Worked Solutions for Sample Examination Questions

Question 1

(a) Acceleration is greatest when the unbalanced restoring force is greatest. This is when the displacement from equilibrium is greatest at position **Q**.

(b) Velocity has its greatest positive value when at zero displacement, travelling in the + displacement direction: **R**.

Question 2

(a) $f = T^{-1} = 1 / 2.4\ \text{s} = 0.42\ \text{Hz}$.

(b) Given $x = A \cos (2\pi f t)$, $A = 0.20$ m and $t = 2.0$ s.
Substituting $x = 0.2 \cos (2\pi \times 0.42\ \text{Hz} \times 2.0\ \text{s}) = 0.107\ \text{m} \approx 0.11\ \text{m}$.
(NB: the term in the round brackets is in radians, so set your calculator accordingly before pressing the 'cos' key.)

Question 3

(a) The period T is the time for one **complete** oscillation, for example between two successive instants at which the displacement is zero and the velocity is in the same direction.

(b) Reading from the graph: length 1.0 m.

(c)(i) Plotting to give a linear (straight line) graph. Either T^2 against L, or (taking the square root of both quantities) T against $L^{1/2}$ are direct proportional relationships. Alternatively, log T against log L will be a straight line of slope 1/2.

(c)(ii) The direct proportion graphs should be straight lines through the origin. The log–log graph is a straight line with slope 1/2.

(d) Loss of gravitational potential energy = $m\, g\, \Delta h = 9\ \text{kg} \times 9.8\ \text{N kg}^{-1} \times 1.2\ \text{m} =$ 105.8 J.
105.8 J / 43 000 oscillations = 0.00246 J per oscillation ≈ 2.5 mJ per oscillation.

(e)(i) A larger drop height stores more potential energy when raised so the clock lasts longer between 'windings'.

(e)(ii) A massive bob means that the fractional energy loss per oscillation is less (air drag has less effect on a more massive bob). The graph shows that the period is less sensitive to changes in length for longer lengths, so the clock is easier to adjust.

Question 4

(a) The time intervals are not equally spaced (they decrease with time).

(b)(i) The rate of fall of height is directly proportional to the present height (NB: the negative sign is taken into account by stating rate of **fall**). The height falls exponentially.

(b)(ii) Any one of:
- diameter of outflow pipe
- viscosity of fluid (different fluids or different temperatures)
- diameter of the container.

Question 5

(a)(i) 2.5 x 10^{18} U-238 atoms and 12 decays per second.

The decay constant λ = the probability of decay per second
= 12 decays s^{-1} / (2.5 x 10^{18} nuclei)
= 4.8 x 10^{-18} s^{-1} ≈ 5 x 10^{-18} s^{-1}

(a)(ii) Half life = $\frac{\ln 2}{\lambda}$ = 0.693 / (4.8 x 10^{-18} s^{-1}) = 1.4438 x 10^{17} s = 4.5 x 10^{9} years.

(a)(iii) Radioactive decay is a random (quantum) process, so all that is known for a given sample in a given time interval is how many **on average** will decay, not which particular atoms will decay.

(b)(i) 1/8 corresponds to 3 half lives, i.e. $(½)^3$ = ⅛. Taking the value for half life from **(a)(ii)**:

3 x (4.5 x 10^{9} year) = 1.3 x 10^{10} year.

(b)(ii) The stars must have taken some time to form after the start of the Universe ('Big Bang'). Thus the universe is older than its oldest stars.

(c)(i)
Minimum age = 1 / H_{max} = 1 / (3.2 x 10^{-18} s^{-1})
= 3.125 x 10^{17} s
= 3.125 x 10^{17} s / 3.2 x 10^{7} s $year^{-1}$
= 9.8 x 10^{9} year.

Maximum age = 1 / H_{min} = 1 / (1.6 x 10^{-18} s^{-1})
= 6.25 x 10^{17} s
= 6.25 x 10^{17} s / 3.2 x 10^{7} s $year^{-1}$
= 2.0 x 10^{10} year.

(c)(ii) The younger age cannot be correct as it is less than the age of the stars obtained from cosmochronometry, so the larger value of H_0 must be wrong.

Question 6

(a) The area under the graph = ½ $Q\,V$ = ½ (3.5×10^{-3} C) x 8 V = 0.014 J.

(b) $Q = C\,V$, so $C = Q / V$ (which is the gradient of the graph):

$C = Q / V$ = (3.5×10^{-3} C) / 8 V
= 4.375×10^{-4} F ≈ 4.4×10^{-4} F.

Question 7

(a) $Q = C\,V$ = (4700×10^{-6} F) x 100 V = 0.47 C.

(b) Time constant $\tau = C\,R$ so:

$R = \tau / C$ = 0.70 s / (4700×10^{-6} F) = 150 Ω.

(c)(i) Energy stored = ½ $C\,V^2$ = 0.5 x (4700×10^{-6} F) x $(72\text{ V})^2$ = 12.18 J ≈ 12 J.

(c)(ii) Power = energy transferred / time to transfer. Thus time = 12 J / 150 W = 0.08 s.

(c)(iii) Time constant to discharge << time constant to charge.
When charging, $\tau = R\,C$ so the resistance of the conducting lamp must be << R.

Question 8

(a) Change in velocity is from 40 m s^{-1}, to 30 m s^{-1} **in the opposite direction**:

Δv = 40 m s^{-1} – (–30 m s^{-1}) = 70 m s^{-1}.

(b) Momentum $p = m\,v$ (units = kg m s^{-1} or N s):

$\Delta p = m\,\Delta v$ = 0.11 kg x 70 m s^{-1} = 7.7 kg m s^{-1} away from the wall.

Question 9

(a) Energy stored = ½ $k\,x^2$
= ½ x 220 N m^{-1} x $(30 \times 10^{-3}\text{ m})^2$
= 0.099 J ≈ 0.1 J.

(NB: Convert mm to m.)

(b) Ignoring air drag, by conservation of energy $m\,g\,h$ = stored elastic energy:

0.099 = $m\,g\,h$

h = 0.099 J / (0.08 kg x 9.8 N kg^{-1}) = 0.13 m.

(A more roundabout way is to find velocity^2 by equating initial kinetic energy to the stored elastic energy. Then use $v^2 = u^2 + 2\,a\,s$ and solve for s.)

Question 10

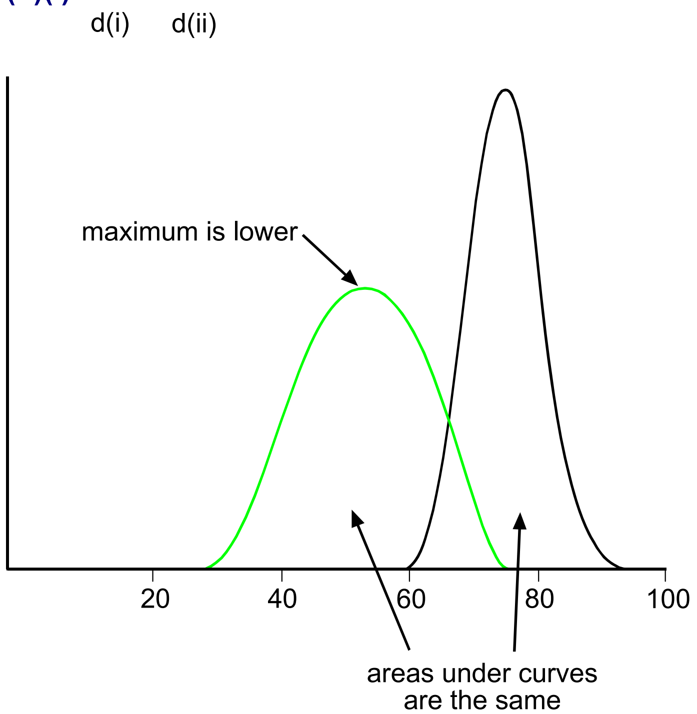
(a) Momentum $p = m\,v$.
Before the crash p = 75 kg x 11 m s^{-1} = 825 kg m s^{-1}.
Final momentum is zero because $v = 0$.
Thus change in momentum = 825 kg m s^{-1}.

(b) Force = $\Delta p / \Delta t$ = 825 kg m s^{-1} / 0.14 s = 5893 N.
Weight = 75 kg x 9.8 N kg^{-1} = 735 N.
Thus average force compared to weight = 5893 N / 735 N = 8.02 ≈ 8 times its weight.

(c) The longer the time for the head to come to rest, the smaller the deceleration and hence (because $F = m\,a$) the smaller the force. A less rigid bag will minimise any bounce; bouncing increases the momentum change and hence the force ($F = \mathrm{d}p / \mathrm{d}t$).

(NB: $F = m\,a$ and $F = \mathrm{d}p / \mathrm{d}t$ are easily derived from each other:
$\mathrm{d}p = m\,v - m\,u$, so $\mathrm{d}p / \mathrm{d}t = m\,(v - u) / t$. But $(v - u) / t$ is equal to the acceleration a, so that $\mathrm{d}p / \mathrm{d}t = m\,a$.)

(d)(i)



For the average force to be lower the maximum force must be lower. The head meets the air bag earlier than it would have hit the steering wheel, so the force starts to rise earlier.

(d)(ii)
Because $F = \Delta p / \Delta t$, $\Delta p = F\,\Delta t$ which is the area under the force–time graph.
Because Δp is the same the area under both graphs must be the same.

Question 11

(a) Y and Z are at the same potential. No atmosphere means no air drag, so the energy required is independent of the path (the force is said to be ‘conservative’). ΔV for X to Y vertically is the same as the path from X to Z.

(b)(i) To move from the surface of the Moon where $V = -2.8 \times 10^6$ J kg^{-1} to ‘infinity’ where $V = 0$ (by definition) requires

$(28 \times 10^{-3}$ kg$) \times (2.8 \times 10^6$ J kg$^{-1}) = 78\,400$ J $\approx 78\,000$J

(b)(ii) Energy needed per mole = 78 400 J. Thus:

energy per molecule = 78 400 J mol^{-1} / (6.0×10^{23} molecule mol^{-1}) = 1.3×10^{-19} J per molecule.

$\frac{1}{2}(m v^2) = 1.3 \times 10^{-19}$ J

thus

$v^2 = [2 \times (1.3 \times 10^{-19}\text{ J})] / (4.7 \times 10^{-26}\text{ kg}) = 5.56 \times 10^6\ (\text{m s}^{-1})^2$

thus

$v = 2360$ m s$^{-1} \approx 2.4 \times 10^3$ m s^{-1}.

(Alternatively just equate the change in gravitational potential energy to kinetic energy: $m\,\Delta V = \frac{1}{2}(m v^2)$. The m cancels to make the calculation slightly quicker.)

(c) The ideal gas law is $p V = n R T$ (n = number of moles).
Given $pV = \frac{1}{3}Nmc^2_{\text{rms}}$ (N = total number of molecules).
Equating the two expressions:

$$nRT = \frac{1}{3}Nmc^2_{\text{rms}}$$

$$c^2_{\text{rms}} = (3nRT)/Nm = (3RT)n/Nm\,.$$

Now $n = N / N_A$ where N_A is the number of molecules in 1 mole.

So

$$\begin{aligned} n / N m &= N / (N_A\, N\, m) \\ &= 1 / (N_A\, m) \\ &= 1 / M_m \end{aligned}$$

where M_m is the mass of one mole. Thus $c^2_{\text{rms}} = 3RT / M_m$.

(d) (NB: Mass of one mole M_m is given in part **(b)(i)** as 28×10^{-3} kg = 28 g.)
Substituting data:

$$c^2_{\text{rms}} = (3 \times 8.31\ \text{J mol}^{-1}\,\text{K}^{-1} \times 290\ \text{K}) / (28 \times 10^{-3}\ \text{kg mol}^{-1}) = 2.58 \times 10^5\ (\text{m s}^{-1})^2$$

thus c_{rms} = 510 m s^{-1}.

(e) Although c_{rms} = 510 m s^{-1} < 2400 m s^{-1}, there will be a distribution of speeds around the r.m.s. speed c_{rms}. Some will have enough energy to escape. Slower molecules will gain energy by collisions with other molecules etc. The energy for a molecule to escape is only about 20 times the typical energy (3/2)kT of a particle. Such events happen relatively easily.

Question 12

(a) $v = d / t$ so

$d = v\,t$
= ½ x 500 s x (3 x 10^{8} m s^{-1}) = 7.5 x 10^{10} m

(b) Take two distance measurements d_1 and d_2 a known time Δt apart.
The (average) approach speed = $(d_1 - d_2) / \Delta t$
or
you could use the Doppler effect $v / c = \Delta\lambda / \lambda$ by measuring the wavelength of the reflected radio waves and comparing them with the wavelength of the transmitted waves. (In fact, this Doppler shift is twice what you might expect, because the comet is receiving waves with Doppler wavelength shift $\Delta\lambda$ and reflecting these, so the observer on Earth gets a second $\Delta\lambda$ wavelength shift.)

Question 13

(a) $d = v\,t$ = (3.0 x 10^{8} m s^{-1}) x (3.2 x 10^{7} s year^{-1})
= 9.6 x 10^{15} m year^{-1} ≈ 1.0 x 10^{16} m.

(b)(i) If the galaxy is 10 000 million light years away, it takes 10 000 million years for light to reach us.

(b)(ii) 10 000 light-year x (1.0 x 10^{16} m light-year^{-1}) = 1.0 x 10^{20} m.

(c)(i) Red-shift refers to an increase in wavelength which (if visible light) will be towards the red end of the spectrum. Now any increase in λ from any part of the electromagnetic spectrum is referred to as 'red-shift'.

(c)(ii) Cosmological red-shift is an increase in λ due to the fact that space itself is expanding thus stretching the waves (as opposed to the movement of the galaxy away from us).

(c)(iii) The light from galaxies that are further away has been travelling for a longer time, and thus more stretching of space-time has occurred.

Question 14

(a) $\frac{v}{c} = 0.4$, giving $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.16}} = 1.09$

(b) $k = \gamma\left(1+\frac{v}{c}\right) = 1.09(1+0.4) = 1.5$

The wavelength would be measured as 1.5 × 21 cm = 32 cm

(c) Assuming that $k = \left(1+\frac{v}{c}\right) = 1.5$ gives v = 0.5c, which is more than the 0.4c specified.

Question 15

(a) The spacecraft travels 4 light years at a speed of 0.50c = 0.50 light years year^{-1}.
Time taken = 4 light years /0.50 light years year^{-1} = 8.0 years, as seen from Earth.

(b) The craft has v/c = 0.50 giving $\gamma = 1/\sqrt{(1-0.25)} = 1.15$.

(c) The measured time on the spacecraft = t/γ where t is measured in the Earth's frame of reference.
Measured time = 8.0 years/1.15 = 6.9 years < 8 years as measured on Earth.

(d) So, which is the correct time for the travel?
8.0 years, as measured from the Earth?
6.9 years, as measured by the travellers in the spacecraft?
Both are correct for the measurer concerned: if there were identical twins, one in the spacecraft and one remaining on Earth, the Earth-bound one would have aged 8.0 years while the traveller aged 6.9 years.

Question 16

(a) Light travels twice the distance d in 40.2 s:

$2\,d = c\,\Delta t$
d = [(3 x 10^{8} m s^{-1}) x 40.2 s] / 2 = 6.03 x 10^{9} m ≈ 6 x 10^{9} m.

Assumptions: speed of light is constant; the change in position of the asteroid during the measurement is negligible.

(b)(i) The difference in reflection times corresponds to twice the distance covered by the asteroid:

$2\,d = \Delta t\,c$
d = [0.2 s x (3 x 10^{8} m s^{-1})] / 2 = 3.00 x 10^{7} m.

(Or a longer method is repeat the calculation as in part **(a)** and find the difference. This is the distance covered in the 14 minutes (= 840 s) between the two distance measurements.)
Thus the speed of the asteroid = 3.00 x 10^{7} m / 840 s = 3.6 x 10^{5} m s^{-1}.

(c) Too long to wait for the return pulse! Pulse too weak to detect.

(d) In each A2 paper, one or two question parts will be introducing 'Stretch and Challenge' by having less structure. You will have to think your own way through the question.

In this case, you must realize that you need to work out how far away the typical distant galaxy lies, and the how long it has taken for the light to reach us. In other easier questions you might be told to calculate these two separately, thus guiding your thinking.

$v = H_0\, d$ thus

$d = v / H_0$
$= 1.0 \times 10^6$ m s^{-1}/ 2.2×10^{-18} s^{-1}= 4.6×10^{23} m.

The light has travelled a distance of 4.6×10^{23} m at 3×10^8 m s^{-1}, so it must have been emitted at a time

$d / c = 4.6 \times 10^{23}$ m/ 3×10^8 m s^{-1} = 1.5×10^{15} s ago
$= 1.5 \times 10^{15}$ s / 3.2×10^7 s year^{-1} ≈ 47 million years ago.

(e) 70 km s^{-1} Mpc^{-1} = (70×10^3 m s^{-1}) / (3.1×10^{22} m)
$= 2.258 \times 10^{-18}$ s^{-1} ≈ 2×10^{-18} s^{-1}

Question 17

(a) The ideal gas law states pV is proportional to T.
If p is constant, V is directly proportional to T, and is represented by graph **A**.

(b) If T is constant then $p\,V$ is constant, so $p \propto 1 / V$, which is represented by graph **B** ('Halving V doubles p').

Question 18

The ideal gas relation is $p\,V$ = (some constant) x T.
The constant depends upon how much gas is present.
Thus $p\,V / T$ is a constant.

Initial conditions of the balloon at ground level:
$V = 5$ m^3, $T = 298$ K and $p = 1.0 \times 10^5$ Pa.

After rising, $V = 10.8$ m^3, $T = 257$ K, p = ?

$$\frac{pV}{T} = \text{a constant}$$

thus

$$\frac{(1.0 \times 10^5\ \text{Pa}) \times 5\ \text{m}^3}{298\ \text{K}} = \frac{p \times 10.8\ \text{m}^3}{257\ \text{K}}.$$

Solving for p gives $p = 4.0 \times 10^4$ Pa.

Question 19

(a) For one mole the ideal gas relationship is $p\,V = R\,T$.
If T remains constant $p\,V$ is a constant (i.e. Boyle's Law):

$(2.0 \times 10^5) \times V = P_{\text{final}} \times V_{\text{final}}$

$V_{\text{final}} = V / 3$

$(2.0 \times 10^5 \text{ Pa}) \times V = P_{\text{final}} \times (V / 3)$

thus

$P_{\text{final}} = 6.0 \times 10^5$ Pa.

(Or, more simply, inverse proportion means that V↓3× $\Rightarrow$ P↑3×.)

(b) $pV = NkT = \frac{1}{3}Nmc_{\text{rms}}^2$ where k is the Boltzmann constant (= R / N_A, i.e. the gas constant per molecule). Thus

$$c_{\text{rms}}^2 = \frac{3kT}{m}.$$

Note that this also comes straight from

$$\frac{3}{2}kT = \frac{1}{2}mc_{\text{rms}}^2.$$

$$\begin{aligned} m &= \frac{4 \times 10^{-3} \text{ kg mol}^{-1}}{N_A} \\ &= \frac{4 \times 10^{-3} \text{ kg mol}^{-1}}{6.0 \times 10^{23} \text{ mol}^{-1}} \\ &= 6.7 \times 10^{-27} \text{ kg} \end{aligned}$$

where N_A is the Avogadro number. So:

$$\begin{aligned} c_{\text{rms}}^2 &= \frac{3kT}{m} \\ &= \frac{3 \times 1.4 \times 10^{-23} \text{ J K}^{-1} \times 300 \text{ K}}{6.7 \times 10^{-27} \text{ kg}} \\ &= 1.88 \times 10^6 \text{ (m s}^{-1})^2 \end{aligned}$$

giving $c_{\text{rms}} = 1.371 \times 10^3 \approx 1.4 \times 10^3$ m s^{-1}.

(c) Using

$$c_{\text{rms}}^2 = \frac{3kT}{m}$$ (derived in part **(b)**)

$c_{rms} = (3kT / m)^{1/2}$

$$\frac{c_{rms} \text{ at } 400\text{K}}{c_{rms} \text{ at } 300\text{K}} = \frac{(3 \times k \times 400\,\text{K} / m)^{1/2}}{(3 \times k \times 300\,\text{K} / m)^{1/2}} = (4/3)^{1/2}.$$

(Or you could calculate c_{rms} at 400 K and show that $c_{rms} = (4/3)^{½} \times 1.4 \times 10^3$ m s^{-1} = 1600 m s^{-1})

Question 20

(a) 0.5 kg of fat stores 1.6 x 10^8 J.
The energy needed to warm the water = $m\ C\ \Delta T$.

Thus

1.6 x 10^8 J = m x 4200 J K^{-1} kg^{-1} (37 – 0) K.

Solving for m = 1.029 x 10^3 kg ≈ 1000 kg.

(b) You cannot possibly drink 1000 kg of water (up to 20 times your body mass!) and hypothermia would soon set in. An amount you could drink, say a few litres, could only reduce fat by 0.5 kg / 1000 for each litre, or 0.5 g per litre drunk.

Question 21

(a) Energy transfer needed = the specific thermal capacity = 4200 J.

N x (3.5 x 10^{-16} J) = 4200 J

therefore N = 1.2 x 10^{19} neutrons.

(b) One interaction would only result in a negligible temperature rise of ΔT where ΔT = 3.5 × 10^{-16} J/(1 kg × 4200 J kg^{-1} K^{-1}) = 8.3 x 10^{-20} K which is undetectable. To accumulate enough energy to raise the temperature by even a microkelvin would take 10^{-6} K/8.3 x 10^{-20} K day^{-1} ≈ 10^{13} days, which is longer than the current age of the Universe.

(c) This question is not structured into parts (i), (ii), (iii) as would be expected in easier questions. This is because it is introducing 'Stretch and Challenge' and expecting you to work out the steps required.

In this case, you must realize that need to find the temperature rise in the helium-3 produced by one particle, and then estimate how many particles this small volume of helium is likely to absorb per day.

$Q = m\ C\ \Delta T$

3.5 x 10^{-16} J = (8.0 x 10^{-6} kg) x (7.0 x 10^{-8} J kg^{-1} K^{-1}) x ΔT

thus

ΔT = 6.3 x 10^{-4} K.

Water absorbs one particle per kg per day. If helium-3 is similar, 8.0×10^{-6} kg will only absorb one particle every $1 / (8.0 \times 10^{-6})$ days = 125 000 days. Detection rates can be improved if many more detectors are used.

Question 22

(a) $kT = (1.4 \times 10^{-23}\ \mathrm{J\ K^{-1}}) \times 300\ \mathrm{K} = 4.2 \times 10^{-21}\ \mathrm{J}$.

(b) Two bonds need $2 \times 3.2 \times 10^{-20}\ \mathrm{J} = 6.4 \times 10^{-20}\ \mathrm{J}$ to separate.

Compared to kT this is

$$\frac{6.4 \times 10^{-20}\ \mathrm{J}}{4.2 \times 10^{-21}\ \mathrm{J}} \approx 15 \text{ times larger,}$$

i.e. to break two bonds needs ~ 15 kT.

(c) Provided E / kT is in the range 15 to 30, processes can occur. kT gives a measure of the average energy. Some molecules will have energies very much greater than this. The fraction which have this large energy is sufficient for many processes to occur on a reasonable time-scale, if E / kT is in the range 15 to 30.

Question 23

(a) $$\varepsilon / kT = \frac{3 \times 10^{-20}\ \mathrm{J}}{(1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}) \times 300\ \mathrm{K}} = 7.2$$

$$\begin{aligned} e^{-\varepsilon / kT} &= e^{-7.2} \\ &= 1 / e^{7.2} \\ &= 1/(2.718)^{7.2} \\ &= 7.5 \times 10^{-4}. \end{aligned}$$

(b) kT has the units of energy J, or eV.

Question 24

(a)(i) The springs are in parallel, so each spring carries only half the load. So for a given load, only half the compression occurs, so k doubles.

(a)(ii) Weight = $m\,g$ = 1000 kg x 9.8 N $\mathrm{kg^{-1}}$= 9.8 kN

$F = -k\,x$

$x = -F / k$ = 9800 N / (5.2×10^{4} N $\mathrm{m^{-1}}$) = 0.188 m ~ 0.2 m = 20 cm.

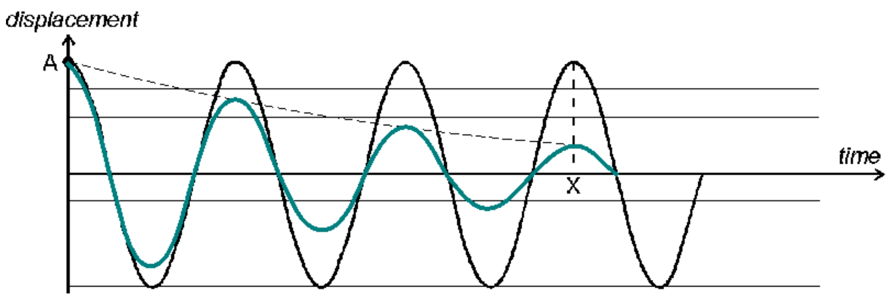
(b)
$$\begin{aligned} T &= 2\pi \sqrt{m / k} \\ &= 2\pi \sqrt{500\ \mathrm{kg} / (5.2 \times 10^{4}\ \mathrm{N\,m^{-1}})} \\ &= 0.616\ \mathrm{s}\ (\text{approx. } 0.6\ \mathrm{s}). \end{aligned}$$

(c) $T \propto \sqrt{m}$, so doubling T means that m is quadrupled (= 2000 kg).
Total mass = truck + maximum load = 500 kg + 1000 kg = 1500 kg < 2000 kg
so the truck is overloaded.

(d) The wheel moves vertically suddenly. The inertia of the supported mass means that the spring will be compressed. The body will start moving upwards as the wheel drops down the other side of the bump.

(e) If the time to travel between the bumps matches the natural period, resonance occurs.

(f) To a good approximation, the period stays the same when damped. The exact amplitude ratios are A, 0.63A, (0.63) x (0.63)A = 0.40A, (0.63) x (0.40)A = 0.25A at the time equal to X, but you would not be expected to do that calculation. You should just draw an exponential decay curve (by eye) starting at A and reaching the first horizontal gridline above X, and sketching a decaying oscillation to stay just inside that curve as shown below.



Question 25

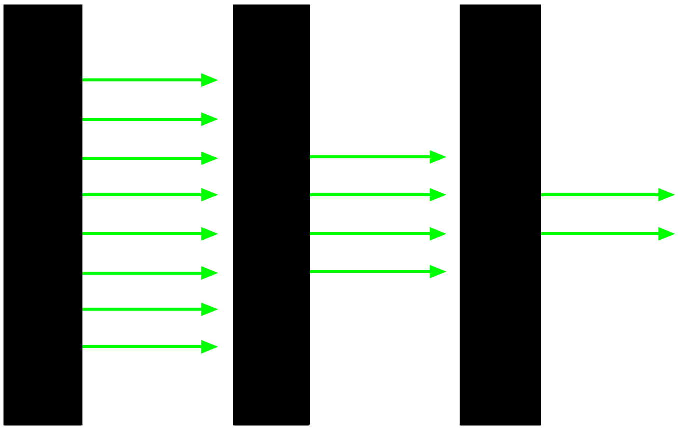
(a) Compute half-thickness x density for the three substances:

- lead 141 600
- steel 171 600
- water 178 000.

Conclusion: the relationship is not bad for steel and water, but lead is somewhat low. On these data, the relationship is not proven.

(b) Tough and strong: for impact survival in the case of a road traffic accident.
Not damaged by radiation: to prevent leaks.
Not inflammable.

(c) (NB: The diagram show sheets of lead, whose half thickness is given in the table before part **(a)**). Each sheet has a thickness equal to the half thickness, so the radiation is reduced by one half after each sheet. Thus after the first there will be 8 arrows, after the second 4 arrows, so emerging after the third sheet will be just two arrows.



(d) Equal increases in thickness absorb the same fraction of incoming radiation, so the fraction absorbed is proportional to the *increase* in thickness:

$I = I_0 \exp(-k\,x)$.

I_0 is initial intensity, I is the intensity after thickness x and
k is the 'absorption constant'.

(e) 120 mm = 10 x 12 or ten half thicknesses.
So the reduction in intensity = $(1/2)^{10}$ = 1/1024 = 0.00098
= 100 × 0.00098 = 0.098% < 0.1%.

Question 26

(a)(i) Density = mass / volume so mass = volume x density.
The volume of a sphere is $4/3\,\pi\,r^3$, so

$m = 4/3\,\pi \times (500\text{ m})^3 \times 5000\text{ kg m}^{-3} = 2.6 \times 10^{12}\text{ kg}$.

(a)(ii) Weight is the force of the gravitational attraction a mass experiences:

$F = G\,M\,m\,/\,r^2$
$= [(6.7 \times 10^{-11}\text{ N m}^2\text{ kg}^{-2}) \times (2.6 \times 10^{12}\text{ kg}) \times 50\text{ kg}] / (500\text{ m})^2$
$= 0.035\text{ N} \approx 0.04\text{ N}$.

(b)(i) The zero of potential is defined to be at infinity. As an object 'falls' onto a planet from infinity it gains kinetic energy at the expense of gravitational potential energy. To reduce from zero implies that the gravitational potential energy becomes negative. To put it another way: to escape, the Little Prince must gain gravitational potential energy. If he escapes and has zero potential energy, he started with less than zero.

(b)(ii) Very little motion in a vertical direction is needed to acquire a kinetic energy that is sufficient to 'escape' (m = 50 kg; kinetic energy = $\frac{1}{2}\,m\,v^2$,
so v is only ~ 1.4 m s^{-1}).

(c)(i) Nitrogen molecules have a greater mass than hydrogen, i.e. (14 + 14) nucleons compared with (1+1) nucleons, a ratio of ~ 14. Notice that 1.7 / 0.12 ≈ 14.

(c)(ii) $kT = (1.4 \times 10^{-23}\text{ J K}^{-1}) \times 93\text{ K} = 1.3 \times 10^{-21}$ J.
For H_2, $E_{grav} \approx 9\,kT$ so escape is possible.
For N_2, $E_{grav} \approx 130\,kT$, so escape is unlikely.

Thus the atmosphere will be nitrogen.

Question 27

(a) The balloon surface represents the entire Universe, i.e. the whole of space-time.

(b) Everything is getting further from everything else, because the space in between is expanding.

(c) The distance between our galaxy O and B increases more than the distance from A in the same time as the balloon is inflated.

(d) The graph shows direct proportion (a straight line through the origin) $v \propto d$ so v = (some constant) x d.
Hubble's Law has just this form: $v = H_0\, d$.

(e) H_0 is the slope of the graph v / d = 2800 km s^{-1} / 40 Mpc
= 70 km s^{-1} Mpc^{-1}

Question 28

(a) Weight = $m\,g$ = (1.4 x 10^3 kg) x 9.8 N kg^{-1} = 1.37 x 10^4 N ≈ 14 kN.

(b) $\rho = m / V$ so

$m = \rho V$ = 1.2 kg m^{-3} x 1.0 x 10^4 m^3 = 1.2 x 10^4 kg.

Upthrust = weight of air displaced = $m\,g$ = (1.2 x 10^4 kg) x 9.8 N kg^{-1}
= 1.176 x 10^5 N ≈ 120 kN.

(c) resultant force $F = ma$, so

F = upthrust – weight of balloon = (1.176 x 10^5 N) – (0.137 x 10^5 N) = 1.039 x 10^5 N.

$$a = \frac{F}{m} = \frac{1.039 \times 10^5 \text{ N}}{1.4 \times 10^3 \text{ kg}} = 74 \text{ m s}^{-2}.$$

(NB: 74 m s^{-2} is very large, but air drag has been ignored in the above calculation.)

(d)(i) The volume of the balloon will increase, thus displacing more air on this account, giving an **increase** in upthrust.

(d)(ii) The weight of the displaced air will decrease, so the upthrust will, on this account, **decrease**.

(d)(iii) As the balloon cools it will contract, so displacing less air, giving **less** upthrust. However, the cooler surrounding air has a higher density, **increasing** the weight of the displaced air and thus the upthrust.

(Whether the balloon rises, falls or stays at the same height depends upon the combined effect of the various effects outlined in the answer, as well as the effects upon air drag!)