gap is made narrower or if the wavelength of the waves is increased.

Diffraction



Monochromatic light passing through a single narrow slit produces a pattern of bright and dark fringes. Intensity **minima** are observed at angles θ given by the equation $d \sin \theta = n\lambda$, where d is the gap width, n is a positive integer and θ is the angle between the incident direction and the direction of diffraction.



phasors add to zero

If the distance across the gap is taken to be a large number of equally spaced point sources, **1**, **2**, **3**, etc, the phasor due to **1** will be a certain fraction of a cycle behind the phasor due to **2**, which will be the same fraction behind the phasor due to **3** etc. The resultant phasor is therefore zero at those positions where the tip of the last phasor meets the tail of the first phasor.

The path difference between the top and bottom of the slit is $d \sin \theta$. If this path difference is equal to a whole number of wavelengths, $n\lambda$, and if the last and first phasors join tip to tail minima occur when

 $d\sin\theta = n\lambda$

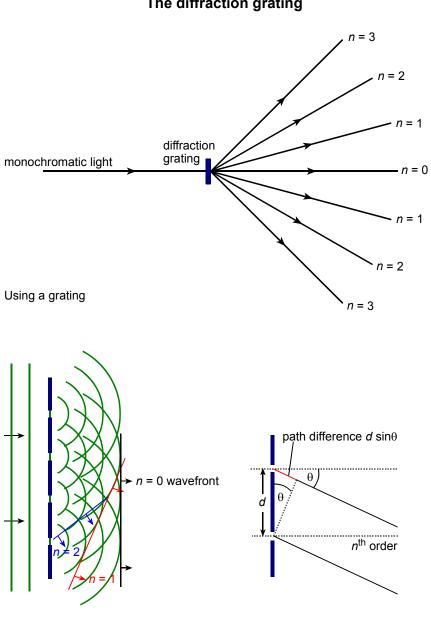
For small angles, sin $\theta = \theta$ giving an angular width 2λ / d for the central maximum.

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Gratings and spectra

A grating is a plate with a large number of parallel grooves ruled on it. A transmission grating transmits and diffracts light into spectra.

When a narrow beam of monochromatic light is directed normally at a transmission grating, the beam passes through and is diffracted into well-defined directions (ie there are **maxima** in theses directions) given by $d \sin \theta = n \lambda$, where d, the grating spacing, is the distance between adjacent slits and n is an integer called the spectral order. The path difference between waves from adjacent slits is $d \sin \theta$ and this must be equal to a whole number n of wavelengths for reinforcement.



The diffraction grating

Formation of wavefronts

Path difference

Using a white light source, a continuous spectrum is observed at each order, with blue nearer the centre and red away from the centre. This is because blue light has a smaller wavelength than red light so is diffracted less. Spectra at higher orders begin to overlap because of the spread.

Using light sources that emit certain wavelengths only, a line emission spectrum is observed which is characteristic of the atoms in the light source.

Phase and phasors

'Phase' refers to stages in a repeating change, as in 'phases of the Moon'.

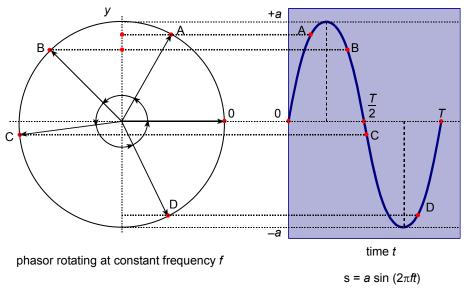
The phase difference between two objects vibrating at the same frequency is the fraction of a cycle that passes between one object being at maximum displacement in a certain direction and the other object being at maximum displacement in the same direction.

Phase difference is expressed as a fraction of one cycle, or of 2π radians, or of 360° .

Phasors are used to represent amplitude and phase in a wave. A phasor is a rotating arrow used to represent a sinusoidally changing quantity.

Suppose the amplitude *s* of a wave at a certain position is $s = a \sin(2\pi ft)$, where *a* is the amplitude of the wave and *f* is the frequency of the wave. The amplitude can be represented as the projection onto a straight line of a vector of length *a* rotating at constant frequency *f*, as shown in the diagram. The vector passes through the +*x*-axis in an anticlockwise direction at time t = 0 so its projection onto the *y*-axis at time *t* later is $a \sin(2\pi ft)$ since it turns through an angle $2\pi ft$ in this time.

Phasors can be used to find the resultant amplitude when two or more waves superpose. The phasors for the waves at the same instant are added together 'tip to tail' to give a resultant phasor which has a length that represents the resultant amplitude. If all the phasors add together to give zero resultant, the resultant amplitude is zero at that point.



Generating a sine wave

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Superposition

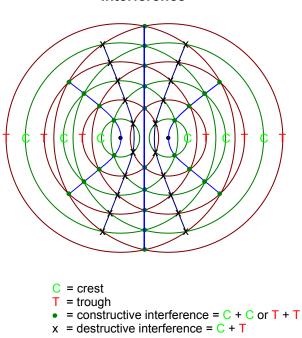
When two or more waves meet, their displacements superpose.

The principle of superposition states that when two or more waves overlap, the resultant displacement at a given instant and position is equal to the sum of the individual displacements at that instant and position.

In simple terms, where a wave crest meets another wave crest, the two wave crests pass through each other, forming a 'super crest' where and when they meet. If a wave trough

meets another wave trough, they form a 'super trough' where they meet. In both cases, the waves reinforce each other to increase the displacement momentarily. If a wave crest meets a wave trough, the waves cancel each other out momentarily.

An example of superposition is the interference pattern produced by a pair of dippers in a ripple tank, as shown below.



Interference

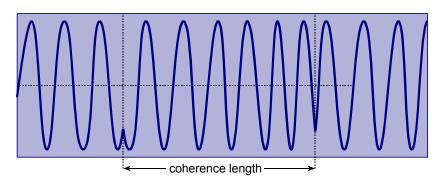
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Coherence

Coherence is an essential condition for observing the interference of waves.

Two sources of waves are coherent if they emit waves with a constant phase difference. Two waves arriving at a point are said to be coherent if there is a constant phase difference between them as they pass that point.

The coherence length of light from a given source is the average length of a wavetrain between successive sudden phase changes.



Coherence along a wave

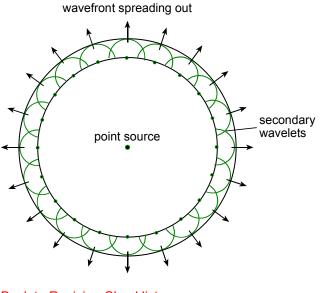
To see interference with light, the two sets of waves need to be produced from a single source, so that they can be coherent. For this, the path difference must not be larger than the coherence length of the source.

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Huygens' wavelets

Huygens' wavelet theory can be used to explain reflection, refraction, diffraction and interference (or superposition) of light.

Huygens' theory of wavelets considers each point on a wavefront as a secondary emitter of wavelets. The wavelets from the points along a wavefront create a new wavefront, so that the wave propagates.



Huygens' wavelets

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Accuracy and precision

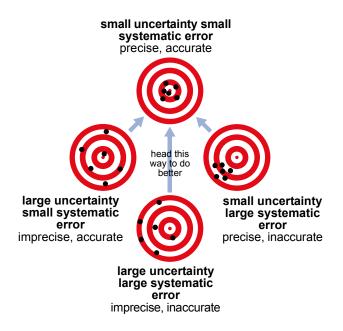
A measurement is accurate if it is close to the true value. A measurement is precise if values cluster closely, with small uncertainty.

A watch with an accuracy of 0.1% could be up to five minutes astray within a few days of being set. A space probe with a trajectory accurate to 0.01% could be more than 30 km off target at the Moon.

Think of the true value as like the bullseye on a target, and measurements as like arrows or darts aimed at the bullseye.

Uncertainty and systematic error

Think of measurements as shots on a target. Imagine the 'true value' is at the centre of the target



An accurate set of measurements is like a set of hits that centre on the bullseye. In the diagram above at the top, the hits also cluster close together. The uncertainty is small. This is a measurement that gives the true result rather precisely.

On the left, the accuracy is still good (the hits centre on the bullseye) but they are more scattered. The uncertainty is higher. This is a measurement where the average still gives the true result, but that result is not known very precisely.

On the right, the hits are all away from the bullseye, so the accuracy is poor. But they cluster close together, so the uncertainty is low. This is a measurement that has a systematic error, giving a result different from the true result, but where other variations are small.

Finally, at the bottom, the accuracy is poor (systematic error) and the uncertainty is large.

A statement of the result of a measurement needs to contain two distinct estimates:

- 1. The best available estimate of the value being measured.
- 2. The best available estimate of the range within which the true value lies.

Note that both are statements of belief based on evidence, not of fact.

For example, a few years ago discussion of the 'age-scale' of the Universe put it at 14 plus or minus 2 thousand million years. Earlier estimates gave considerably smaller values but with larger ranges of uncertainty. The current (2008) estimate is 13.7 ± 0.2 Gy. This new value lies within the range of uncertainty for the previous value, so physicists think the estimate has been improved in precision but has not fundamentally changed.

Fundamental physical constants such as the charge of the electron have been measured to an astonishing small uncertainty. For example, the charge of the electron is $1.602\ 173\ 335 \times 10^{-19}$ C to an uncertainty of $0.000\ 000\ 005 \times 10^{-19}$ C, better than nine significant figures.

There are several different reasons why a recorded result may differ from the true value:

1. **Constant systematic bias**, such as a zero error in an instrument, or an effect which has not been allowed for.

Constant systematic errors are very difficult to deal with, because their effects are only observable if they can be removed. To remove systematic error is simply to do a better experiment. A clock running slow or fast is an example of systematic instrument error. The effect of temperature on the resistance of a strain gauge is an example of systematic experimental error.

2. **Varying systematic bias**, or drift, in which the behaviour of an instrument changes with time, or an outside influence changes.

Drift in the sensitivity of an instrument, such as an oscilloscope, is quite common in electronic instrumentation. It can be detected if measured values show a systematic variation with time. Another example: the measured values of the speed of light in a pipe buried in the ground varied regularly twice a day. The cause was traced to the tide coming in on the nearby sea-shore, and compressing the ground, shortening the pipe a little.

 Limited resolution of an instrument. For example the reading of a digital voltmeter may change from say 1.25 V to 1.26 V with no intermediate values. The true potential difference lies in the 0.01 V range 1.25 V to 1.26 V.

All instruments have limited resolution: the smallest change in input which can be detected. Even if all of a set of repeated readings are the same, the true value is not exactly equal to the recorded value. It lies somewhere between the two nearest values which can be distinguished.

4. Accidental momentary effects, such as a 'spike' in an electrical supply, or something hitting the apparatus, which produce isolated wrong values, or 'outliers'.

Accidental momentary errors, caused by some untoward event, are very common. They can often be traced by identifying results that are very different from others, or which depart from a general trend. The only remedy is to repeat them, discarding them if further measurements strongly suggest that they are wrong. Such values should never be included in any average of measurements, or be used when fitting a line or curve.

5. **Human errors**, such as misreading an instrument, which produce isolated false recorded values.

Human errors in reading or recording data do occur, such as placing a decimal point wrongly, or using the wrong scale of an instrument. They can often be identified by noticing the kinds of mistake it is easy to make. They should be removed from the data, replacing them by repeated check observations.

6. **Random fluctuations**, for example noise in a signal, or the combined effect of many unconnected minor sources of variation, which alter the measured value unpredictably from moment to moment.

Truly random variations in measurements are rather rare, though a number of unconnected small influences on the experiment may have a net effect similar to random variation. But because there are well worked out mathematical methods for dealing with random variations, much emphasis is often given to them in discussion of the estimation of the uncertainty of a measurement. These methods can usually safely be used when inspection of the data suggests that variations around an average or a fitted line or curve are small and unsystematic. It is important to look at visual plots of the variations in data before deciding how to estimate uncertainties.

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Systematic error

Systematic error is any error that biases a measurement away from the true value.

All measurements are prone to systematic error. A systematic error is any biasing effect, in the environment, methods of observation or instruments used, which introduces error into an

experiment. For example, the length of a pendulum will be in error if slight movement of the support, which effectively lengthens the string, is not prevented, or allowed for.

Incorrect zeroing of an instrument leading to a **zero error** is an example of systematic error in instrumentation. It is important to check the zero reading during an experiment as well as at the start.

Systematic errors can change during an experiment. In this case, measurements show trends with time rather than varying randomly about a mean. The instrument is said to show **drift** (e.g. if it warms up while being used).

Systematic errors can be reduced by checking instruments against known standards. They can also be detected by measuring already known quantities.

The problem with a systematic error is that you may not know how big it is, or even that it exists. The history of physics is littered with examples of undetected systematic errors. The only way to deal with a systematic error is to identify its cause and either calculate it and remove it, or do a better measurement which eliminates or reduces it.

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Uncertainty

The uncertainty of an experimental result is the range of values within which the true value may reasonably be believed to lie. To estimate the uncertainty, the following steps are needed.

- 1. Removing from the data **outlying** values which are reasonably suspected of being in serious error, for example because of human error in recording them correctly, or because of an unusual external influence, such as a sudden change of supply voltage. Such values should not be included in any later averaging of results or attempts to fit a line or curve to relationships between measurements.
- 2. Estimating the possible magnitude of any systematic error. An example of a constant systematic error is the increase in the effective length of a pendulum because the string's support is able to move a little as the pendulum swings. The sign of the error is known (in effect increasing the length) and it may be possible to set an upper limit on its magnitude by observation. Analysis of such systematic errors points the way to improving the experiment.
- 3. Assessing the **resolution** of each instrument involved, that is, the smallest change it can detect. Measurements from it cannot be known to less than the range of values it does not distinguish.
- 4. Assessing the magnitude of other small, possibly random, unknown effects on each measured quantity, which may include human factors such as varying speed of reaction. Evidence of this may come from the spread of values of the measurement conducted under what are as far as possible identical conditions. The purpose of repeating measurements is to decide how far it appears to be possible to hold conditions identical.
- 5. Determining the combined effect of possible **uncertainty** in the result due to the limited resolution of instruments (3 above) and uncontrollable variation (4 above).

To improve a measurement, it is essential to identify the largest source of uncertainty. This tells you where to invest effort to reduce the uncertainty of the result.

Having eliminated accidental errors, and allowed for systematic errors, the range of values within which the true result may be believed to lie can be estimated from (a) consideration of the resolution of the instruments involved and (b) evidence from repeated measurements of the variability of measured values.

Most experiments involve measurements of more than one physical quantity, which are combined to obtain the final result. For example, the length *L* and time of swing *T* of a simple pendulum may be used to determine the local acceleration of free fall, *g*, using

$$T=2\pi\sqrt{\frac{L}{g}}$$

so that

$$g=\frac{4\pi^2 L}{T^2}.$$

The range in which the value of each quantity may lie needs to be estimated. To do so, first consider the resolution of the instrument involved – say ruler and stopwatch. The uncertainty of a single measurement cannot be better than the resolution of the instrument. But it may be worse. Repeated measurements under supposedly the same conditions may show small and perhaps random variations.

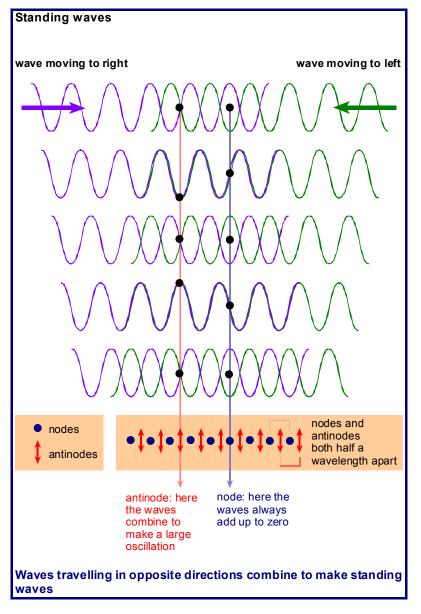
If you have repeated measurements, 'plot and look', to see how the values vary. A simple estimate of the variation is the spread = $\pm \frac{1}{2}$ range.

A simple way to see the effect of uncertainties in each measured quantity on the final result is to recalculate the final result, but adding or subtracting from the values of variables the maximum possible variation of each about its central value. This is pessimistic because it is unlikely that 'worst case' values all occur together. However, pessimism may well be the best policy: physicists have historically tended to underestimate uncertainties rather than overestimate them. The range within which the value of a quantity may reasonably be believed to lie may be reduced somewhat by making many equivalent measurements, and averaging them. If there are *N* independent but equivalent measurements, with range *R*, then the range of their average is likely to be approximately *R* divided by the factor \sqrt{N} . These benefits are not automatic, because in collecting many measurements conditions may vary.

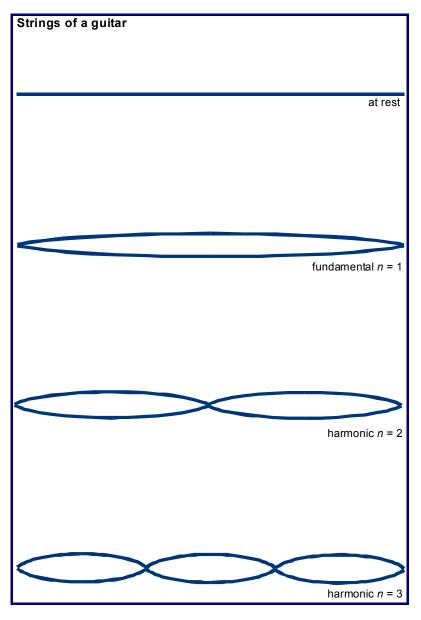
Summary Diagrams

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Standing waves



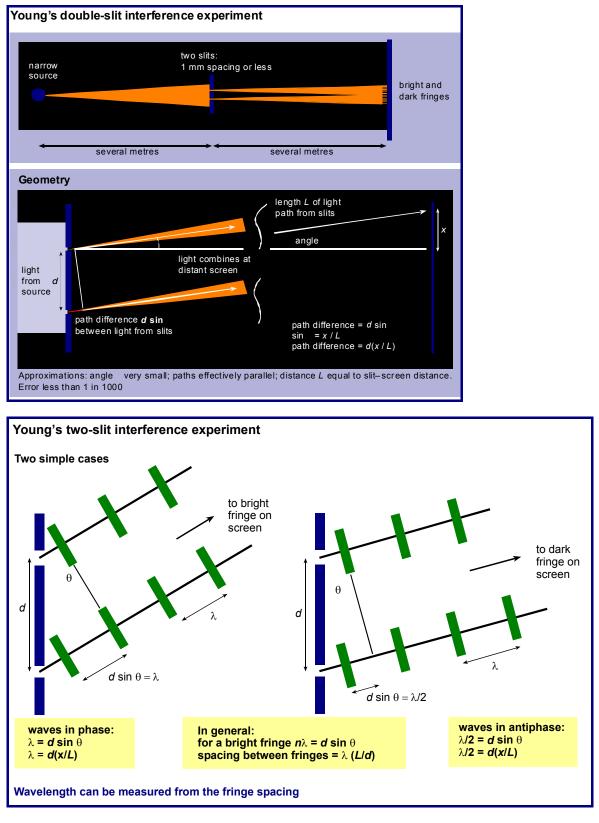
Standing waves on a guitar An example of superposition.



Standing waves in pipes An example of superposition.

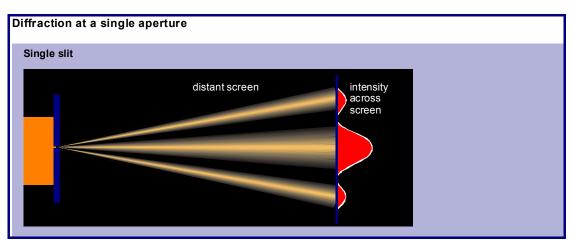
Standing waves in	pipes		
Olaand alaan			
Closed pipes		A loudspeaker sends a sound into a long tube.	
		Dust in the tube can show nodes and antinodes. Nodes are half a wavelength apart. So are antinodes. Dark colour shows maximum	
loudspeaker	12	pressure variation and minimum motion of air (pressure antinode). Light colour shows	
At a lower frequency, the	wavelength is longer	minimum pressure variation and maximum motion of air (pressure node).	
	/2		
e	/2	The fundamental: The lowest frequency which can form a standing wave has wavelength equal to twice the length of the tube	
Pipes open at both ends		Sound can be reflected from an open end as well as from a closed end.	
		This is how open organ pipes and flutes work.	
Pipes closed at one end		Pipes closed at one end are shorter, for the same note. A clarinet is like this. An oboe is too, but with a tapered tube. Some organ pipes are stopped at one end.	
Frequencies of standing waves	5		
	pipes open or closed at bot strings fixed at both ends	n ends pipes open at one end	
length L	L = n / 2	L = (2n-1) / 4	
fundamental	f = v / 2L	f = v / 4L	
harmonics	2f 3f	3f 5f	
	nf	 (2 <i>n</i> -1) <i>f</i>	
The frequencies sounded by a pipe depend on the pipe's length			

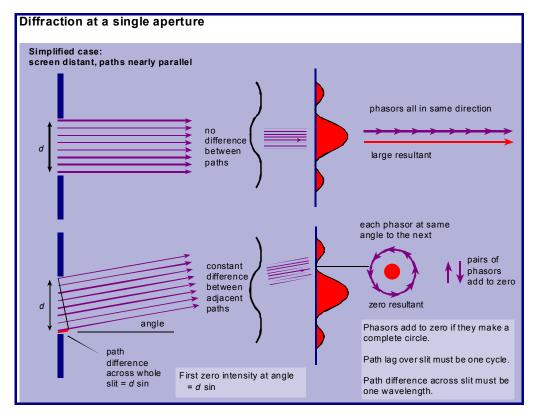
Two-slit interference

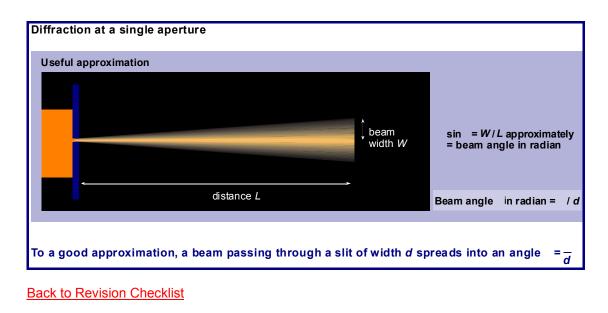


Diffraction

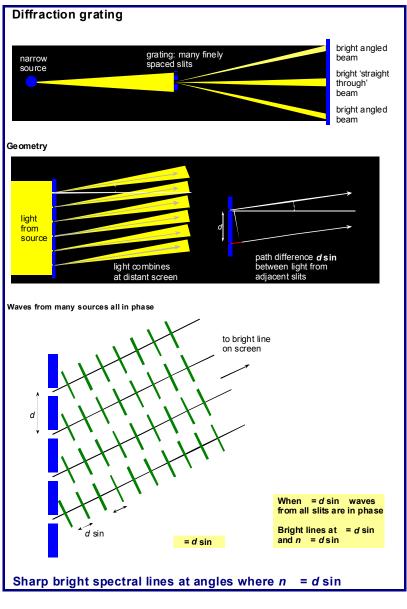
A single hole produces interesting patterns.





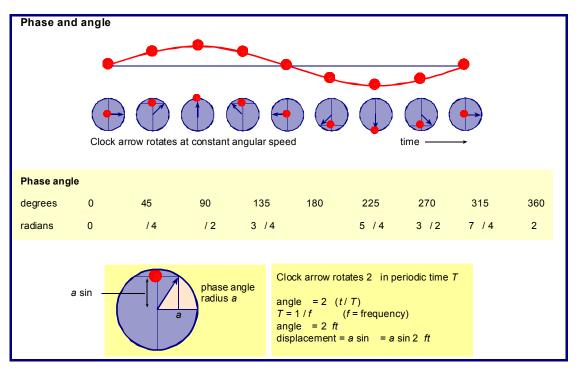


A transmission grating Many slits produce bright, sharp beams.



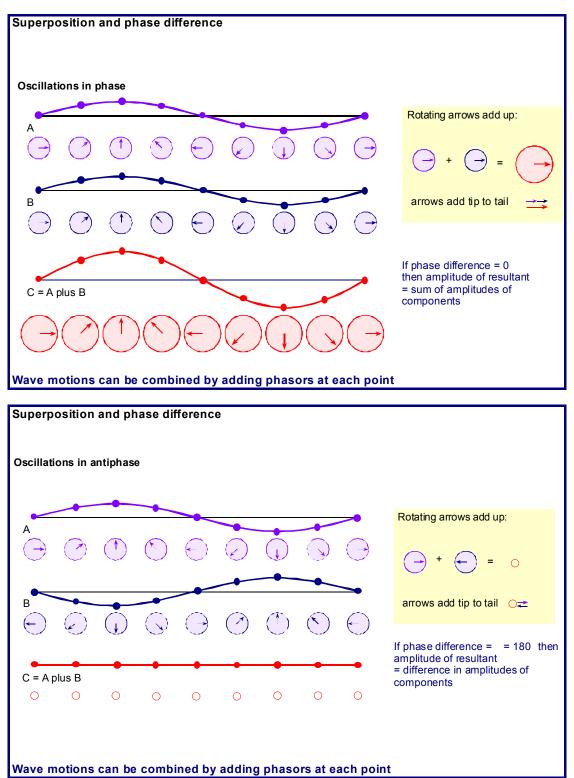
Phase and angle

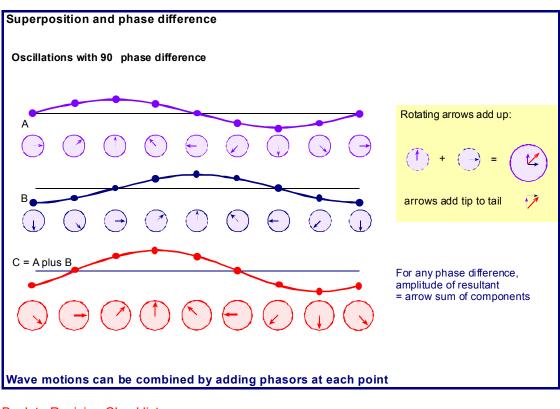
The phase of a wave motion gives where it is in its wave cycle.



Adding oscillations using phasors

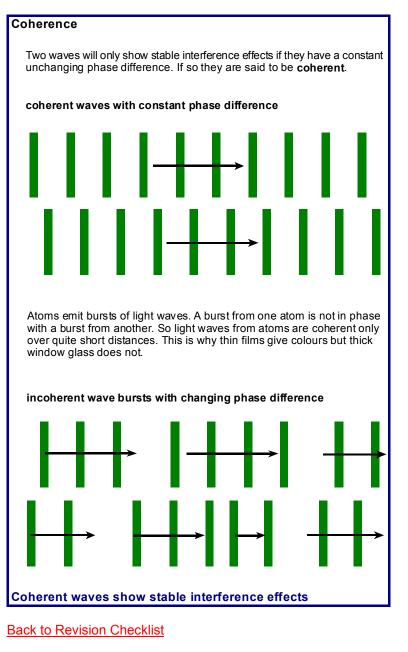
The diagrams show pairs of oscillations being added (superposed), with different phases between the two oscillations.





Coherence

Here phase differences do not change over time.



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