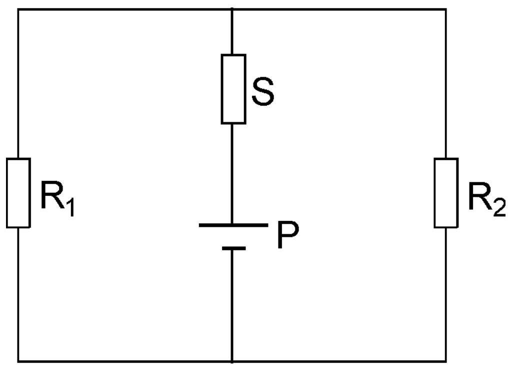
Worked Solutions for Sample Examination Questions

Question 1

(a) The answer is **A**.

Explanation
This magnetic circuit of the transformer is like this parallel electrical circuit:



where the cell P represents the primary coil which is producing the flux, much as a cell produces current in an electrical circuit, the resistor S represents the secondary coil, and the two resistors R_1 and R_2 are the side-arms of the transformer core.

Just as, in the electrical circuit above, the current in S is the same as the current in P, so the flux through the primary coil and the flux through the secondary coil are the same, assuming there is no leakage of flux. This assumption is reasonable, as iron has a high magnetic permeability – it is a good 'conductor of flux'.

The currents through R_1 and R_2 are clearly less than the current in S, because they must add up to give the current through P. If the secondary coil had been wound on one of the side-limbs of the transformer, the flux through it would be less than the flux through the primary coil.

(b)

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$V_s = V_p \frac{N_s}{N_p}$$

$$= 400\text{ V} \times \frac{200\text{ turns}}{1000\text{ turns}}$$

$$= 80\text{ V}.$$

The frequency must be the same at 50 Hz.

Question 2

(a) Rate of change of magnetic flux linkage = $\dfrac{90 \times 10^{-3}\ \text{Wb}}{450 \times 10^{-6}\ \text{s}} = 200\ \text{Wb s}^{-1}$.

(N.B. flux linkage = flux × number of turns on the coil; this is already included in the data given.)

(b) Induced emf = rate of change of magnetic flux linkage,
so emf = 200 V.

Question 3

(a) Flux linkage = flux × number of turns, so

flux Φ linking one turn of the coil = $\dfrac{4.0 \times 10^{-4}\ \text{Wb turns}}{400\ \text{turns}} = 1.0 \times 10^{-6}\ \text{Wb}$.

(b) Flux Φ = BA where B is the flux density, so

$$B = \frac{\Phi}{A} = \frac{1.0 \times 10^{-6}\ \text{Wb}}{1.25 \times 10^{-5}\ \text{m}^2} = 8.0 \times 10^{-2}\ T.$$

Question 4

(a) Symbol ϕ or Φ, weber (Wb) ($\phi = B\,A$).

(b) Symbol B, measured in tesla (T) (= ϕ / A).

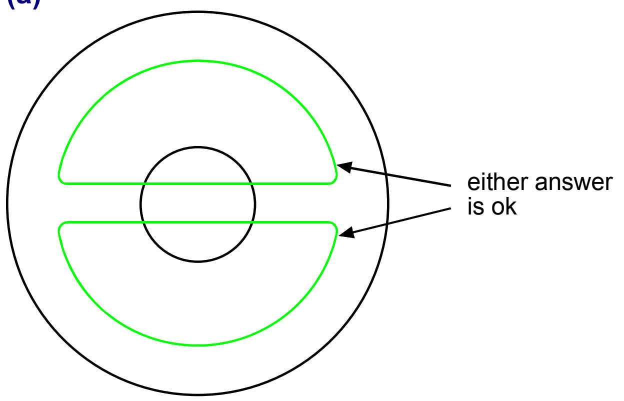
(c) Rate of change of magnetic flux linkage is the emf (Faraday's Law) so it can be measured in volts (V).

Question 5

Rotating twice as fast doubles the frequency to 20 Hz, and cuts flux lines twice as fast so the emf will double to 8 V: therefore the answer is **C**.

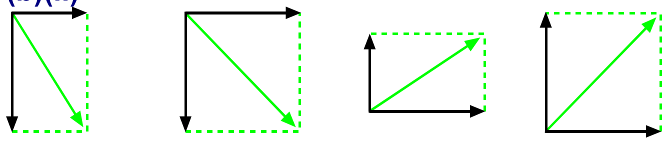
Question 6

(a)



(b)(i) reading from the graph one period = 16 ms. Time difference between the signals = 4 ms = 4/16 = ¼ of a period, which corresponds to a phase difference of 90° or $\pi/2$.

(b)(ii)



(b)(iii) The resultant is rotating anticlockwise.

(b)(iv) The changing flux induces an emf in the rotor which sets up a current (an 'eddy current'). The current interacts with the magnetic field to give a force ('$F = I\,L\,B$'). You could also answer this in terms of the induced currents in the rotor, making it into an electromagnet which aligns itself with the field so that, as the field rotates, it follows.

Question 7

$E = -\dfrac{dV}{dx}$ so the slope (gradient) of the graph at r gives the electric field.

Question 8

$F = q\,v\,B = (3.2 \times 10^{-19}\ \text{C}) \times (1.5 \times 10^{7}\ \text{m s}^{-1}) \times 0.25\ \text{T}$
$= 1.2 \times 10^{-12}\ \text{N}$.

Question 9

By definition $V \to 0$ as $d \to \infty$; only graphs A or B fit this fact. A proton has a positive charge, so the answer is B.

Question 10

Field strength = force per unit charge:

$$E = \frac{kQ}{r^2}$$
$$= \frac{(8.98 \times 10^{9}\ \text{N C}^{-2}\ \text{m}^2) \times (1.92 \times 10^{-18}\ \text{C})}{(5.0 \times 10^{-11}\ \text{m})^2}$$
$$= 6.9 \times 10^{12}\ \text{N C}^{-1}.$$

(V m^{-1} is also an acceptable unit for electric field strength.)

Question 11

(a) Current = 20 μA = 20 x 10^{-6} A = 20 x 10^{-6} C s^{-1}. Thus:

ions per second = $\dfrac{20 \times 10^{-6}\ \text{C s}^{-1}}{1.6 \times 10^{-19}\ \text{C ion}^{-1}} = 1.3 \times 10^{14}\ \text{ion s}^{-1}$.

(b)(i)

$$E_K = \frac{mv^2}{2}.$$

Therefore the change in E_K is

$$\frac{m}{2}[(3.0\times10^5\ \mathrm{m\,s^{-1}})^2-(100\ \mathrm{m\,s^{-1}})^2]=\frac{3.32\times10^{-26}\ \mathrm{kg}}{2}[(9\times10^{10}\ \mathrm{m^2\,s^{-2}})-(1.0\times10^4\ \mathrm{m^2\,s^{-2}})]$$
$$=1.49\times10^{-15}\ \mathrm{J}.$$

(NB: The 1.0 x 10^4 is too small to make any significant difference to the answer: the kinetic energy that the ion started with could have been ignored, which simplifies the calculation.)

(b)(ii) $q\ V$ = 1.49 x 10^{-15} J. Thus

$$V=\frac{1.49\times10^{-15}\ \mathrm{J}}{1.6\times10^{-19}\ \mathrm{C}}=9.3\ \mathrm{kV}$$

so V must be greater than 5 kV.

(c)(i) The magnetic force is always at right angles to the direction of motion of the ion beam (~current). This produces circular motion similar to the motion of the Earth around the Sun under the influence of the Sun's gravitational attraction which is always at right angles to the direction of motion of the Earth.

(c)(ii) The radius of the circular path, r = 0.125 m as stated in the question.

$$F=\frac{mv^2}{r}=\frac{(3.32\times10^{-26}\ \mathrm{kg})\times(3.0\times10^5\ \mathrm{m\,s^{-1}})^2}{125\times10^3\ \mathrm{m}}$$
$$=2.39\times10^{-14}\ \mathrm{N}.$$

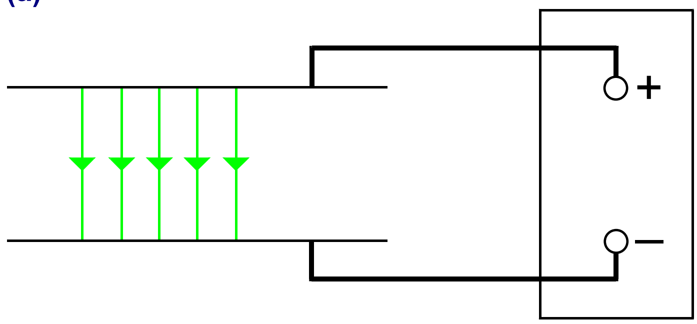
(c)(iii) $F = e\ v\ B$ so

$$B=\frac{F}{ev}=\frac{2.39\times10^{-14}\ \mathrm{N}}{(1.6\times10^{-19}\ \mathrm{C})\times(3.0\times10^5\ \mathrm{m\,s^{-1}})}=0.49\ \mathrm{T}.$$

(d) Neon-22 has a mass 10% larger than neon-20. Thus for a given accelerating force its final speed will be slower. Thus a smaller value of B would be needed to bend it into the same orbit so it will bend with a different radius and not go through the detector slit.

Question 12

(a)



(NB: The question asks for 5 lines: they must be equally spaced because the field is uniform.)

(b) (Note that the set-up is just that for a Millikan's apparatus to find the elementary charge on the electron.) If the sphere does not move the force due to the electric field must act upwards to oppose the sphere's weight due to gravity. As the upper plate is positively charged, the sphere must have a negative charge (unlike charges attract).

(c) $n\,e = 4.8 \times 10^{-14}$ C, so

$$n = \frac{4.8 \times 10^{-14}\ \text{C}}{1.6 \times 10^{-19}\ \text{C per electron}} = 3.0 \times 10^{5}\ \text{electrons.}$$

(NB: The value of *e* is in the Data, Formulae and Relationships Booklet used in all written exams.)

(d)(i) Force = $q\,E = m\,g$:

$$E = \frac{mg}{q} = \frac{(7.4 \times 10^{-9}\ \text{kg}) \times 9.8\ \text{N kg}^{-1}}{4.8 \times 10^{-14}\ C}$$
$$= 1.5 \times 10^{6}\ \text{V m}^{-1}\ \text{(or N C}^{-1}\text{)}.$$

(d)(ii) $E = V / d$, so:

$V = E\,d = (1.5 \times 10^{6}\ \text{V m}^{-1}) \times (10 \times 10^{-3}\ \text{m}) = 1.5 \times 10^{4}\ \text{V}$ (or 15 kV).

(e) Beta radiation will ionize the air molecules. Positive ions will be attracted to the negatively charged sphere thus reducing its negative charge. If the sphere intercepts an actual beta particle it will become more negatively charged.

Question 13

Greater energy means that the proton can get closer to the nucleus because it can climb further up the electrostatic potential energy 'hill'. The distance of closest approach *d* is given by:

$$E_K \propto \frac{qQ}{d}$$

where q is the charge on the proton and Q the charge on the nucleus, and thus $E_K\,d$ is a constant. If E_K increases, d must get smaller.

Question 14

(a) Energy difference for a transition from **D** to **B** = 5.0 eV – 2.5 eV = 2.5 eV.
Thus the energy left with the electron = 3.0 eV – 2.5 eV = 0.5 eV.

(b) A transition to **A** requires 5.0 eV – 1.5 eV = 3.5 eV, which is more than the electron energy, so this transition is impossible.

Question 15

This question incorporates 'Stretch and Challenge', so it is important that you are able to plan the stages needed in an answer. It is not structured into different stages, so you have to think the way through for yourself.

The stages in this process are:

- protons and neutrons are the same size, closely packed into a spherical nucleus
- the fraction of empty space between nuclei is constant, so the volume of nucleus $\propto$ number of nucleons present
- substitute in the equation for the volume of a sphere
- re-arrange to make the radius the subject.

(a)(i) The nucleons are 'close packed' and touching each other like water molecules in a drop of water.

If volume is proportional to the number of nucleons, we can write this as

$$\frac{4}{3}\pi r^3 \propto A$$

thus

$$\frac{4}{3}\pi r^3 = KA \text{ where } K \text{ is a constant.}$$

$$r = \left(\frac{3}{4\pi}\right) K^{1/3} A^{1/3}$$

As $\left(\frac{3}{4\pi}\right) K^{1/3}$ is a constant, which can be written as r_0,
$r = r_0\, A^{1/3}$ where r_0 is some constant.

(In fact, by putting A = 1 you should see that r_0 is the radius of a proton or neutron.)

(a)(ii) For neon-20 A = 20:

r = (1.2 x 10^{-15} m) x $20^{1/3}$ = 3.2 x 10^{-15} m,

so the diameter is

2 x (3.25 x 10^{-15} m) = 6.5 x 10^{-15} m.

(b) Electrons have a well-defined de Broglie wavelength, hence a well-defined frequency that can be represented by a rotating phasor. The electron waves explore all possible paths to the detector. The phasor for each path will arrive with a different angle (phase differences). Superposition of the phasors will give different amplitudes resulting in maxima and minima. The behaviour is similar to the diffraction of light around a small obstacle, which is itself rather like the diffraction of light by a slit.

(c)(i) θ_{min} from the graph = 5°.
Diameter of neon-20 = 6.5 x 10^{-15} m (see **(a)(ii)**) thus

λ = 1.2 x (6.5 x 10^{-15} m) x sin 5° = 6.8 x 10^{-16} m ≈ 7 x 10^{-16} m.

(c)(ii) $\lambda = h / p$ so

$p = h / \lambda$ = (6.63 x 10^{-34} J s) / (6.8 x 10^{-16} m) = 9.8 x 10^{-19} N s.

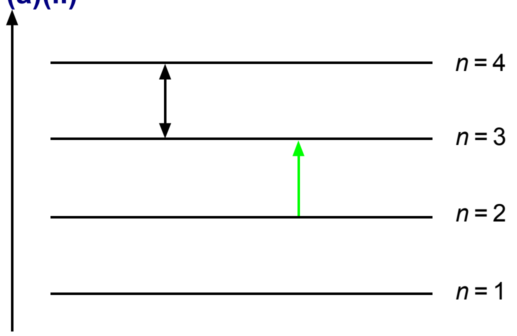
(d) The argon-40 graph has two differences from the neon-20 graph: (a) the diffracted signal is larger and (b) the minimum is shifted to a smaller angle.
Argon-40 will make a larger diameter target than the neon-20: sin $\theta \propto \lambda / b$ so as b increases, sin θ, and hence θ, gets smaller.
The argon nucleus has more protons and hence a larger charge than the neon, so more electrons in the beam will be affected to give a stronger signal.

Question 16

(a)(i) $c = f\lambda$ so $f = c / \lambda$. From the graph λ = 73 nm = 73 x 10^{-9} m:

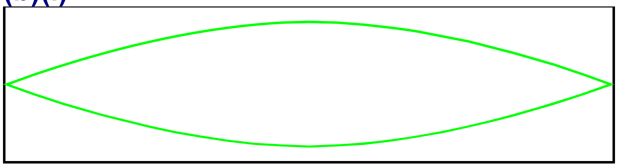
f = (3 x 10^{8} m s^{-1}) / (73 x 10^{-6} m) = 4.1 x 10^{12} Hz.

(a)(ii)



(a)(iii) $\varepsilon = h f$ = (6.6 x 10^{-34} J s) x (4.1 x 10^{12} Hz) = 2.7 x 10^{-21} J.

(b)(i)



(b)(ii) $E = p^2 / 2m$ so

$p = \sqrt{2mE}$

and then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{\sqrt{h^2}}{\sqrt{2mE}} = \sqrt{\frac{h^2}{2mE}}.$$

(b)(iii)

$$\lambda = (6.6 \times 10^{-34}\ \text{J s})^2 / \sqrt{2 \times (5.1 \times 10^{-26}\ \text{kg}) \times (1.35 \times 10^{-21}\ \text{J})}$$
$$= 5.6 \times 10^{-11}\ \text{m.}$$

(c)(i) When n = 1, the successive nodes at the ends of the box correspond to half a wavelength. So the length of the box = (5.6 x 10^{-11} m) / 2 = 2.8 x 10^{-11} m.

(c)(ii) For n = 3:

$$E = (1.35 \times 10^{-21}\ \text{J}) + 2\varepsilon$$
$$= 1.35 \times 10^{-21}\ \text{J} + [2 \times (2.7 \times 10^{-21}\ \text{J})]$$
$$= 6.75 \times 10^{-21}\ \text{J}$$

and

$$\lambda = \sqrt{h^2 / 2mE} = 2.5 \times 10^{-11}\ \text{m.}$$

For n = 3, box length = 1.5 λ = 3.8 x 10^{-11} m, which is larger than for the ground state.

Question 17

Equating baryon numbers 239 +1 = 100 + 134 + n.
Thus n = 240 – 234 = 6 (minus the original inducing neutron, = 5 neutrons emitted by the plutonium-239).

Question 18

(a) $T_{1/2}\,\lambda$ = ln 2 thus

λ = ln 2 / (9.4 x 10^3 s) = 7.37 x 10^{-5} s^{-1} ≈ 7 x 10^{-5} s^{-1}.

(b) Given

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

$$\lambda = \frac{-\Delta N}{N\,\Delta t}.$$ (The minus sign is there because ΔN is a decrease.)

ΔN and N have no units (they are just numbers),
thus λ has the same units as 1 / Δt = s^{-1}.

(c) Using the radioactive exponential decay equation (see **(b)**):

3×10^3 Bq = (7.37×10^{-5} s^{-1}) N

giving $N = 4.1 \times 10^7$ atoms.

Question 19

Risk per X-ray = probability per unit dose x dose equivalent
= 3% Sv^{-1} × (2×10^{-4} Sv year^{-1})
= 6×10^{-4} % per year.

Over 25 years the risk = (6×10^{-4} % per year) × 25 years = 1.5×10^{-2} %.

Question 20

$E = m c^2$

$m = E / c^2$

E = 1.2 MeV = (1.2×10^6 eV) x (1.6×10^{-19} J eV^{-1}) = 1.92×10^{-13} J.

Thus

$$m = \frac{1.92 \times 10^{-13} \text{ J}}{(3 \times 10^8 \text{ m s}^{-1})^2} = 2.1 \times 10^{-30} \text{ kg.}$$

Note: this is of the same order of magnitude as the mass of an electron.

Question 21

(a) This part of the question incorporates 'Stretch and Challenge', and is not structured, so that you must think the way through the answer yourself.

The steps to the solution are:

- find the mass difference in u.
- convert to kg
- use $E = m c^2$ to find the energy difference

Initial mass = 2 x 2.0141 u = 4.0282 u.

Final mass = 3.0160 u + 1.0087 u = 4.0247 u.

Difference = 4.0282 u – 4.0247 u = 0.0035 u.

0.0035 u x (1.66×10^{-27} kg u^{-1}) = 5.81×10^{-30} kg.

$E = m c^2$ = (5.81×10^{-30} kg) x (3.0×10^8 m s^{-1})^2 = 5.2×10^{-13} J.

(b)(i) We need to represent the process proton → neutron + positron + neutrino:

$${}^{1}_{1}\text{p} \rightarrow {}^{1}_{0}\text{n} + {}^{0}_{1}\text{e} + {}^{0}_{0}\nu.$$

Notice that the electric charge (proton) numbers and mass (nucleon) numbers must balance: the neutrino is uncharged and almost massless.

(b)(ii) 1.0073 u – 1.0087 u – 0.00055 u = –0.00195 u.

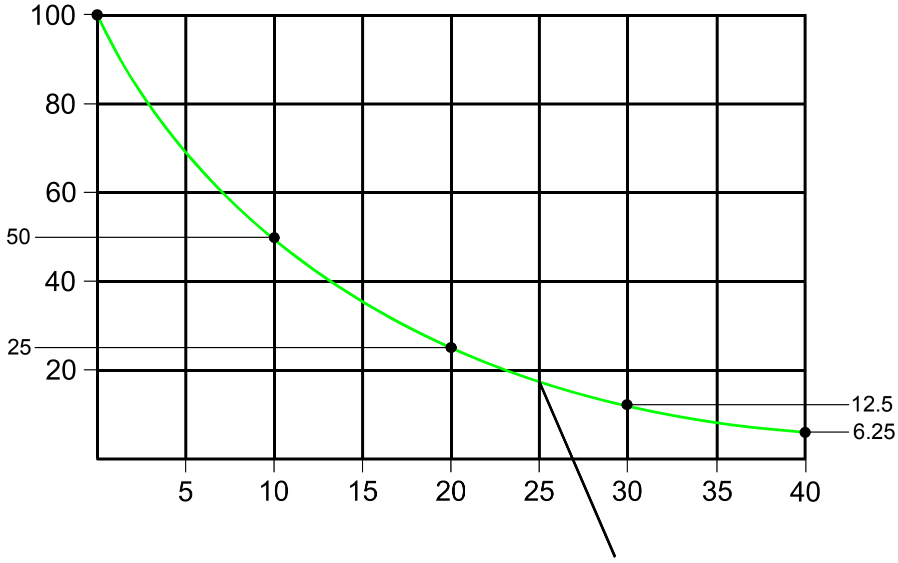
(b)(iii) The mass of the products is greater than the mass of the proton, so the process would require an energy input.

Question 22

(a) Activity $\Delta N / \Delta t = -\lambda N$

$T_{1/2}\,\lambda = \ln 2$, thus a longer half-life $T_{1/2}$ implies a smaller decay constant λ. So for λN to be the same (= the activity), the longest half-life requires the largest number of active particles N.

(b)(i)



(b)(ii) 25 mm gives 18% transmission:

$$\frac{18}{100} \times (4 \times 10^4) = 7.2 \times 10^3 \text{ Bq.}$$

(c)(i) Radiation is emitted in all directions, so at most only half will be towards the student's body.

(c)(ii) Exposure is for 1 hour = 3600 s:

dose = absorbed energy = ½ [(4 x 10^4 Bq) x 3600 s x (8.8 x 10^{-14} J)] = 6.3 x 10^{-6} J.

(c)(iii) Beta radiation will be absorbed only fairly close to the source, so will not be absorbed evenly by the whole body. Thus the dose equivalent to the body tissue around the source will be much higher than 0.1 μSv. If only 1 kg of body tissue absorbed the radiation, then the dose equivalent for that kg is (0.1 x 10^{-6} Sv) x 65 = 6.5 μSv, which is still much smaller than that due to background radiation, so the risk is still small.

Question 23

(a)(i) The rotating coil cuts line of magnetic flux, so the flux linked is changing. Changing magnetic flux induces an emf in the windings of the coil.

(a)(ii) Note that the car is **not** being slowed down by ordinary brakes. It is using the induced emf to give energy to the batteries. The car is slowing down, so the emf cannot be constant (so the answer is not A). The rate of rotation will decrease, thus the rate of cutting magnetic flux lines will also decrease, so B and C which show initial increases are not relevant. This leaves D as the correct answer.

(b) Either relatively large total mass (due to batteries, etc.) so since $a = F / m$, the acceleration is poor. Also, maximum power from batteries is less than available from burning petrol in a petrol engine. Both of these factors are likely to change rapidly due to electric car development in light of the mounting price of oil.

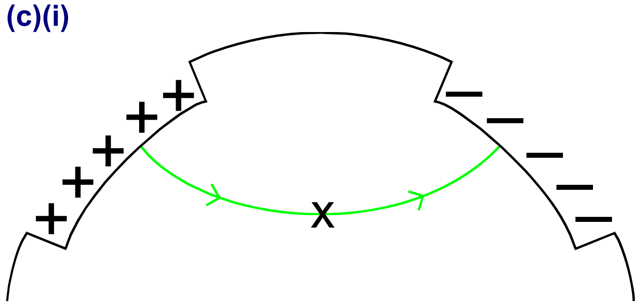
Question 24

(a)(i) The electrical resistance of the larger wires will be less, because although they are twice as long, their cross-sectional area is four times larger. Thus for a given p.d., the electric current will be greater. A larger current gives a larger magnetic flux.

(a)(ii) The magnetic circuit of the larger device is twice as long but has four times the cross-sectional area. Thus a given number of current turns produces a larger flux density.

(b) Centripetal force mv^2/r increases $\propto v^2$. This force is supplied by the tensile stress in the rotor arm, and in faster rotors the breaking stress may be exceeded.

(c)(i)



(c)(ii) The electric field exerts a force on the electric charge (in this case towards the + and away from the –), thus causing the rotor to rotate (anticlockwise).

Question 25

(a) To remove photons that have not come from the sodium iodide crystal.

(b)(i)

$$E = -\frac{dV}{dx},$$

i.e. E is the slope (gradient) of the V against distance graph. The slope is constant, so E is constant.

(b)(ii) Slope of the graph = 80 V / (5 x 10^{-3} m) = 16 000 V m^{-1}.

(b)(iii) Force = $e\,E$ = (1.6 x 10^{-19} C) x (16 x 10^{3} N C^{-1}) = 2.56 x 10^{-15} N
≈ 2.6 x 10^{-15} N.
(NB: The value of *e* is in the Data, Formulae and Relationships booklet used in all written exams.)

(b)(iv) $F = m\,a$, thus:

$$a = \frac{F}{m} = \frac{2.56 \times 10^{-15}\ \text{N}}{9.11 \times 10^{-31}\ \text{kg}}$$
$$= 2.8 \times 10^{15}\ \text{m s}^{-2}.$$

This is enormously greater than *g* (9.8 m s^{-2}).
(NB: the value of *m* is in the Data, Formulae and Relationships booklet.)

Question 26

(a) Fact 1: curves the opposite way, thus the positron must be travelling in the opposite direction from D to C.
Fact 2: the track is less curved, thus the positron is travelling faster.

(b) The process must obey conservation of electric charge. The gamma photon has zero charge, so the pair of particles produced must have zero net charge, i.e. opposite and equal charges.

(c) The gamma photon, being uncharged, does not ionise gas molecules very readily.

(d) Because they are both electrically charged and readily ionise the molecules (thus leaving tracks) they are losing energy and thus are slowing down.

(e) The positron meets an electron and annihilates into a pair of uncharged gamma photons which leave no tracks.

Question 27

(a) Solve these type of problems by applying the conservation laws: e.g. the decay must balance electric charge, nucleon number, lepton number etc.
Nucleon number does balance (14 = 14), so X is not a nucleon (i.e. a neutron or proton).
Electric charge will balance if X has a charge of –1 (6 = 7 – 1).
Lepton number will balance if X has a lepton number of +1 (0 = 1 – 1, the anti-neutrino = –1).
Hence X is a lepton, with one unit of negative electric charge, and is thus a beta particle (electron).

(b) $T_{1/2}\,\lambda = \ln 2$. Hence:

$$T_{1/2} = \frac{\ln 2}{3.8 \times 10^{-12}\ \text{s}^{-1}}$$
$$= 1.8 \times 10^{11}\ \text{s}$$
$$= \frac{1.8 \times 10^{11}\ \text{s}}{3.2 \times 10^{7}\ \text{s year}^{-1}}$$
$$= 5.70 \times 10^{3}\ \text{years} \approx 6000\ \text{years.}$$

(c) This part of the question incorporates Stretch and Challenge, so you have to think through the steps to a solution.

The first thing that you must do is to realize that the solution has to follow these steps:

- find the mass of one carbon-14 atom in kg
- find the number of carbon-14 atoms in 1.3 x 10^{-11} kg
- use the decay constant to find the activity using $\Delta N / \Delta t = -\lambda\ N$.

Mass of one carbon atom = 14 x (1.7 x 10^{-27}) kg.

Thus the number of carbon atoms in 1.3 x 10^{-11} kg is

$$\frac{1.3 \times 10^{-11}\ \text{kg}}{14 \times (1.7 \times 10^{-27})\ \text{kg atom}^{-1}} = 5.46 \times 10^{14}\ \text{atoms .}$$

$\Delta N / \Delta t$ is the activity= $-\lambda\ N$ = (3.8 x 10^{-12} s^{-1}) x (5.5 x 10^{14} atoms) = 2.09 x 10^{3} Bq
≈ 2 kBq.

(d)(i) 5000 years is almost one half-life, so the activity will have dropped by about ½, to about 1 kBq.

(d)(ii) The mass required = 10 / 1000 = 1 / 100 which corresponds to 0.65 kg which if removed will substantially damage the specimen.

(e)(i) Energy absorbed per second = activity x energy per decay
= 2000 Bq x (2.5 x 10^{-14} J decay^{-1})
= 5.0 x 10^{-11} J s^{-1}.

(e)(ii) 1 Gy is the absorbed dose per kg. Thus absorbed dose is:

$$\frac{5.0 \times 10^{-11}\ \text{J s}^{-1}}{65\ \text{kg}} = 7.69 \times 10^{-13}\ \text{Gy s}^{-1}$$
$$= (7.69 \times 10^{-13}\ \text{Gy s}^{-1}) \times (3.2 \times 10^{7}\ \text{s year}^{-1})$$
$$= 2.5 \times 10^{-5}\ \text{Gy year}^{-1}.$$

(e)(iii) There are four possible explanations for this. You are not expected to know which are correct – the word ‘suggest’ makes this clear.

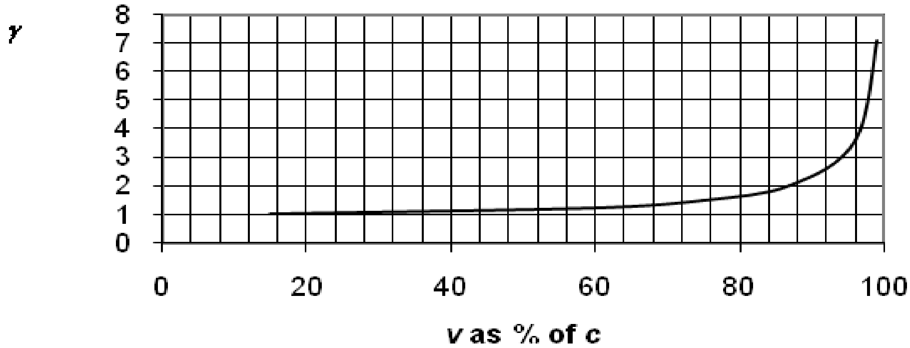
The possibilities are:

- there may be much more potassium-40 in the body than carbon-14
- potassium-40 may be more active than carbon-14 (larger value of λ)
- radiation from potassium-40 may be more energetic than that from carbon-14

- the radiation from potassium-40 may be alpha radiation, which is more readily absorbed by the body than the beta radiation from carbon-14.

Question 28

(a)(i) At these high speeds, the Newtonian equations do not hold, and you have to use $p = \gamma mv$ and $E = \gamma mc^2$. If you calculate γ for different values of v, then you will see that it becomes significantly greater than 1 only at very high speeds indeed:



(a)(ii) v = 99.9% of c, so $v/c = 0.999$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.999)^2}} = \frac{1}{\sqrt{1.999\times10^{-3}}} = 22$$

$p = \gamma mv = 22 \times 1.7 \times 10^{-27}$ kg $\times 3.0 \times 10^{8}$ m s^{-1} $= 1.1 \times 10^{-17}$ kg m s^{-1}

total energy $E = \gamma mc^2$ = rest energy + kinetic energy

rest energy = mc^2, so kinetic energy = $\gamma mc^2 - mc^2 = 22mc^2 - mc^2 = 21mc^2$

Note how the kinetic energy overwhelms the rest energy at this value of γ and how very much larger it is than the $½mc^2$ used by the student.

Kinetic energy = $21 \times 1.7 \times 10^{-27}$ kg $\times (3.0 \times 10^{8}$ m s$^{-1})^2 = 3.2 \times 10^{-9}$ J

(b)(i) Assuming the protons and antiprotons each have a total energy of 50 GeV, 50 GeV $= 50 \times 10^{9} \times 1.6 \times 10^{-19}$ J $= 8.0 \times 10^{-9}$ J $= E = \gamma mc^2$.

$$\gamma = \frac{8.0\times10^{-9}}{mc^2} = \frac{8.0\times10^{-9}}{1.7\times10^{-27}\times(3.0\times10^{8})^2} = 52$$

(b)(ii) This is a case, which sometimes happens in ‘show that’ questions, where it is easier to use the given answer (v) to calculate the quantity already calculated (γ) and show that it is about the same size.

v = 99.98% of c, so $v/c = 0.9998$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-(0.9998)^2}} = \frac{1}{\sqrt{4.00\times 10^{-4}}} = 50 \approx 52 \text{ as calculated above.}$$

Section C questions

The questions in this section are based on the advance notice article 'Taming Nuclear Fusion'.

Question 29

(a)(i) A logarithmic scale does not have equal differences in value between equally-spaced axis units, as the x-axis of this graph, which is a linear scale, does. It has a constant ratio between values that are equally spaced: note how the grid-lines labelled 1 × 10^9, 2× 10^9, 4 × 10^9, 8 × 10^9 are equally spaced, although each value is double the previous one.

(a)(ii) Plotting *N* on a logarithmic scale has the same effect as plotting log(*N*) on ordinary graph paper: it produces a straight line graph. It is much easier to recognise and analyse a straight line on a graph than a curve.

(a)(iii) From the graph, it is clear that the graph has two regions which are roughly linear, and that the change occurred at about 1950. After 1950, the graph is significantly steeper, showing that the exponential factor increased greatly. If you use the graph to estimate the time taken for the population to double, you will find that before 1950 this was about 100 years, while after 1950 it was about 40 years.

(b)(i) From the graph, extrapolation shows that the population in 2007 would have been about 7 × 10^9.
This value gives the energy consumption in the year as
$7 \times 10^9 \times 68 \times 10^9$ J = 4.8×10^{20} J ≈ 5×10^{20} J

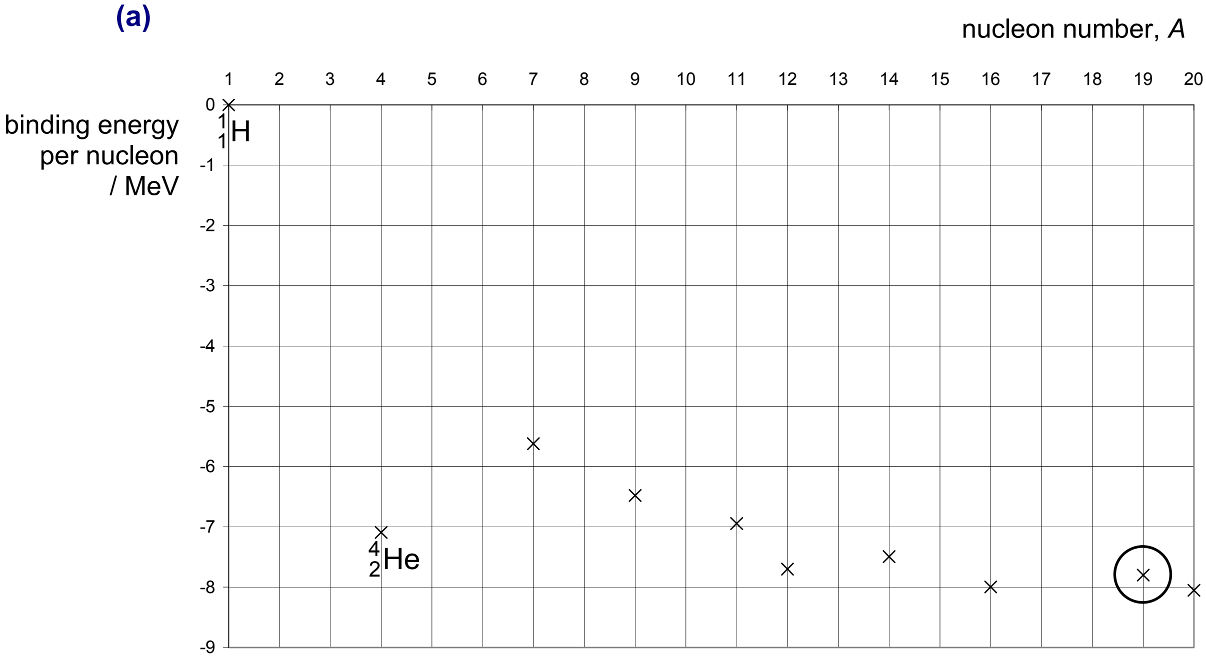
(b)(ii) Lifetime = 4×10^{22} J/5×10^{20} J year^{-1} = 80 years

(b)(iii) There are many factors here which are likely to change. 'Suggest' means that any reasonable suggestion is acceptable. Some possibilities are:

- energy demand per capita likely to increase
- population is also likely to increase
- price increases may cut the energy consumption per capita
- different energy sources may be found.

Question 30

(a)



(b) Because the hydrogen-1 nucleus contains only one nucleon, there cannot be any binding energy.

(c) The binding energy per nucleon, from the graph, is about –7.1 MeV.
Helium-4 has four nucleons, so total binding energy = 4 × binding energy per nucleon = 4 × –7.1 MeV = 28.4 MeV ≈ –28 MeV

(d) Δm = mass of helium nucleus – (mass of 2 protons and two neutrons)
= 6.6240×10^{-27} kg – ($2 \times 1.6693 \times 10^{-27}$ kg + $2 \times 1.6675 \times 10^{-27}$ kg)
= -4.96×10^{-29} kg

Binding energy = $mc^2 = -4.96 \times 10^{-29}\ \text{kg} \times (3.0 \times 10^8\ \text{m s}^{-1})^2$
= $-4.5 \times 10^{-12}\ \text{J} \approx 4 \times 10^{-12}\ \text{J}$

Question 31

(a) If potential energy $= \dfrac{\text{constant}}{r}$, then potential energy × r = constant.

From the graph, PE = 1.6 MeV at r = 0.9 fm, and 0.6 MeV at r = 2.4 fm.

Units are not important in checking the relationship:

1.6 × 0.9 = 1.44 and 0.6 × 2.4 = 1.44

so the relationship does hold.

(You could verify this inverse proportion relationship more simply by observing that, as r doubles from 0.9 fm to 1.8 fm, the potential energy halves from 1.6 MeV to 0.8 MeV.)

(b)(i) 1.44 MeV = $1.44 \times 10^{6} \times 1.6 \times 10^{-19}$ J = 2.3×10^{-13} J for the two protons

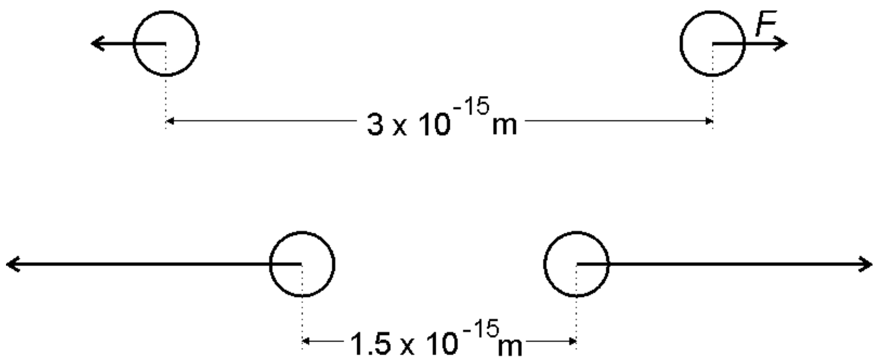
Energy needed by each proton = 2.3×10^{-13} J / 2
= 1.2×10^{-13} J $\approx 1 \times 10^{-13}$ J

(b)(ii) $kT = 1.4 \times 10^{-23} \times 1.5 \times 10^{7} = 2.1 \times 10^{-16}$ J

(b)(iii) $E/kT = 1.2 \times 10^{-13} / 2.1 \times 10^{-16} = 570$
Processes are likely to occur on a time scale of seconds or so if the energy required is in 15 – 30 kT range. Here the energy is 570 kT, so the reaction is extremely slow: effectively it does not occur at all.

(c) Each pair of protons should have equal and opposite forces acting on them, by Newton's Third Law.

Treating each proton as a point source, the force between them will vary as the inverse square of the distance between them. Thus halving the distance will increase the size of the forces by $2^2 = 4\times$.



(d) This question is not structured, as it is intended to pose 'Stretch and Challenge' to the student.

The first stage in tackling it is to plan the different steps in which the calculation must be done:

- find the volume of one mole of ideal gas under these conditions
- use the volume of 1 mole to find the mean volume of 1 particle
- imagine the whole of the volume divided into small cubical boxes of side d
- this distance d is the mean separation of particles in the gas.

For 1 mole, $pV=RT$

$V = RT/p = (8.3 \times 1.5 \times 10^{7}) / 3.4 \times 10^{16}$
$= 3.7 \times 10^{-9}\ \text{m}^3$

A mole contains N_A particles,

so mean volume of 1 particle = $3.7 \times 10^{-9}\ \text{m}^3 / 6.0 \times 10^{23}$
= $6.1 \times 10^{-33}\ \text{m}^3$

if this is a cubical box of side d, $d^3 = 6.1 \times 10^{-33}\ \text{m}^3$
$d = \sqrt[3]{6.1 \times 10^{-33}\ \text{m}^3} = 1.8 \times 10^{-11}$ m.

Question 32

(a) The tokamak needs to produce a large changing flux in the plasma.
To do this, the magnetic circuit must have as large a permeance ('magnetic conductance') as possible. This is done by making the cross-sectional area of the core large, the length of the magnetic circuit as small as possible and making the core from a good 'magnetic conductor', i.e. iron.

(b) The plasma current is produced by an induced emf. To produce an induced emf, you need a changing flux. As the flux is produced by the current in the primary coil, that current must be changing.

(c) The graph of Fig. 32.2 shows that the flux is changing at a constant rate. This means that the induced emf (= rate of change of flux linkage) is constant, so a constant current is produced (graph B).

(d)(i) Ions moving parallel to flux lines have no force acting on them, so there will be no effect on their motion.

(d)(ii) Ions moving at an angle to the flux lines have forces acting on them which are perpendicular to the magnetic field and also perpendicular to ion velocity, so the direction of movement will change. (In the simple case of ions moving at right-angles to the field direction, the resulting motion is circular; if the angle is not 90°, a spiral path is produced.)

Question 33

(a)(i) $P = I^2R = (3 \times 10^6 \text{A})^2 \times 5.0 \times 10^{-7} \text{R} = 4.5 \times 10^6$ W, which is 'a few megawatts'.

(a)(ii) Each $^2_1H^+$ ion has a charge of 1.6×10^{-19} C

number of ions $s^{-1} = 1.0 \times 10^6 \text{ C s}^{-1}/1.6 \times 10^{-19} \text{ C} = 6.3 \times 10^{24} \text{ s}^{-1}$

(b) $f = 1/T = 1/(4 \times 10^{-8} \text{ s}) = 2.5 \times 10^7$ Hz (25 MHz), which is a radio frequency.

(c)(i) Singly-charged ions require 60 kV to give them energy of 60 keV, so the p.d. required is 60 000 V.

(c)(ii) Ions entering the magnetic field in the tokamak will experience *Bqv* forces and be deflected off course, so they must be neutralised if they are to hit their target.