Worked Solutions for Sample Examination Questions

Question 1

(a)(i) Condenser lens: the condenser lens **adds** curvature. So the negatively curved waves become flat (or have zero curvature) to lie on the plane of the transparent negative.

(a)(ii) The lens equation is:

$$\frac{1}{v}=\frac{1}{u}+\frac{1}{f}.$$

Zero curvature means the rays are parallel.
Parallel rays require v = infinity, so that u = **minus** f.
Thus to result in parallel rays, the lamp must be at the focus of the condenser lens,

f = 1 / power = 1 / 4 m

thus u = (–) 0.25 m.

(b)(i) Projecting lens:

magnification $M = 6 = v / u$

thus $u = v / M$

v = 0.35 m (see diagram)

u = 0.35 m / 6 = 0.058 m.

(b)(ii) Rearranging the thin lens equation:

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}.$$

Magnification $M = 6 = v / u$, so $v = 6u$.

Cartesian sign convention: u is negative, v is positive so

$$\begin{aligned} f &= \frac{1}{v}-\frac{1}{-v/6} \\ &= \frac{1}{v}+\frac{6}{v} \\ &= \frac{7}{v} \end{aligned}$$

and

$$\frac{1}{f} = \frac{7}{0.35\text{ m}} = 20\text{ D}$$

(D = dioptres, the strength of the lens) and

$$f = \frac{1}{20\text{ D}} = 0.05\text{ m}.$$

Question 2
The samples are measured to one part in 2^{16} = 65536.
0.1 mm in 5 m is 1 part in 50000, which is larger than 1 part in 65536.

Question 3
The noise voltage must be less than the total voltage divided by 2^{16} = 65536.
The noise voltage must be less than 1.5μV.

Question 4
There are two equations you can use here:

the exponent form

$$2^b = \frac{V_{total}}{V_{noise}}$$

and the logarithmic form

$$b = \log_2\left(\frac{V_{total}}{V_{noise}}\right),$$

but the former is easier with most calculators (calculators with $\log_n$ functions need to be in a Maths mode).

(a)

$$2^b = \frac{V_{total}}{V_{noise}} \rightarrow 2^8 = \frac{0.200\text{V}}{V_{noise}}$$

$$V_{noise} = \frac{0.200\text{V}}{2^8} = \frac{0.200\text{V}}{256}$$

= 7.8×10^{-4} V = 0.78 mV

(b)(i)

$$b = \log_2\left(\frac{V_{total}}{V_{noise}}\right) \text{ where } \frac{V_{total}}{V_{noise}} = \frac{100\%}{0.5\%} = 200$$

$b = \log_2(200) = 7.6$

This means that 8 bit encoding measures intervals smaller than the noise variation, so 7 bits is the maximum giving useful information.

Alternative working:

$$2^b = \frac{V_{total}}{V_{noise}}$$

and $\frac{V_{total}}{V_{noise}} = 200$ as above.

By trial and error, $2^6 = 54$, $2^7 = 128$ and $2^8 = 256$, so it can be seen that 2^7 is the largest value which does not exceed the critical 200 in this case, and therefore 7 bits is the maximum.

(b)(ii) Whichever form of calculation you used, it should be clear that the extra information encoded when more than 7 bits are used cannot be detected out of the noise: the noise will swamp the fine detail encoded. Both 8-bit and 16-bit encoded signals would sound the same.

Question 5

(a) A 4 bit binary number allows for 2^4 alternatives = 16 digits.
0 to 9 inclusive = 10 digits which is less than 16, so 4 bits is sufficient.

(b) One hundred 12-digit numbers requires 100 x 12 x 4 = 4800 bits.
Number of bytes = bits / 8 = 600.

(c) The English alphabet has 26 letters. 4 bits can only code $2^4 = 16$ alternatives.
26 alternatives requires 5 bits ($2^5 = 32 > 26$).

(d)(i) Highest frequency = ½ of the sample rate = 10 000 / 2 = 5000 Hz = 5 kHz.

(d)(ii) Any two of the following:

- music may contain a wider frequency spectrum than speech
- lower quality reproduction of speech is still heard correctly
- only a poor quality speaker is used
- stereo is often used for music
- music suffers more from high-frequency cut-off, aliasing, spurious frequencies.

Question 6

(a) UV is absorbed by the ozone layer.

(b) 1.2×10^8 m diameter is imaged using 512 pixels.
Distance per pixel = $(1.2 \times 10^8 \text{ m}) / 512 = 2.34 \times 10^5$ m resolution

Question 7

(a) The wavelengths of Long Wave stations (calculated from $v = f\lambda$, where $v = 3.0 \times 10^8 \text{ m s}^{-1}$) ranged from 1 km to 10 km and were thus long when compared with Medium Wave transmissions, which range from 100 m to 1 km. VHF (part d) has wavelengths in the range 1 m to 10 m.

(b) The bandwidth (9 kHz) is a thirtieth of the total LW radio band.

Only 30 stations can be transmitted if their signals are to be separate. By a similar calculation, the MW band has 300 possible stations.

(c) 200 kHz is a much smaller fraction of the range of frequencies in the VHF band: about a three-thousandth. This means that the VHF band can support ten times as many stations as MW, and a hundred times as many as LW.
Quality of sound: the much greater bandwidth of a VHF station allows a much greater frequency range to be transmitted giving higher quality sound.
It is also a stereo transmission, which requires extra bandwidth.

(d)(i) The rate of sampling = 2 × 15 kHz = 30 kHz.
With 16 bits per sample, bit rate of 30 kHz x 16 bit sample^{-1} = 480 kbit s^{-1}.
Data compression is needed to reduce this to 128 kbit s^{-1}.

(d)(ii) A reason to limit the rate of transmission of digital information is that the bandwidth needed to transmit the information is the same order of magnitude as the bit rate (actually equal to half the bit rate). Limiting the bit rate and so the carrier bandwidth required makes it possible to fit more digital stations into a given frequency band. But this has to be done without excessively degrading the quality of the signal, so a compromise is needed.

Question 8

(a) The ratio $\frac{V_{total}}{V_{noise}} = 5000$, so 2^b cannot exceed 5000.
2^{12} = 4096, while 2^{13} = 8192, so the largest value of b which gives less than 5000, is 12.

(b) Number of bits required for the two stereo channels = 24,
and the sampling frequency = 44 100 Hz.

Rate of information transfer = 44 100 × 24 = 1.1×10^6 bit s^{-1}

$$= \frac{1.1 \times 10^6}{8} \text{ byte s}^{-1} = 1.3 \times 10^5 \text{ bytes s}^{-1}$$

(c) As 1.1×10^6 bit s^{-1} < 2.0×10^6 bit s^{-1}, this could be listened to ‘live’.

A video signal needs a sound signal as calculated above (with possibly only one channel of sound), but also requires all the data for each screen image, refreshed quite frequently (e.g. every 0.04 s). This is such a large quantity of data that it increases the bandwidth requirements considerably. The way to do this with limited bandwidth is to download all the data for the film rather more slowly than it will eventually be played at.

Question 9

(a)

	conductor	**insulator**
electrical conductivity	very high	very low
suitable material	copper	plastic (or rubber)

(b)(i) Conductance $G = (\sigma A) / L$.

Total length of the two cables in series = 60 m.

$$G = \frac{(5.9 \times 10^{7}\ \text{S m}^{-1}) \times (1.8 \times 10^{-6}\ \text{m}^{2})}{60\ \text{m}} = 1.77\ \text{S}.$$

Substituting given data gives G = 1.8 S to 2 significant figures (the precision of the data).

(b)(ii) You do not need to work out the resistance from the conductance.

$R = 1 / G$

$V = I R = I / G$.

$$V = \frac{13\ \text{A}}{1.8\ \text{S}} = 7.2\ \text{V}.$$

Substituting data gives 7.2 V.

(b)(iii) Power = $I V$ = 13 A x 7.2 V = 93.6 W (~100 W)

(b)(iv) When coiled, the cooling of the cable is reduced (less surface area in contact with the air). So the power dissipated will raise the temperature of the inner coils, possibly melting the insulation and and/or becoming a fire hazard.

Question 10

(a)(i) $I = V / R$ = (200 x 10^{-3} V) / (5 x 10^{6} Ω) = 4.0 x 10^{-8} A. (NB: mV are converted to V. The equation is a re-arrangement of the definition of resistance, not an application of Ohm's law.)

(a)(ii) The total resistance in the circuit is now 10 MΩ.
$I = V$ / (total R) = 2.0 x 10^{-8} A.

(a)(iii) $V = I R$ = (2.0 x 10^{-8} V) x (5 x 10^{6} Ω) = 0.1 V. (NB: A suitable device to measure 0.1 V with a reasonable precision might be a potential divider.)

(b)(i) The best choice is the cathode ray oscilloscope.

(b)(ii) There are two reasons. The cathode ray oscilloscope has the highest internal resistance, so the output voltage is not dropped too much when connected to the external load presented by the cathode ray oscilloscope. The variable sensitivity allows a near full-scale deflection (FSD) setting to take data.

(b)(iii) Cathode ray oscilloscope:

$$V_{out} = V_{in} \times \frac{R_{LOAD}}{R_{LOAD} + R_{DEVICE}}$$

$$= 0.2 \text{ V} \times \frac{25 \text{M}\,\Omega}{25 \text{ M}\Omega + 5\text{M}\,\Omega}$$

$$= 0.167 \text{ V}$$

$$= 167 \text{ mV.}$$

If you had chosen the digital voltmeter a similar calculation gives 57 mV.

If you had chosen the ammeter, the current I that would flow would be

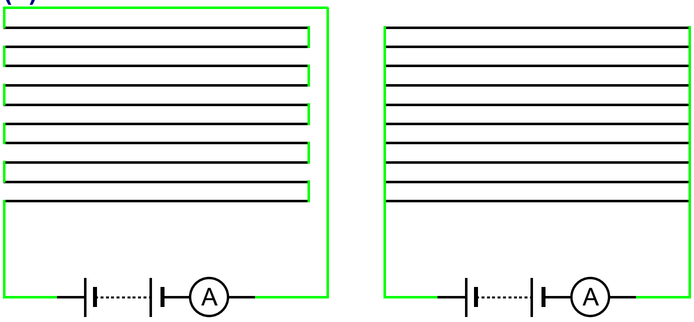
V / (total R) = 0.2 V / [15 Ω + (5 x 10^6 Ω)] = 4 x 10^{-8} A,

which when flowing through the ammeter only corresponds to a voltage drop of

$I\,R$ = (4 x 10^{-8} A) x 15 Ω = 0.6 microvolts!

Question 11

(a)



(b)

	series connection	parallel connection
p.d. across each track / V	1.2	12
current in each track / A	20	2
conductance of each track / S	17	0.17

The question states that **each** track must dissipate 24 W.

Power = $V \times I$

Series connection: the total resistance in the circuit is 10 times the resistance of a single track. So (ignoring the internal resistance of the battery as instructed in the question) the p.d. across each track will be 1/10 of the total p.d. or 1.2 V.

For a series connection

current I = power / V = 24 W / 1.2 V = 20 A.

Conductance $G = 1 / R = I / V$ = 20 A / 1.2 V = 17 S (to 2 significant figures)

Parallel connection: each track is connected directly to the battery, so the p.d. across each track = 12 V.

For a parallel connection

current I = power / 12 V = 24 W / 12 V = 2 A.

Conductance $G = I / V$ = 2 A / 12 V = 0.17 S (to 2 significant figures).

(c)(i) Start from the familiar

$$R = \frac{\rho L}{A}$$

where L = length, A = cross sectional area and ρ = resistivity.

$$G = \frac{1}{R} = \frac{A}{\rho L} = \frac{\sigma A}{L}$$

where $\sigma = 1 / \rho$ = conductivity.

Now

A = width w x thickness t.

Substituting the above for A and re-arranging for w gives

$$w = \frac{GL}{t\sigma}.$$

$$w = \frac{0.17\ \text{S} \times 1.0\ \text{m}}{(2.0 \times 10^{-4}\ \text{m}) \times (4.2 \times 10^{5}\ \text{S m}^{-1})}$$
$$= 2.0 \times 10^{-3}\ \text{m}.$$

Substituting the given data gives w = 2.0 mm (to 2 significant figures).

(c)(ii) The best choice is the parallel connection. The tracks are quite narrow and so do not obscure too much of the view. If one track breaks, the remaining nine will still work.

With the series connection, the conductance needs to be 100 times larger, which for the same thickness of track implies that the width is 100 times wider (= 20 cm each, so 10 tracks will obscure the whole view!). Furthermore, if one track fails, no track receives any current.

The parallel arrangement uses much less conducting material and should be cheaper.

Question 12

(a) Any situation where the variation in time of the frequencies present in the sound signal is needed, e.g. analysing the sound from a piece of machinery, quality control in the manufacture of musical instruments.

(b) Features of the whistle to choose from are:

- Qualitative observation – frequency falls gently then rises more rapidly. This general observation could be quantified thus: frequency starts around 1.5 kHz, falls to 1.0 kHz then ends near to 2.0 kHz.
- The whistle frequencies at any instant have a range of about ±0.2 kHz = 200 Hz bandwidth.
- Whistle frequencies are higher than those of the speech.

Question 13

(a)

angle of rotation	received signal
0°	maximum
90°	minimum (ideally zero)
180°	maximum

(b) Electromagnetic waves are transverse and thus can be plane polarised. The plane of polarisation is taken to be that of the electric field. When the metal rods of the t.v. aerial are parallel to the direction of polarisation the signal is a maximum. When at 90º there is no component of the electric field in the direction of aerial rods so the signal is zero.

(c)(i) The wave speed equation is $v = f\lambda$:

$3.0 \times 10^{8}\ \mathrm{m\ s^{-1}} = f \times 0.14\ \mathrm{m}$

thus

$f = 2.14 \times 10^{9}\ \mathrm{Hz} \approx 2\ \mathrm{GHz}$.

(c)(ii) Transmission rate = frames/second x pixels/frame x bits/pixel
= $25 \times 2 \times 10^{6} \times 8$

(NB: the information in the question is 1 **byte** = 8 bits)

so the rate = 400×10^{6} bits $\mathrm{s^{-1}}$.

(c)(iii) There are two ways to calculate this.

Either:

the time interval per bit = 1 / (bits per the time interval)

where ‘bits per the time interval’ is the transmission rate, so

time per bit = $1 / (400 \times 10^6$ bit $s^{-1}) = 2.5 \times 10^{-9}$ s bit^{-1} = 2.5 ns bit^{-1}.

Or:

the carrier wave frequency [i.e. the answer to part **(c)(i)**] is 2.14 GHz.
One bit corresponds to 5.5 cycles (count them on the diagram). Thus the time for 5.5 cycles at 2.14 GHz is

$5.5 \times (1 / 2.14 \times 10^9$ Hz) = 2.6 ns

(to 2 significant figures).

Question 14

(a) Advantages of digital form (answer only requires one to be discussed):

- less noise
- easy removal of noise
- can be reproduced (and thus re-transmitted easily)
- can be processed by a computer (‘signal processing’)
- many kinds of information can be represented (text, vision, music etc).

Disadvantages of digital form:

- sampling reduces the frequency range (equivalent to a loss of information) because of upper sampling frequency limit
- introduces low frequency aliases that distort the signal
- loss of resolution due to limited (i.e. finite number of) bits per sample.

(b)(i) Count them on the diagram: 15 levels (including zero).

(b)(ii) 4 bits are needed to encode 15 levels:

4 bits can encode $2^4 = 16$ alternatives (more than the 15 needed); 3 bits would only encode $2^3 = 8$ (less than the 15 needed).

(b)(iii) The diagram shows 16 samples in 1 ms equivalent to $16 / 10^{-3}$ = 16 kHz.

(b)(iv) Information transmission rate in bits per second
= sample rate x bits per sample

= answer to **(b)(iii)** x answer to **(b)(ii)**

= $(16 \times 10^3$ Hz) x 4 bit = 64 kbit s^{-1}.

Question 15

(a) Weight = $m\,g$

$= 300\text{ kg} \times 9.8\text{ N kg}^{-1} = 2940\text{ N}$

$\approx 2900\text{ N}$ (to 2 significant figures).

(b)

$$\text{Stress} = \frac{\text{force}}{\text{area}}.$$

The force is just the weight of the freezer calculated in part **(a)**. Thus:

$$\text{stress} = \frac{2940\text{ N}}{8.0 \times 10^{-4}\text{ m}^2} = 3.7\text{ MPa}\ \text{(to 2 significant figures)}$$

(to be consistent with the precision of the data given in the question) which is greater than the stated yield stress of 3.5 MPa and can thus cause permanent damage.

Question 16

(a) Any of the following statements would get the mark:

- extension is directly proportional to the force
- extension varies linearly through the origin
- as the force doubles so does the extension.

(b)(i) The ‘area under the graph’ is shaped like a triangle.

The area of a triangle = ½ base x height.

Base = the extension = 4 mm = 4×10^{-3} m.

Height = force = 90 N.

Thus:

energy stored = ½ (4×10^{-3} m) x 90 N = 0.18 J.

(b)(ii) Young modulus E = stress / strain:

$$\begin{aligned}\text{stress} &= \frac{\text{force}}{\text{area}}\\ &= \frac{90\text{ N}}{2.5 \times 10^{-7}\text{ m}^2}\\ &= 3.6 \times 10^{8}\text{ N m}^{-2}\ (=\text{Pa})\end{aligned}$$

$$\text{strain} = \frac{\text{extension}}{\text{original length}}$$

$$= \frac{4 \times 10^{-3}\ \text{m}}{2.0\ \text{m}}$$

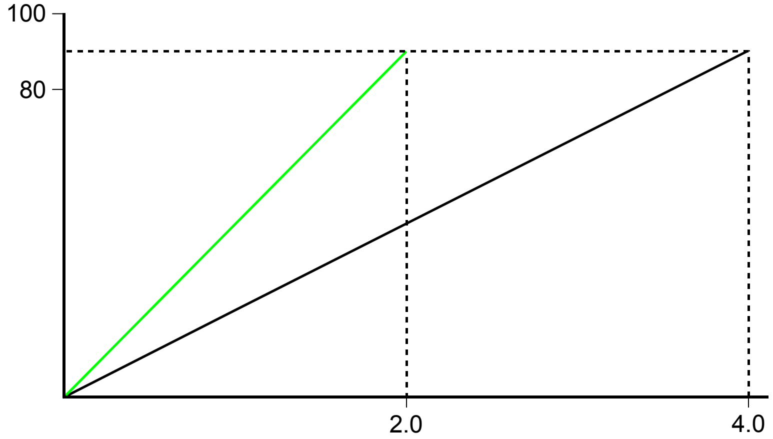
$$= 2 \times 10^{-3}.$$

Thus

$$E = \frac{3.6 \times 10^{8}\ \text{N m}^{-2}}{2 \times 10^{-3}}$$

$$= 1.8 \times 10^{11}\ \text{N m}^{-2}.$$

(c)(i)



A straight line graph from the origin to the point (2.0, 90), i.e. with double the gradient of the first wire.

(c)(ii) The Young modulus is a property of the material and does not depend upon the shape of a particular sample, so it is the same. [Note the question is illustrated by a force–extension graph and **not** a stress–strain graph.]
Half the original length under the same stress will result in half the original extension, so the strain remains the same.

Question 17

(a) **A** shows ductile or plastic fracture: possible materials are metals, some plastics.
B shows brittle fracture: possible materials are glass, ceramics, some alloys (e.g. cast iron).

(b) Material **A** has close-packed planes of atoms. Single atoms can slide over each other when a relatively low stress is applied, thus giving rise to 'dislocation' movement.
Material **B** is amorphous like glass and cannot break and re-form bonds easily to allow slip of whole planes of atoms. Brittle materials fail by small surface cracks or imperfections that concentrate stress at the crack tip allowing the crack to grow.

Question 18

(a) Frequency is the number of (any one of the following will do) oscillations / waves / cycles / vibrations / samples per second.

(b) Digital samples are samples of the signal taken at discrete intervals, and quantised on a digital scale composed of a sequence of binary digits.

(c) A bit of information is a binary digit (the 0 or 1); one bit is 1 / 8th of a byte

Question 19

(a)(i) Each pixel only needs 2 alternative values (black or white). A single bit encodes 2^1 alternatives: $2^1 = 2$.

(ii) The letter M is 7 pixels tall, so each pixel covers
2.6 mm / 7 = 0.37 mm ~ 4×10^{-4} m.

(iii) (Converting mm to m) the area of page = $(300 \times 10^{-3}\ \text{m}) \times (200 \times 10^{-3}\ \text{m})$
= $6.00 \times 10^{-2}\ \text{m}^2$

Area per pixel = $(3.7 \times 10^{-4}\ \text{m})^2 = 1.37 \times 10^{-7}\ \text{m}^2\ \text{pixel}^{-1}$.

Thus pixels per covered page = $6.00 \times 10^{-2}\ \text{m}^2 / 1.37 \times 10^{-7}\ \text{m}^2\ \text{pixel}^{-1}$
= 4.3×10^5 pixels.

One pixel only needs one bit, so bits / page = 4.3×10^5 bits.

(b) At this lower resolution the M and the y may be confused with letters of a similar shape (e.g. N and H, q and g). A block of 24 pixels would not allow for a clear column between the letters.

Question 20

(i) 80 A h = 80 C s^{-1} for 60 x 60 seconds = 288 000 C.

(ii) 10 batteries in parallel can deliver 10 x 80 A for 1 hour = 800 A h.
Thus in 4 hours they will deliver 800 A h / 4 h = 200 A.

(iii) Power = $I\ V$ = 200 A x 24 V = 4.8 kW.

(iv) Delivering the power will also heat the wiring, and result in heating of the batteries themselves due to their internal resistance. (Also, electric motors are not 100% efficient, and work must be done against air drag and friction between all moving parts in the gear box etc etc.)