Revision Guide for Chapter 11

Contents

Student's Checklist

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I can show my understanding of effects, ideas and relationships by describing and explaining cases involving:

momentum as the product of mass × velocity force as rate of change of momentum conservation of momentum when objects interact Revision Notes: Momentum; Newton's Laws of motion Summary Diagrams: Conservation of momentum; Collisions from different viewpoints; Examples of collisions; Momentum and force; Jets and rockets	
work done (*as force × distance moved in the direction of the force:* including cases where the force does not act in the direction of the resulting motion) changes of gravitational potential energy and kinetic energy when objects move in a gravitational field motion in a uniform gravitational field See Revision Guide Chapter 9: Work; kinetic energy; potential energy; free fall; projectile	
the gravitational field and gravitational potential due to a point mass Revision Notes: Gravitational field; Gravitational potential Summary Diagrams: Field direction and equipotentials;	
motion in a horizontal circle and in a circular gravitational orbit about a central mass Revision Notes: Motion in a circle Summary Diagrams: Centripetal acceleration; Kepler's laws; Satellites and Kepler's third law	

I can use the following words and phrases accurately when describing effects and observations:

Momentum Revision Notes: Momentum	
kinetic energy and potential energy See Revision Guide Chapter 9: kinetic energy; potential energy	
gravitational field, gravitational potential	

Revision Notes: Gravitational field; Gravitational potential	

I can sketch, plot and interpret:

graphs showing the variation of a gravitational field with distance, and know that the area under the graph shows the change in gravitational potential Revision Notes: Gravitational field	
graphs showing the variation of gravitational potential with distance, and know that the tangent to the curve gives the gravitational field strength Revision Notes: Gravitational potential	
diagrams illustrating gravitational fields and the corresponding equipotential surfaces Summary Diagrams: Field direction and equipotentials	

I can make calculations and estimates involving:

kinetic energy $\frac{1}{2} mv^2$, gravitational potential energy change mgh energy transfers and exchanges using the idea: work done $\Delta E = Fs \cos\theta$, (no work is done when F and s are perpendicular) See Revision Guide Chapter 9: Work; kinetic energy; potential energy; free fall; projectile	
momentum $p = mv$ and $F = \Delta(mv) / \Delta t$ Revision Notes: Momentum; Newton's Laws of motion	
circular and orbital motion: $a = v^2/r$; $F = mv^2/r$ Revision Notes: Motion in a circle Summary Diagrams: Centripetal acceleration;	
gravitational fields: $F_{grav} = -\frac{GmM}{r^2}$, $g = \frac{F_{grav}}{m} = -\frac{GM}{r^2}$ gravitational potential energy $-\frac{GmM}{r}$ gravitational potential $V_{grav} = \frac{E_{grav}}{m} = -\frac{GM}{r}$ Revision Notes: Gravitational field; Gravitational potential Summary Diagrams: Field direction and equipotentials;	

Revision Notes

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Momentum

Momentum is *mass* x *velocity*. Momentum is a **vector quantity**. The SI unit of momentum is kg m s^{-1}.

Newton's second law defines force as the rate of change of momentum

$$F = \frac{\Delta(mv)}{\Delta t}.$$

If the mass is constant this can be expressed as 'force = mass × acceleration' because acceleration is rate of change of velocity.

The change of momentum of an object acted on by a force is:

$\Delta(mv) = F\Delta t$.

The product $F\Delta t$ is called the impulse of the force.

The thrust on a rocket of the jet of gases that it ejects is equal to the rate at which the jet carries away momentum. This is given by the *mass ejected per second* x *the velocity of the jet*.

When two objects interact, for example in a collision, one object loses an amount of momentum and the other object gains an equal amount. The total momentum of the two objects is the same after the interaction as before. This is the principle of **conservation of momentum**.

Since the time of interaction Δt is the same for both objects, the forces acting on the objects are equal and opposite. This is **Newton's Third Law**. It is a consequence of the conservation of momentum.

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Newton's laws of motion

Newton's first law of motion states that an object remains at rest or moves with constant velocity unless acted on by a resultant force.

Newton's first law defines what a force is, namely any physical effect that is capable of changing the motion of an object. If an object is at rest or in uniform motion, either no force acts on it or forces do act on it and the resultant force is zero.

Newton's second law of motion states that the rate of change of momentum of an object is equal to the resultant force on the object.

That is, $F = \mathrm{d}p / \mathrm{d}t$, where $p = mv$ is the momentum of an object acted on by a resultant force F.

For an object of constant mass m, acted on by a force F

$$F = m\frac{\mathrm{d}v}{\mathrm{d}t} = ma$$

The SI unit of force is the newton (N). 1 N is the force that gives a 1 kg mass an acceleration of 1 m s^{-2}.

Newton's third law of motion states that when two objects interact, there is an equal and opposite force on each.

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Gravitational field

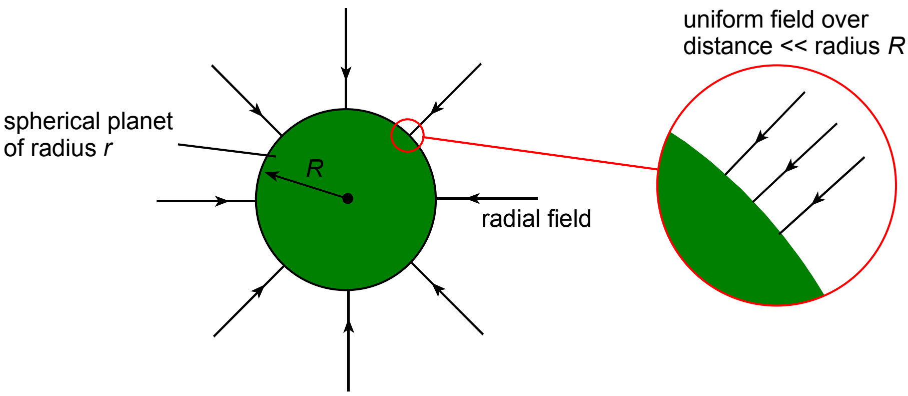
The strength *g* of a gravitational field at a point is the gravitational force per unit mass acting on a small mass at that point. Gravitational field strength is a **vector quantity** in the direction of the gravitational force.

The SI unit of gravitational field strength is N kg^{-1} or equivalently m s^{-2}.

The force *F* on a point mass *m* at a point in a gravitational field is given by $F = m\,g$, where *g* is the gravitational field strength at that point.

Close to the surface of the Earth, the gravitational field is almost uniform. The lines of force are parallel and at right angles to the Earth's surface.

A uniform gravitational field



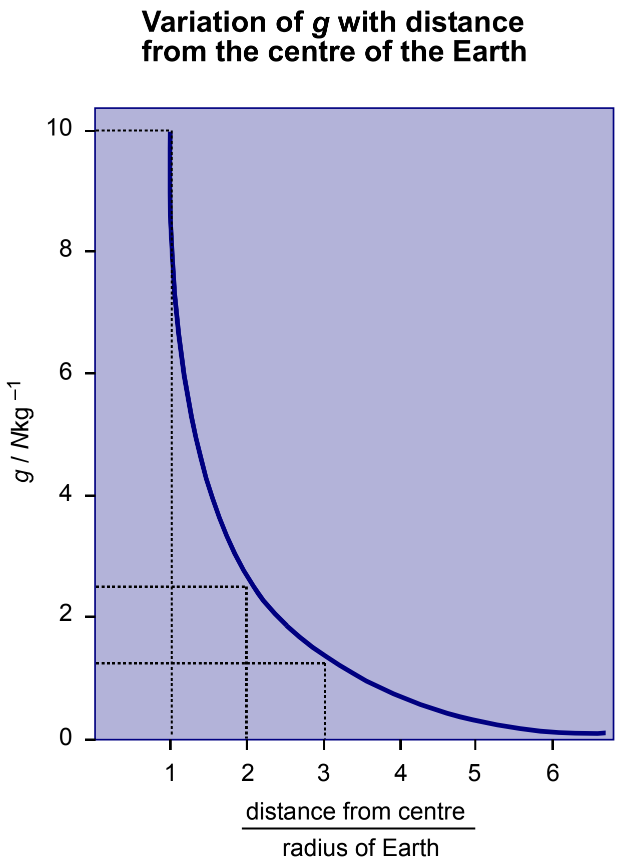
On a large scale, the gravitational field is radial.

Newton's law of gravitation states that the force of gravitational attraction *F* of a mass *M* on another mass *m* obeys an inverse square law:

$$F = -\frac{GMm}{r^2}$$

where *r* is the distance from the centre of *M* to *m* and the minus sign indicates that the force acts towards the mass *M*. The measured value of the Universal Gravitational Constant *G* is 6.67×10^{-11} N m^2 kg^{-2}.

The gravitational field strength $g = F / m = -G M / r^2$ at distance *r* from the centre of the mass *M.*



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Gravitational potential

The gravitational potential at a point is the potential energy per unit mass of a small object placed at that point. This is the work done per unit mass to move a small object from infinity to that point.

The gravitational potential energy E_P of a point mass m is given by $E_P = m\ V_G$, where V_G is the gravitational potential at that point.

The SI unit of gravitational potential is J kg^{-1}. Gravitational potential is a **scalar quantity**.

An **equipotential** is a surface of constant potential. No change of potential energy occurs when an object is moved along an equipotential. The lines of force are therefore always perpendicular to the equipotential surfaces.

The gravitational field strength at a point in a gravitational field is the negative of the potential gradient at that point. In symbols $g = -dV_G / dx$.

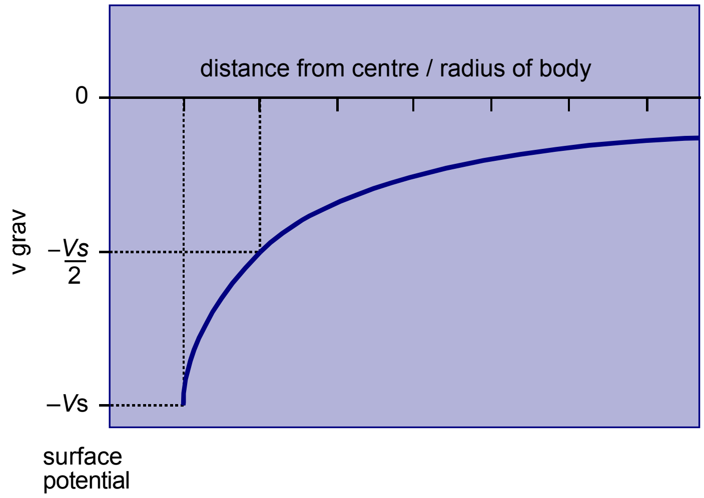
In an inverse square gravitational field, the field strength is:

$$g = -\frac{GM}{r^2}.$$

and the gravitational potential is:

$$V_G = -\frac{GM}{r}$$

Variation of gravitational potential with distance from the centre of a spherical body



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Motion in a circle

An object moving in a horizontal circle at constant speed changes its direction of motion continuously. Its velocity is not constant because its direction of motion is not constant. The resultant force is directed towards the centre of the circle. It is called the **centripetal force**.

For an object moving at constant speed v along a circular path of radius r , the acceleration towards the centre is:

$$a = \frac{v^2}{r}$$

and the centripetal force F acting on it is:

$$F = ma = \frac{mv^2}{r}$$

where m is the mass of the object.

The centripetal force does no work on the moving mass because the force is always at right angles to the direction of motion. The energy of the motion is therefore constant.

The time T taken to move once round the circular path is
$T = 2\pi r / v$

For a proof that $a = \frac{v^2}{r}$ see Summary diagrams: Centripetal acceleration

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Summary Diagrams (OHTs)

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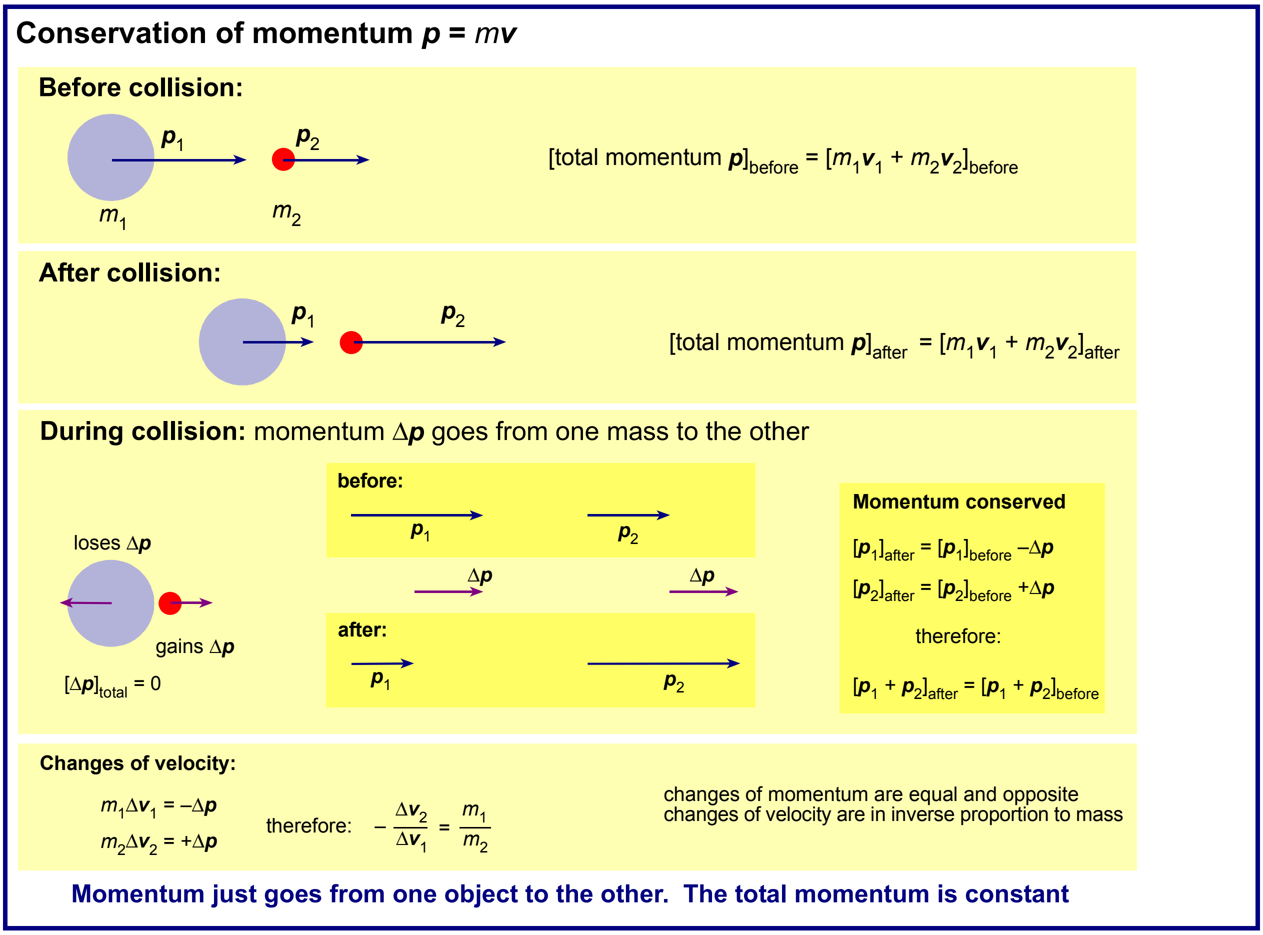
Conservation of momentum



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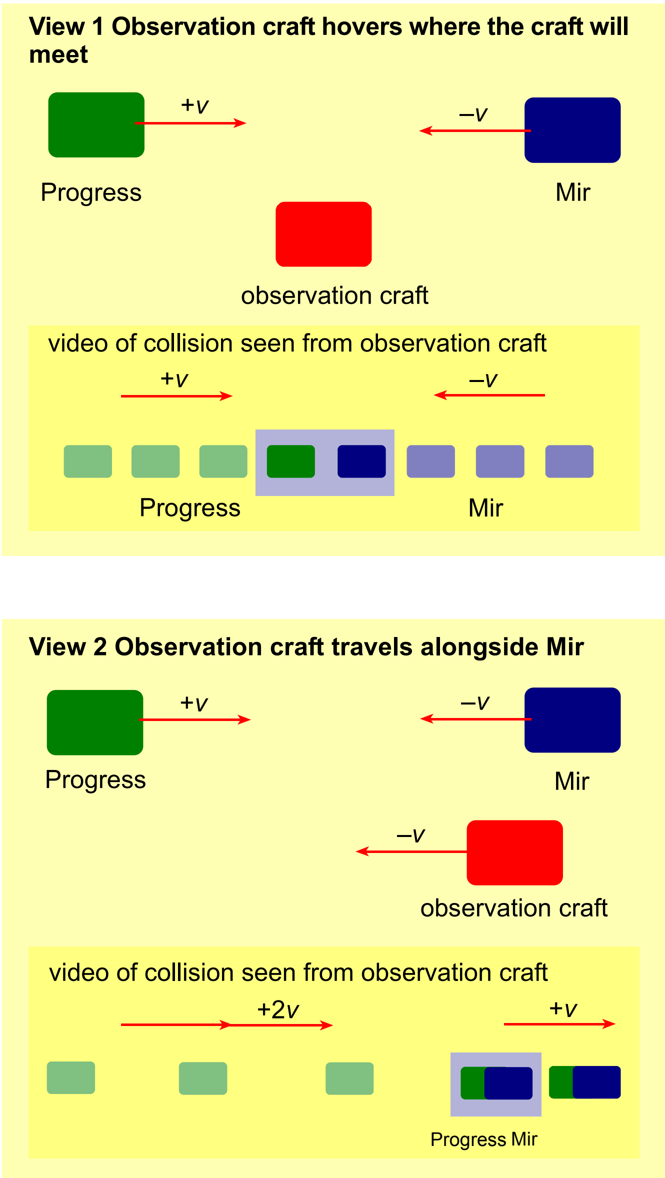
Collisions from different viewpoints

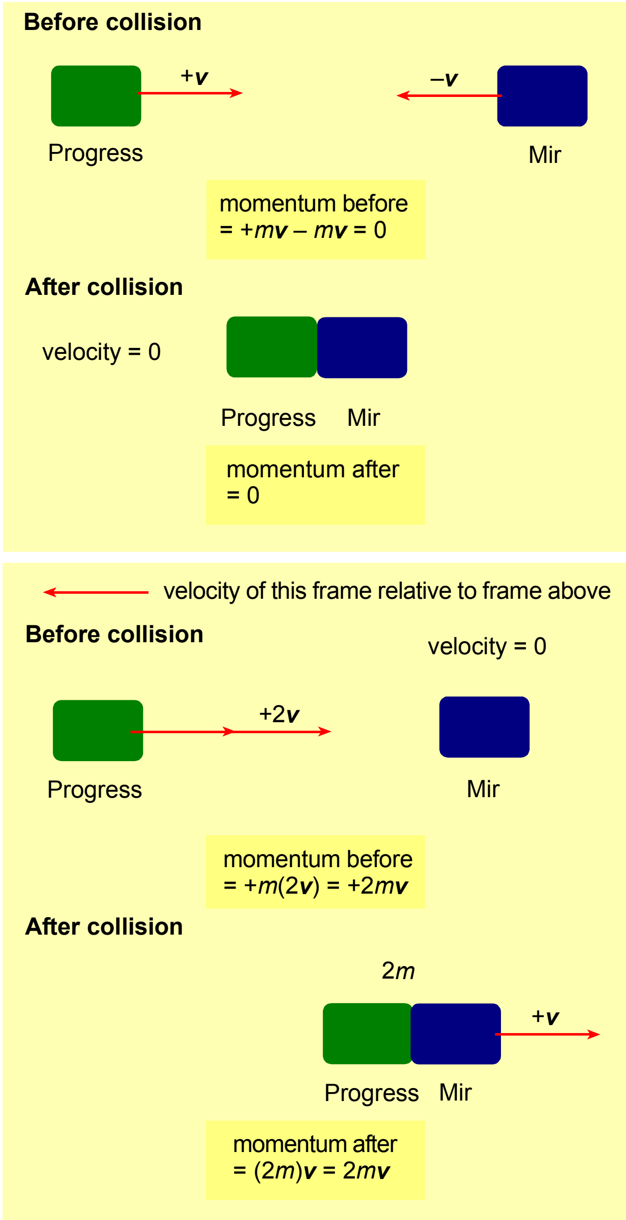
Two equally massive spacecraft dock together and join. The collision is seen from two different moving points of view. Momentum is conserved from both points of view

Two crafts approach one another and dock together



The same event looks different from two different points of view

One event seen from two points of view



Momentum is different in the two views of the same event, but in each case: momentum after = momentum before

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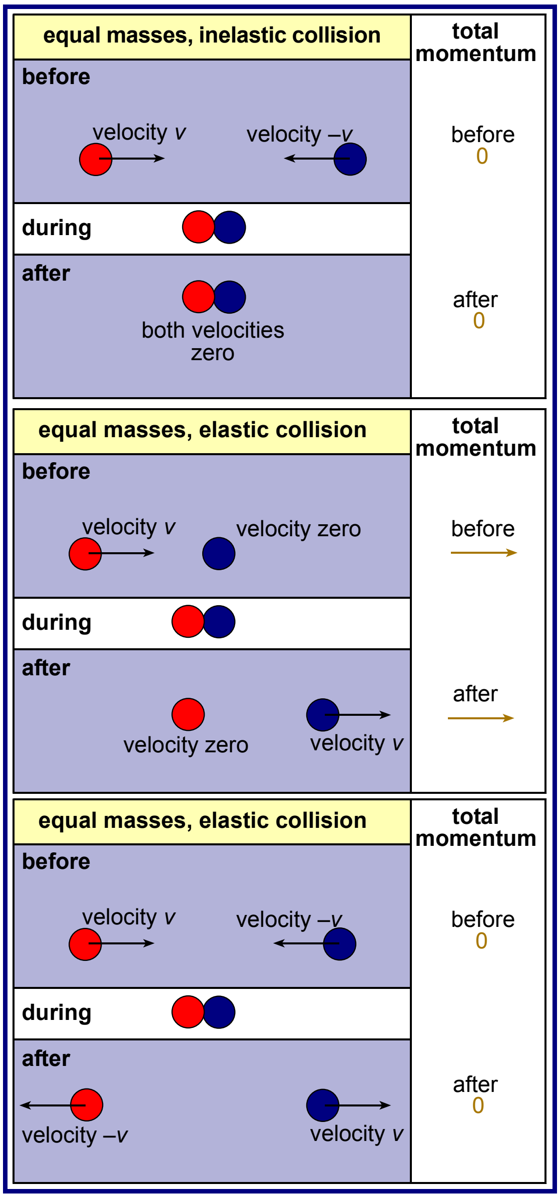
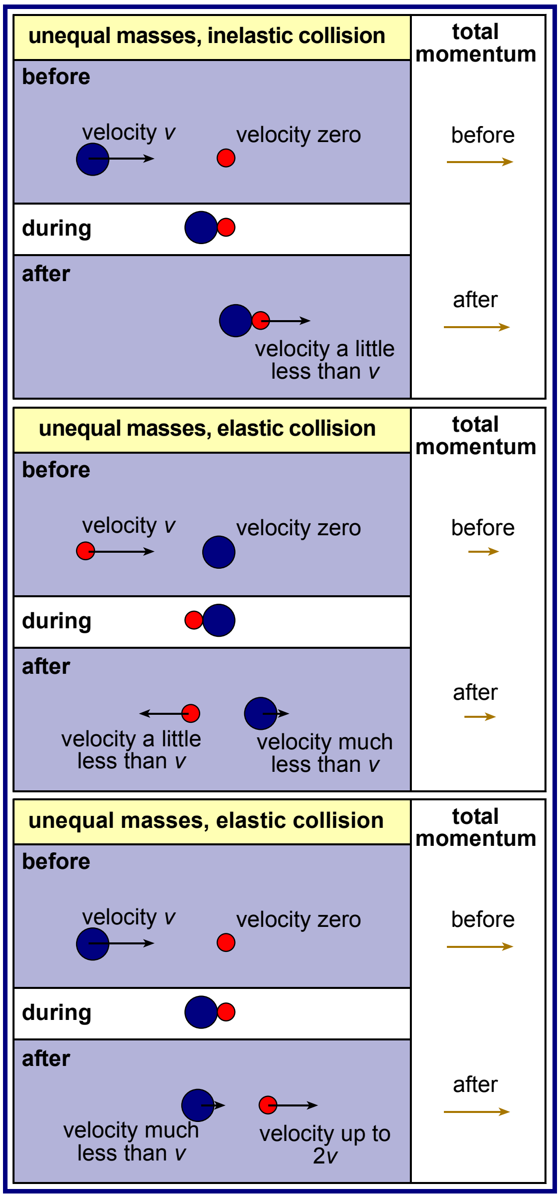
Examples of collisions

Here are six collisions. Notice that the total momentum before is always equal to the total momentum after.





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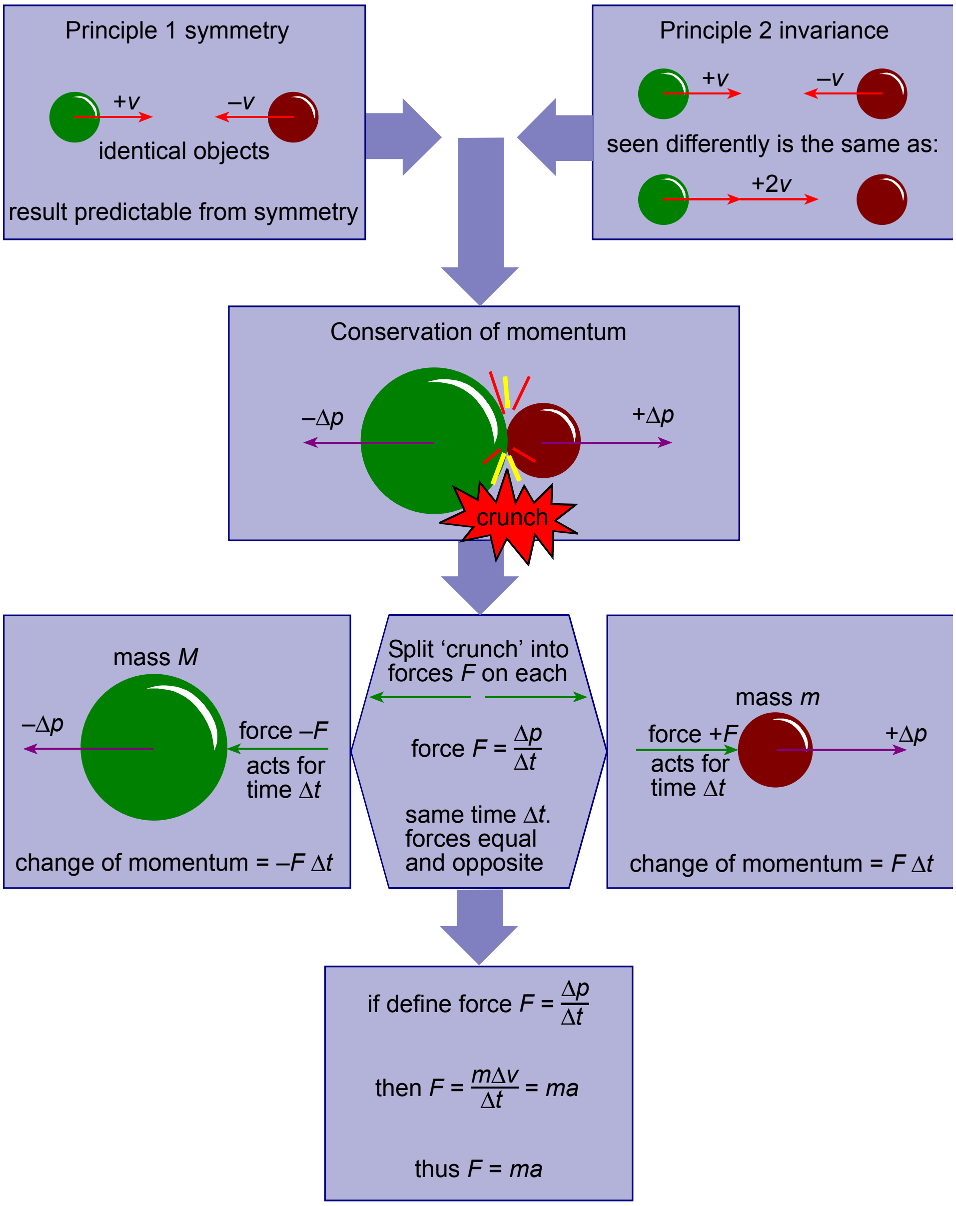
Momentum and force

Thinking about momentum and forces



From symmetry and invariance (looking differently can't change events):

1. momentum is conserved
2. define mass from change of velocity in collision
3. define force as rate of change of momentum, giving $F = ma$
4. forces on interacting objects act in equal and opposite pairs

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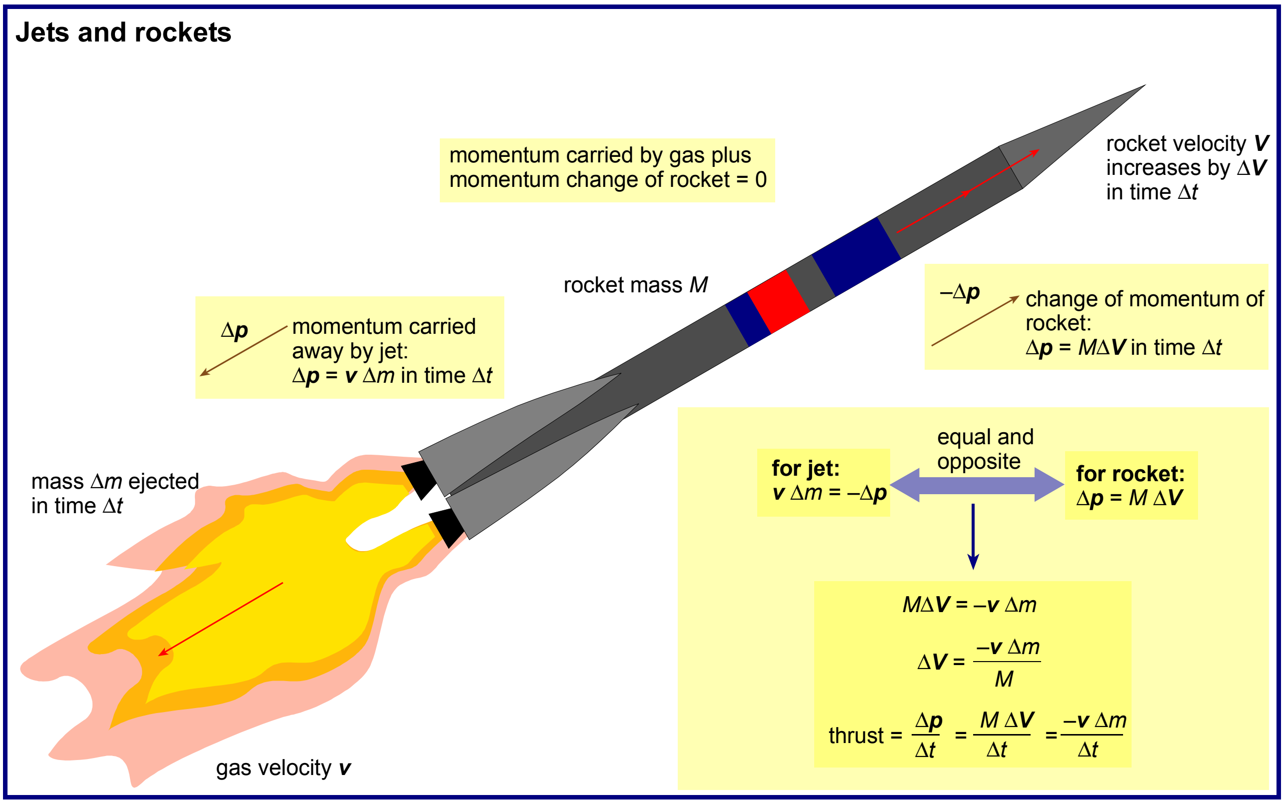
Jets and rockets



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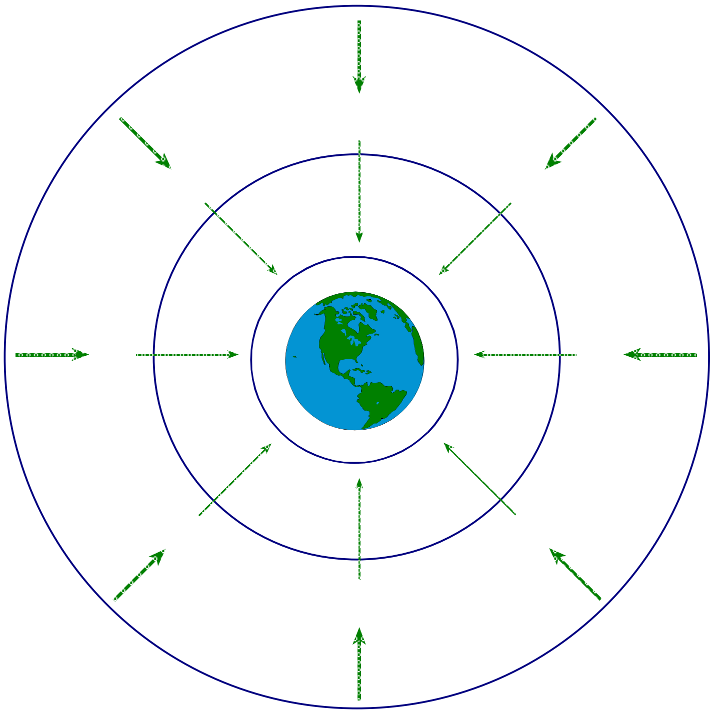
Field direction and equipotentials

Gravitational gradients around Earth

Equipotentials are spheres



contour of constant gravitational potential

gravitational field

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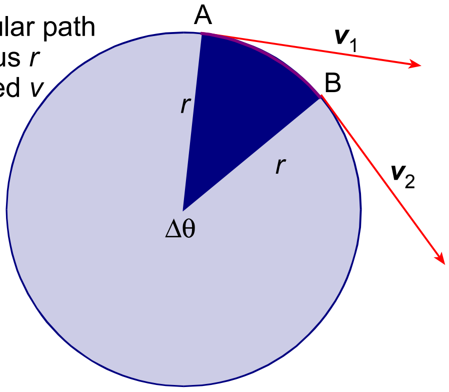
Centripetal acceleration

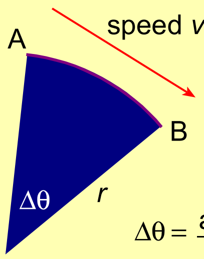
Acceleration towards centre of circular orbit

circular path
radius r
speed v



velocity turns through angle Δθ as planet goes along circular path in short time Δt

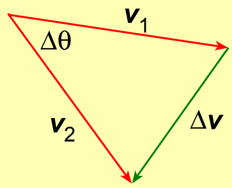
radius turns through Δθ



$$\Delta\theta = \frac{\text{arc AB}}{r}$$

arc AB = distance in time Δt at speed v
arc AB = $v\,\Delta t$

velocity turns through Δθ



change of velocity Δv towards centre of circle

$$\frac{v\,\Delta t}{r} = \Delta\theta \quad \Leftrightarrow \quad \Delta\theta \approx \frac{\Delta v}{v}$$

$$\frac{v\,\Delta t}{r} \approx \frac{\Delta v}{v}$$

multiply by v: $v\,\dfrac{v\,\Delta t}{r} = \Delta v$

divide by Δt: $\dfrac{v^2}{r} = \dfrac{\Delta v}{\Delta t}$ = acceleration

Acceleration towards centre = $\dfrac{v^2}{r}$

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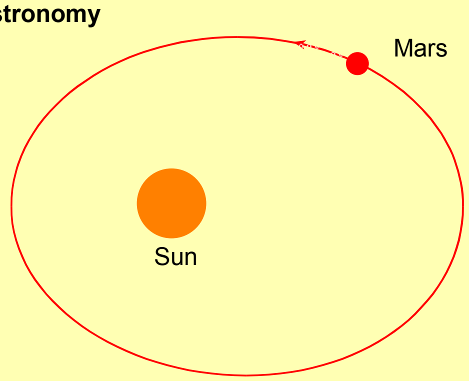
Kepler: Geometry rules the Universe

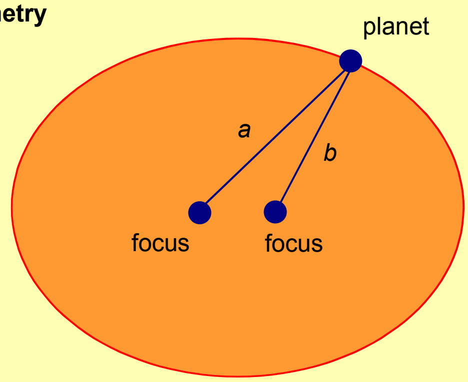
Law 1: a planet moves in an ellipse with the Sun at one focus

Astronomy



Orbit of Mars an ellipse with Sun at a focus

Geometry

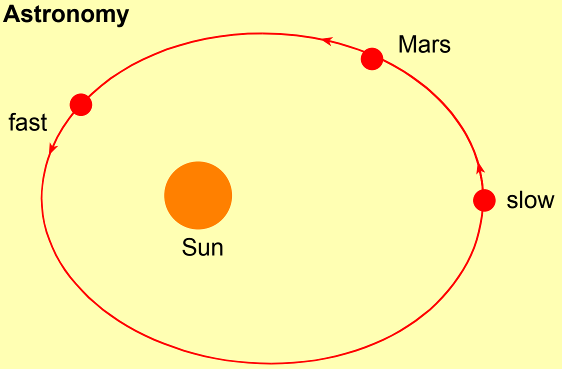


Ellipse: curve such that sum of *a* and *b* is constant

Kepler: Geometry rules the Universe

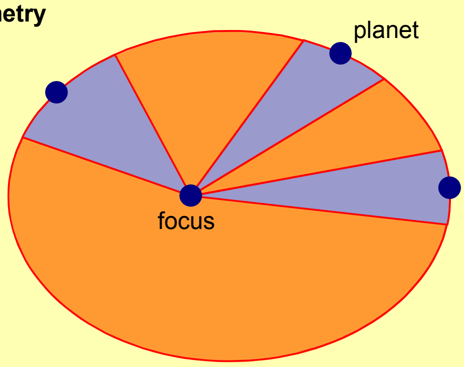
Law 2: the line from the Sun to a planet sweeps out equal areas in equal times

Astronomy



Speed of planet large near Sun, smaller away from Sun

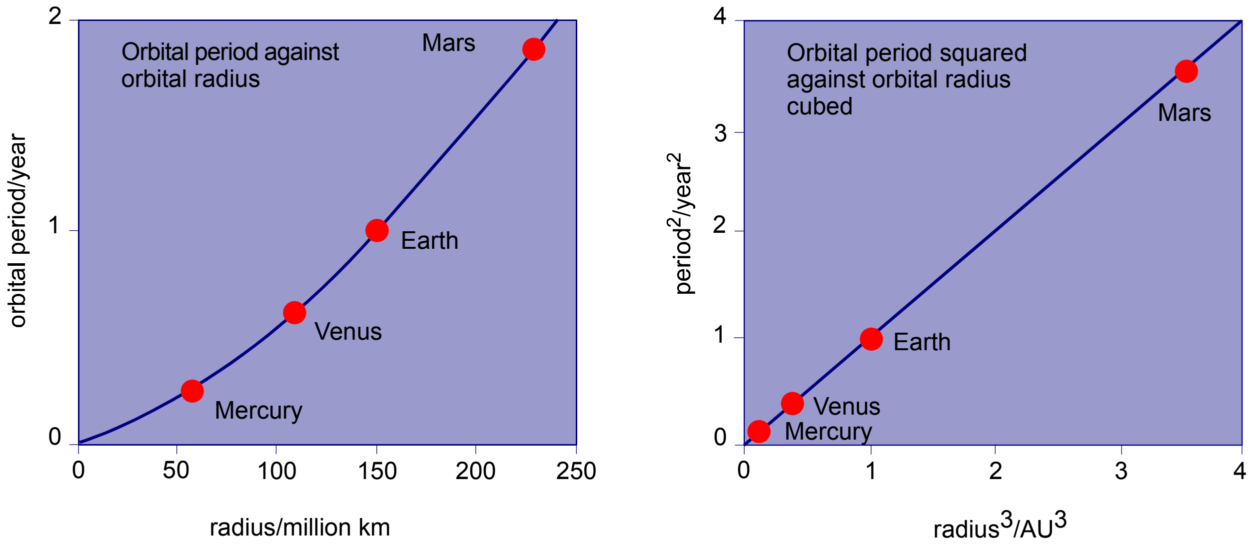
Geometry



Areas swept out in same time are equal

Kepler: Geometry rules the Universe

Law 3: square of orbital time is proportional to cube of orbital radius



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Satellites and Kepler's third law

The example of a geostationary satellite is used to derive Kepler's third law.

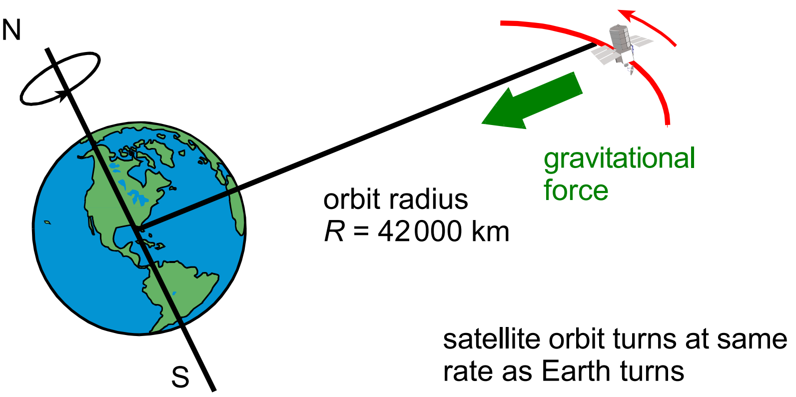
Geostationary satellite

m = mass of satellite
R = radius of satellite orbit
v = speed in orbit

G = gravitational constant
$= 6.67 \times 10^{-11}$ N kg^{-2} m^2

M = mass of Earth
$= 5.98 \times 10^{24}$ kg

T = time of orbit
= 24 hours = 86400 s



Calculating the radius of orbit

Force producing acceleration to centre $\frac{mv^2}{R}$ — equal — Gravitational force on satellite $\frac{GMm}{R^2}$

Forces are equal: $\frac{mv^2}{R} = \frac{GMm}{R^2}$

divide by m: $\frac{v^2}{R} = \frac{GM}{R^2}$

multiply by R: $v^2 = \frac{GM}{R}$

equal

speed in orbit depends on time of orbit and radius

$v = \frac{2\pi R}{T}$

$v^2 = \frac{4\pi^2 R^2}{T^2}$

equate expressions for v^2: $\frac{GM}{R} = \frac{4\pi^2 R^2}{T^2}$

rearrange to calculate R: $\frac{GMT^2}{4\pi^2} = R^3$

Kepler's third law deduced

insert values of G, M and T: $R = 4.2 \times 10^4$ km

$R = 6.6 \times$ radius of Earth (6400 km)

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