Revision Guide for Chapter 10

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Student's Checklist

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I can show my understanding of effects, ideas and relationships by describing and explaining cases involving:

capacitance as the ratio $C = Q/V$ the energy stored on a capacitor $E = \frac{1}{2}QV$ Revision Notes: Capacitance Summary Diagrams: Energy stored on a capacitor	
decay of charge on a capacitor modelled as an exponential relationship between charge and time; with the rate of removal of charge proportional to the quantity of charge remaining Revision Notes: Exponential decay processes Summary Diagrams: Exponential decay of charge	
radioactive decay modelled as an exponential relationship between the number of undecayed atoms, with a fixed probability of random decay per atom per unit time Revision Notes: Exponential decay processes Summary Diagrams: Radioactive decay; Half-life and time constant	
simple harmonic motion of a mass m subject to a restoring force $F = -kx$ proportional to the displacement Revision Notes: Simple harmonic motion Summary Diagrams: Describing oscillations; Motion of simple harmonic oscillator; Graphs of SHM; Computing oscillator motion step by step	
changes of kinetic energy $\frac{1}{2}mv^2$ and potential energy $\frac{1}{2}kx^2$ during simple harmonic motion Summary Diagrams: Elastic potential energy; Energy flows in an oscillator	
free and forced vibrations (oscillations) of an object damping of oscillations resonance (i.e. when natural frequency of vibration matches the driving frequency) Revision Notes: Damping and resonance Summary Diagrams: Resonance	

I can use the following words and phrases accurately when describing effects and observations:

for capacitors: half-life, time constant for radioactivity: half-life, decay constant, random, probability Revision Notes: Exponential decay processes	
simple harmonic motion, amplitude, frequency, period, resonance Revision Notes: Simple harmonic motion; Damping and resonance Summary Diagrams: Describing oscillations; Resonance	
relationships of the form $dx/dt = -kx$, i.e. where a rate of change is proportional to the amount present Revision Notes: Exponential decay processes	

I can sketch, plot and interpret graphs of:

radioactive decay against time (plotted both directly and logarithmically) Summary Diagrams: Radioactive decay; Half-life and time constant	
decay of charge, current or potential difference with time for a capacitor (*plotted both directly and logarithmically*) Summary Diagrams: Exponential decay of charge	
charge against voltage for a capacitor as both change, and know that the area under the curve gives the corresponding energy change Summary Diagrams: Energy stored on a capacitor	
displacement–time, velocity–time and acceleration–time for simple harmonic motion (showing phase differences and damping where appropriate) Summary Diagrams: Graphs of SHM	
variation of potential and kinetic energy in simple harmonic motion Summary Diagrams: Energy flows in an oscillator	
variation in amplitude of a resonating system as the driving frequency changes Summary Diagrams: Resonance	

I can make calculations and estimates making use of:

small difference methods to build a numerical model of a decay equation small difference methods to build a model of simple harmonic motion Revision Notes: Exponential decay processes Summary diagrams: Computing oscillator motion step by step	
data to calculate the time constant $\tau = RC$ of a capacitor circuit data to calculate the half-life of a radioactive source Revision Notes: Exponential decay processes Summary Diagrams: Exponential decay of charge; Half-life and time constant	
the relationships for capacitors: $C = Q/V$ $I = \Delta Q/\Delta t$ $E = (1/2)\, QV = (1/2)\, CV^2$ Revision Notes: Capacitance Summary Diagrams: Energy stored on a capacitor	
the basic relationship for simple harmonic motion $\frac{d^2x}{dt^2} = a = -(\frac{k}{m})x$ the relationships $x = A \sin 2\pi ft$ and $x = A \cos 2\pi ft$ for harmonic oscillations the period of simple harmonic motion: $T = 2\pi\sqrt{\frac{m}{k}}$ and the relationship $F = -kx$ Revision Notes: Simple harmonic motion Summary Diagrams: Describing oscillations; Motion of simple harmonic oscillator; Graphs of SHM; Computing oscillator motion step by step	
the conservation of energy in undamped simple harmonic motion: $E_{total} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ Summary Diagrams: Energy flows in an oscillator	

Revision Notes

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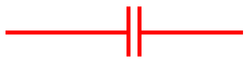
Capacitance

Capacitance is charge stored / potential difference, $C = Q/V$.
The SI unit of capacitance is the farad (symbol F).

Capacitor symbol



One farad is the capacitance of a capacitor that stores a charge of one coulomb when the potential difference across its terminals is one volt. This unit is inconveniently large. Thus capacitance values are often expressed in microfarads (μF) where 1 μF = 10^{-6} F.

Relationships
For a capacitor of capacitance C charged to a potential difference V:
Charge stored $Q = C\,V$.
Energy stored in a charged capacitor $E = 1/2\ Q\,V = 1/2\ C\,V^2$.

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Exponential decay processes

In an exponential decay process the rate of decrease of a quantity is proportional to the quantity remaining.

Capacitor discharge
For capacitor discharge through a fixed resistor, the current I at any time is given by $I = V / R$, where $V = Q / C$. Hence $I = Q / RC$.

Thus the rate of flow of charge from the capacitor is

$$I = \frac{dQ}{dt} = -\frac{Q}{RC}$$

where the minus sign represents the decrease of charge on the capacitor with increasing time.

The solution of this equation is

$$Q = Q_0 e^{-t/RC}.$$

The time constant of the discharge is RC.

Radioactive decay
The disintegration of an unstable nucleus is a random process. The number of nuclei δN that disintegrate in a given short time δt is proportional to the number N present:
$\delta N = -\lambda N \delta t$, where λ is the decay constant. Thus:

$$\frac{\delta N}{\delta t} = -\lambda N.$$

If there are a very large number of nuclei, the model of the differential equation

$$\frac{dN}{dt} = -\lambda N$$

can be used. The solution of this equation is

$$N = N_0 e^{-\lambda t}.$$

The time constant is 1 / λ. The half-life is $T_{1/2}$ = ln 2 / λ.

Step by step computation
Both kinds of exponential decay can be approximated by a step-by-step numerical computation.

1. Using the present value of the quantity (e.g. of charge or number of nuclei), compute the rate of change.
2. Having chosen a small time interval d*t*, multiply the rate of change by d*t*, to get the change in the quantity in time d*t*.
3. Subtract the change from the present quantity, to get the quantity after the interval d*t*.
4. Go to step 1 and repeat for the next interval d*t*.

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Simple harmonic motion

Simple harmonic motion is the oscillating motion of an object in which the acceleration of the object at any instant is proportional to the displacement of the object from equilibrium at that instant, and is always directed towards the centre of oscillation.

The oscillating object is acted on by a restoring force which acts in the opposite direction to the displacement from equilibrium, slowing the object down as it moves away from equilibrium and speeding it up as it moves towards equilibrium.

The acceleration $a = F/m$, where $F = -ks$ is the restoring force at displacement *s*. Thus the acceleration is given by:
$a = -(k/m)s$,

The solution of this equation takes the form
$s = A\sin(2\pi ft + \phi)$ where the frequency *f* is given by $(2\pi f)^2 = k/m$, and ϕ is a phase angle.

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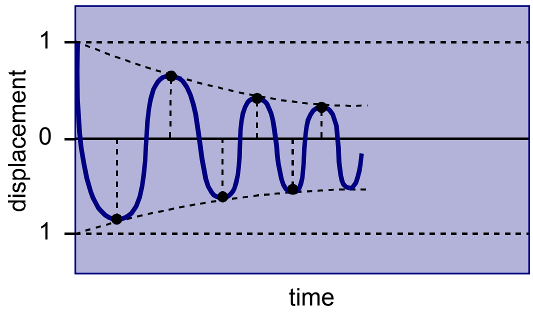
Damping and resonance

In any oscillating system, energy is passed back and forth between parts of the system:

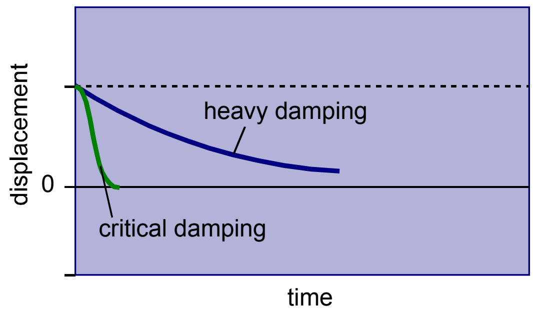
1. If no damping is present, the total energy of an oscillating system is constant. In the mechanical case, this total energy is the sum of its kinetic and potential energy at any instant.
2. If damping is present, the total energy of the system decreases as energy is passed to the surroundings.

If the damping is light, the oscillations gradually die away as the amplitude decreases.

Damped oscillations

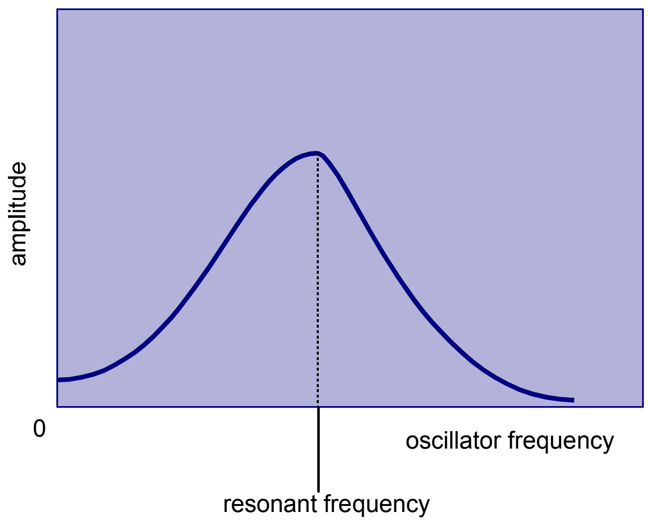


Lightly damped oscillations



Increased damping

Forced oscillations are oscillations produced when a periodic force is applied to an oscillating system. The response of a resonant system depends on the frequency *f* of the driving force in relation to the system's own natural frequency, f_0. The frequency at which the amplitude is greatest is called the resonant frequency and is equal to f_0 for light damping. The system is then said to be in resonance. The graph below shows a typical response curve.



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Summary Diagrams (OHTs)

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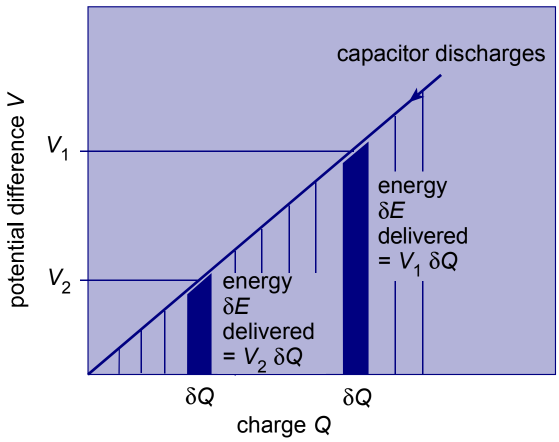
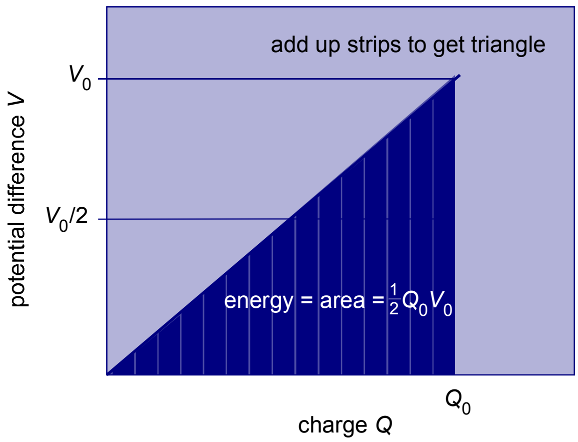
Energy stored on a capacitor

Energy stored on capacitor = $\frac{1}{2}QV$





Energy delivered at p.d. V when a small charge δQ flows $\delta E = V\ \delta Q$

Energy δE delivered by same charge δQ falls as V falls

Energy delivered = charge × average p.d.

Energy delivered = $\frac{1}{2}\ Q_0\ V_0$

Capacitance, charge and p.d.

$C = Q/V$ → $Q_0 = CV_0$, $V_0 = Q_0/C$

Equations for energy stored

$E = \frac{1}{2}\ Q_0V_0$

$E = \frac{1}{2}\ CV_0^2$

$E = \frac{1}{2}\ Q_0^2/C$

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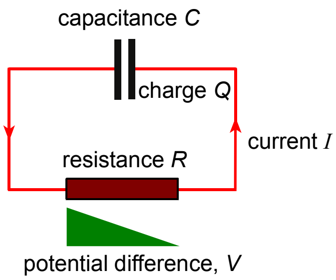
Exponential decay of charge

Exponential decay of charge

What if....
current flowing through resistance is proportional to potential difference and potential difference is proportional to charge on capacitor?



current $I = dQ/dt$

Potential difference V proportional to charge Q

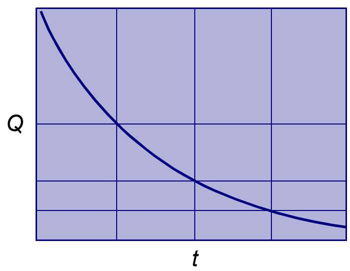
$V = Q/C$

Rate of flow of charge proportional to potential difference

$I = dQ/dt = V/R$

flow of charge decreases charge
rate of change of charge proportional to charge

$dQ/dt = -Q/RC$



time for half charge to decay is large if resistance is large and capacitance is large

Charge decays exponentially if current is proportional to potential difference, and capacitance *C* is constant

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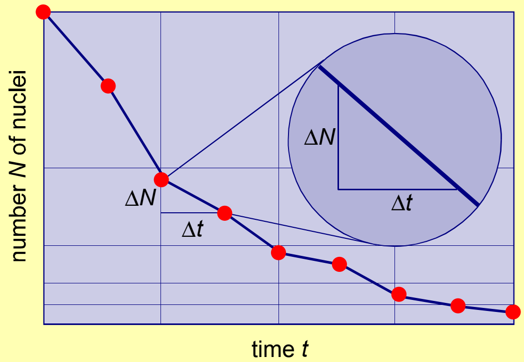
Radioactive decay

Smoothed out radioactive decay

Actual, random decay



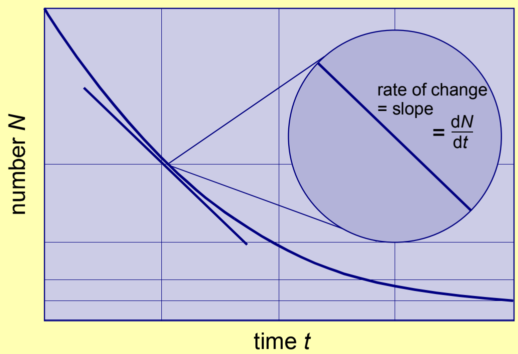
probability p of decay in short time Δt is proportional to Δt:

$p = \lambda \, \Delta t$
average number of decays in time Δt is pN
Δt short so that ΔN much less than N
change in $N = \Delta N = –$number of decays

$\Delta N = -pN$
$\Delta N = -\lambda N \, \Delta t$

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

Simplified, smooth decay



Consider only the smooth form of the average behaviour.
In an interval dt as small as you please:

probability of decay $p = \lambda \, dt$
number of decays in time dt is pN
change in $N =$ d$N = –$number of decays

$dN = -pN$
$dN = -\lambda N \, dt$

$$\frac{dN}{dt} = -\lambda N$$

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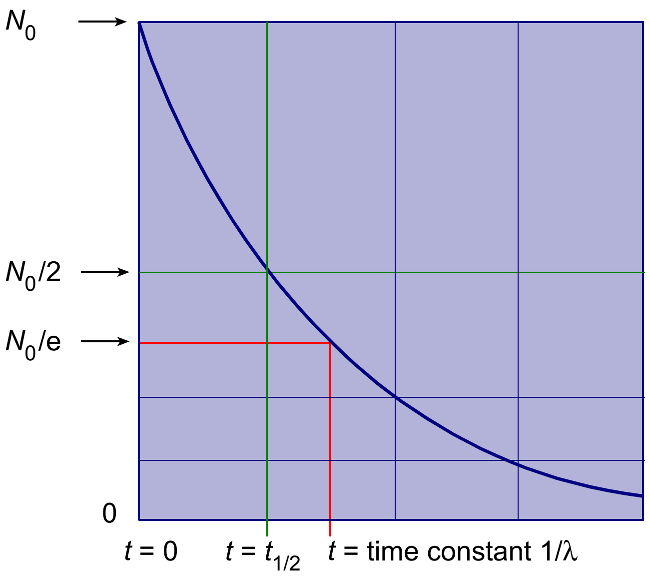
Half-life and time constant

Radioactive decay times

$dN/dt = -\lambda N$ $\qquad N/N_0 = e^{-\lambda t}$



Time constant $1/\lambda$
at time $t = 1/\lambda$
$N/N_0 = 1/e = 0.37$ approx.
$t = 1/\lambda$ is the time constant of the decay

Half-life $t_{1/2}$

at time $t_{1/2}$ number N becomes $N_0/2$

$N/N_0 = \frac{1}{2} = \exp(-\lambda\, t_{1/2})$

$\ln \frac{1}{2} = -\lambda t_{1/2}$

$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$

$\ln 2 = \log_e 2$

Half-life is about 70% of time constant $1/\lambda$. Both indicate the decay time

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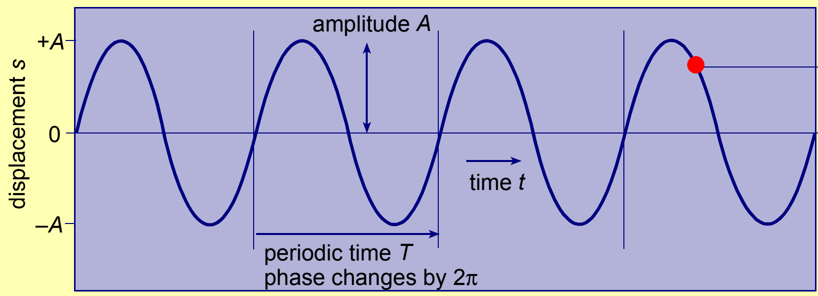
Describing oscillations

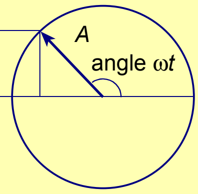
Language to describe oscillations

Sinusoidal oscillation



Phasor picture

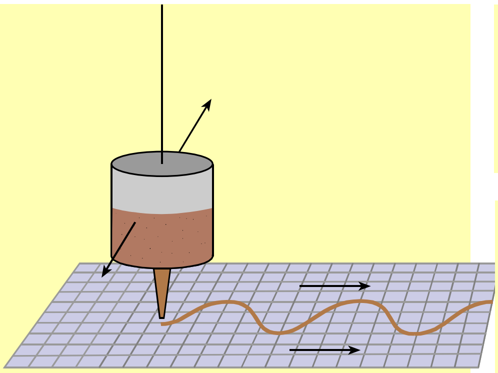
$s = A \sin \omega t$



f turns per second

2π radian per turn

$\omega = 2\pi f$ radian per second



sand falling from a swinging pendulum leaves a trace of its motion on a moving track

Periodic time T, frequency f, angular frequency ω:

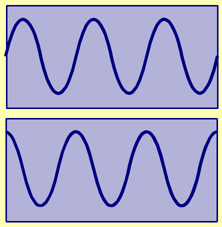
$f = 1/T$ unit of frequency Hz

$\omega = 2\pi f$

Equation of sinusoidal oscillation:

$s = A \sin 2\pi ft$

$s = A \sin \omega t$



Phase difference $\pi/2$

$s = A \sin 2\pi ft$
$s = 0$ when $t = 0$

$s = A \cos 2\pi ft$
$s = A$ when $t = 0$

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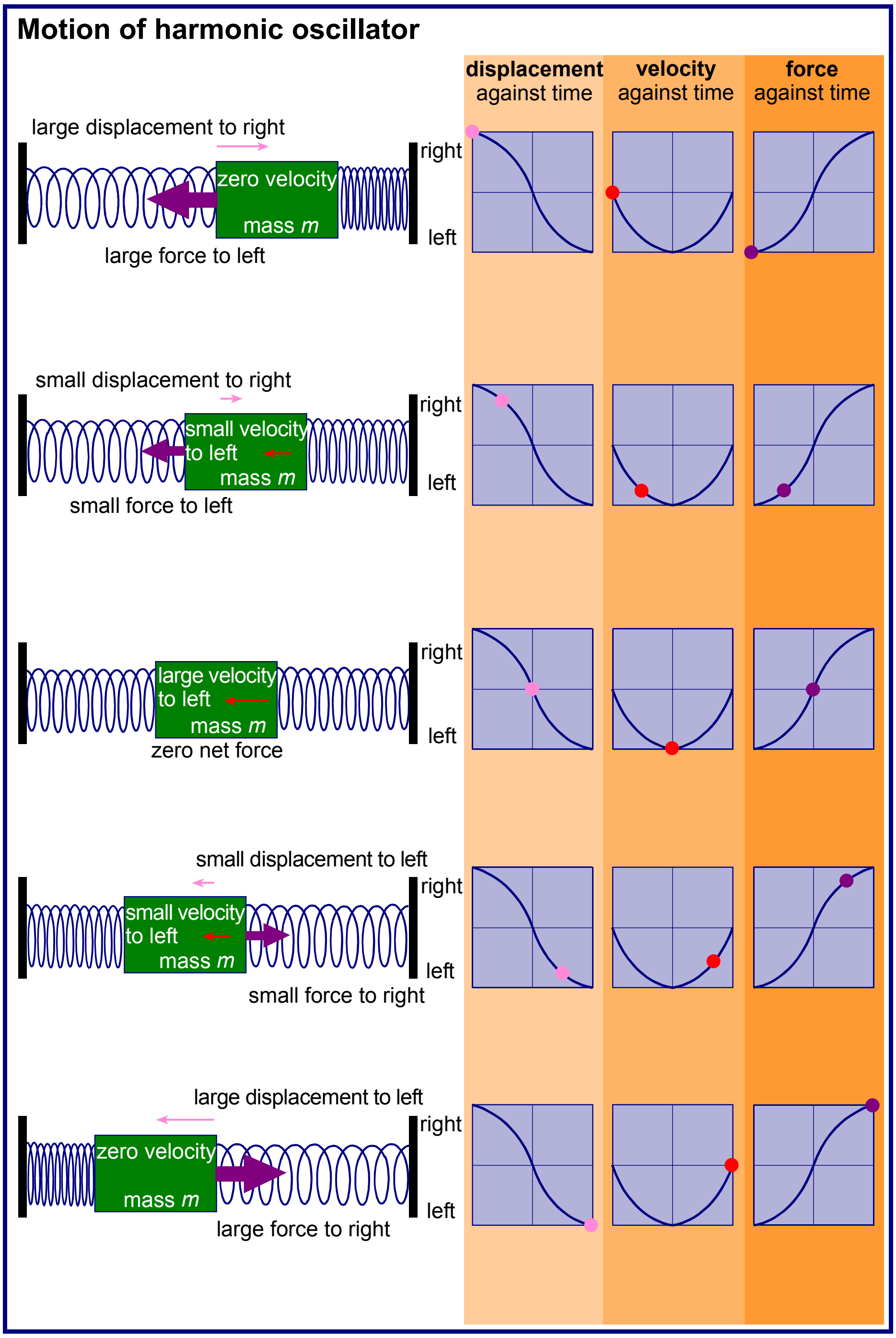
Motion of a simple harmonic oscillator



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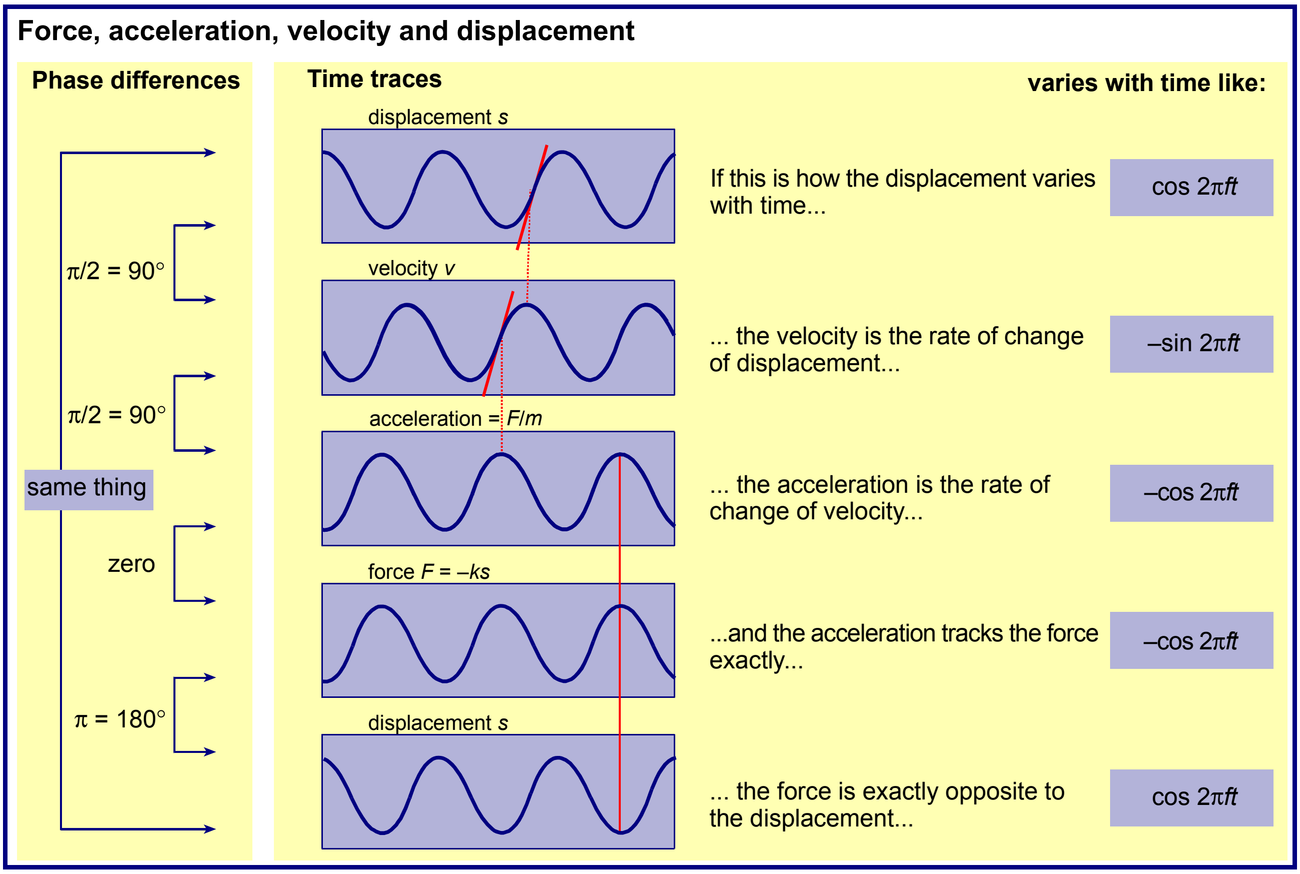
Graphs of simple harmonic motion



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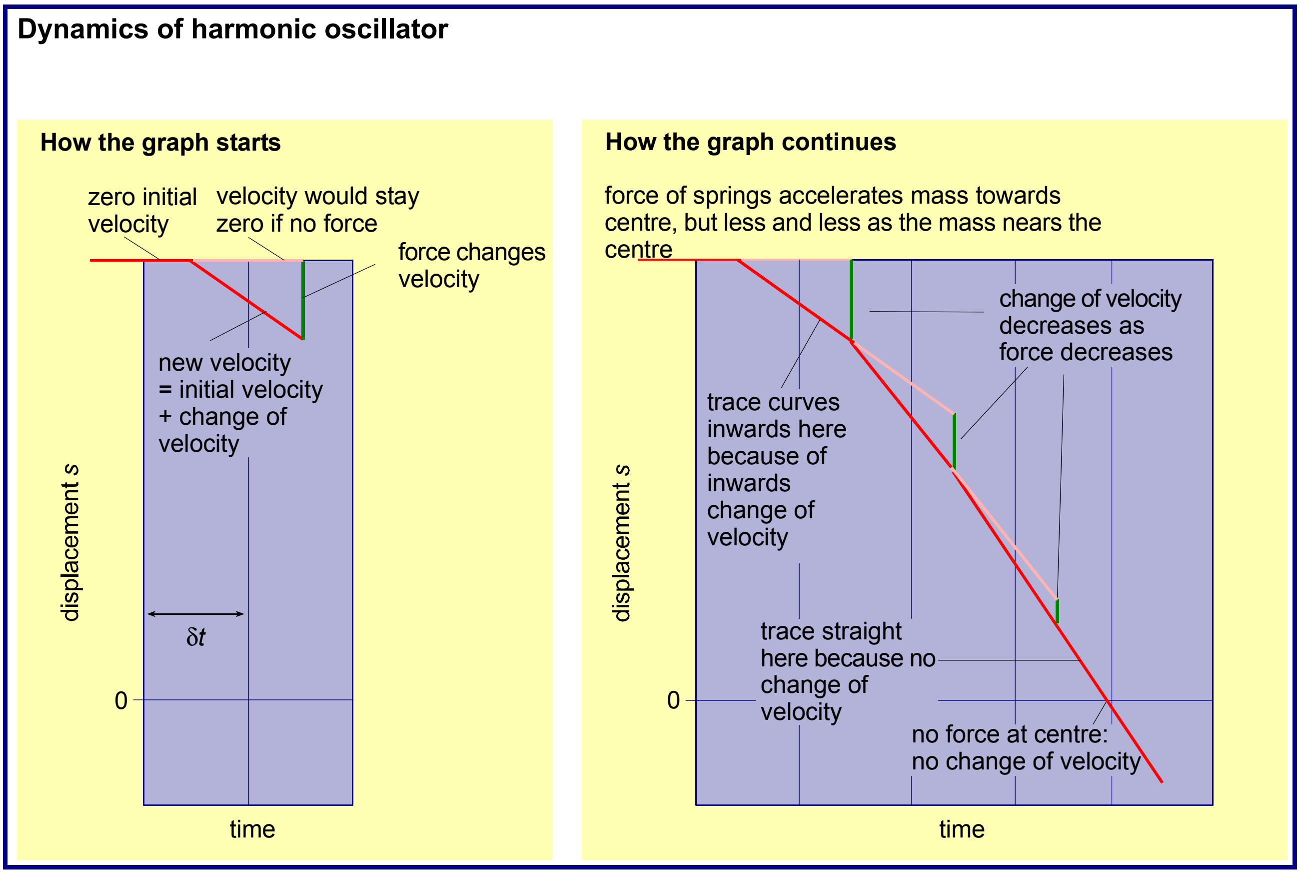
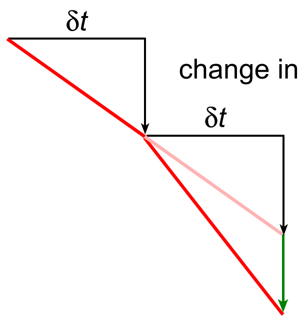
Computing oscillator motion step by step

These two diagrams show the computational steps in solving the equation for a harmonic oscillator.



Constructing the graph



change in displacement = $v\ \delta t$

if no force, same velocity and same change in displacement
plus
extra change in displacement from change of velocity due to force

because of springs:
force $F = -ks$

acceleration = F/m
acceleration = $-(k/m)\ s$

change of velocity δv
= acceleration × δt
$\delta v = -(k/m)\ s\ \delta t$

extra displacement
= $\delta v\ \delta t$

extra displacement = $-(k/m)\ s\ (\delta t)^2$

Health warning! This simple (Euler) method has a flaw. It always changes the displacement by too much at each step. This means that the oscillator seems to gain energy!

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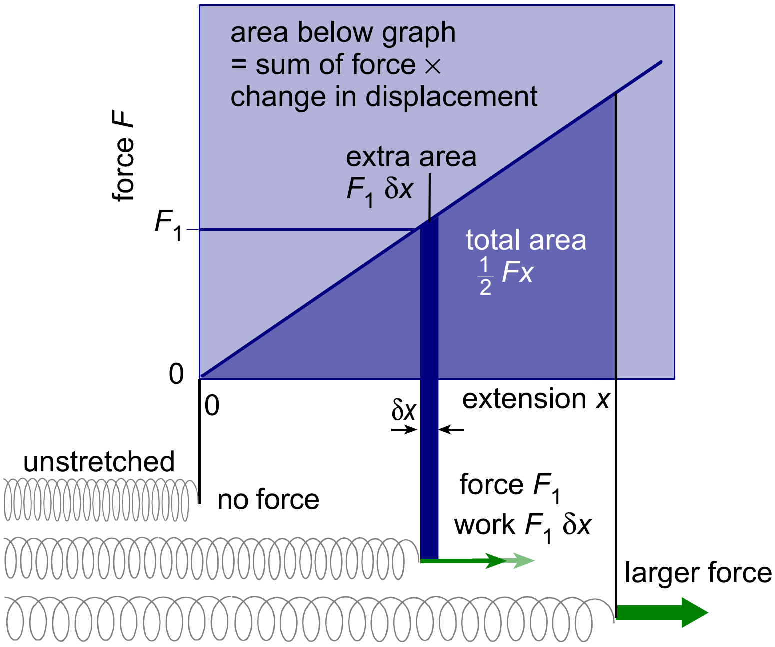
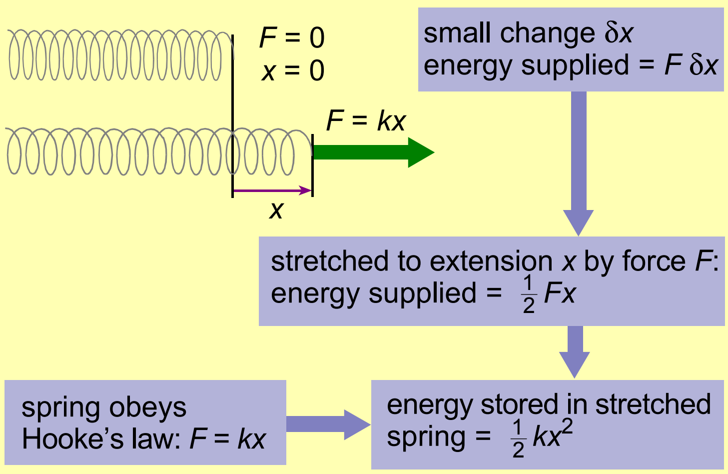
Elastic potential energy

The relationship between the force to extend a spring and the extension determines the energy stored.

Energy stored in a stretched spring



Energy supplied



Energy stored in stretched spring is $\frac{1}{2}kx^2$

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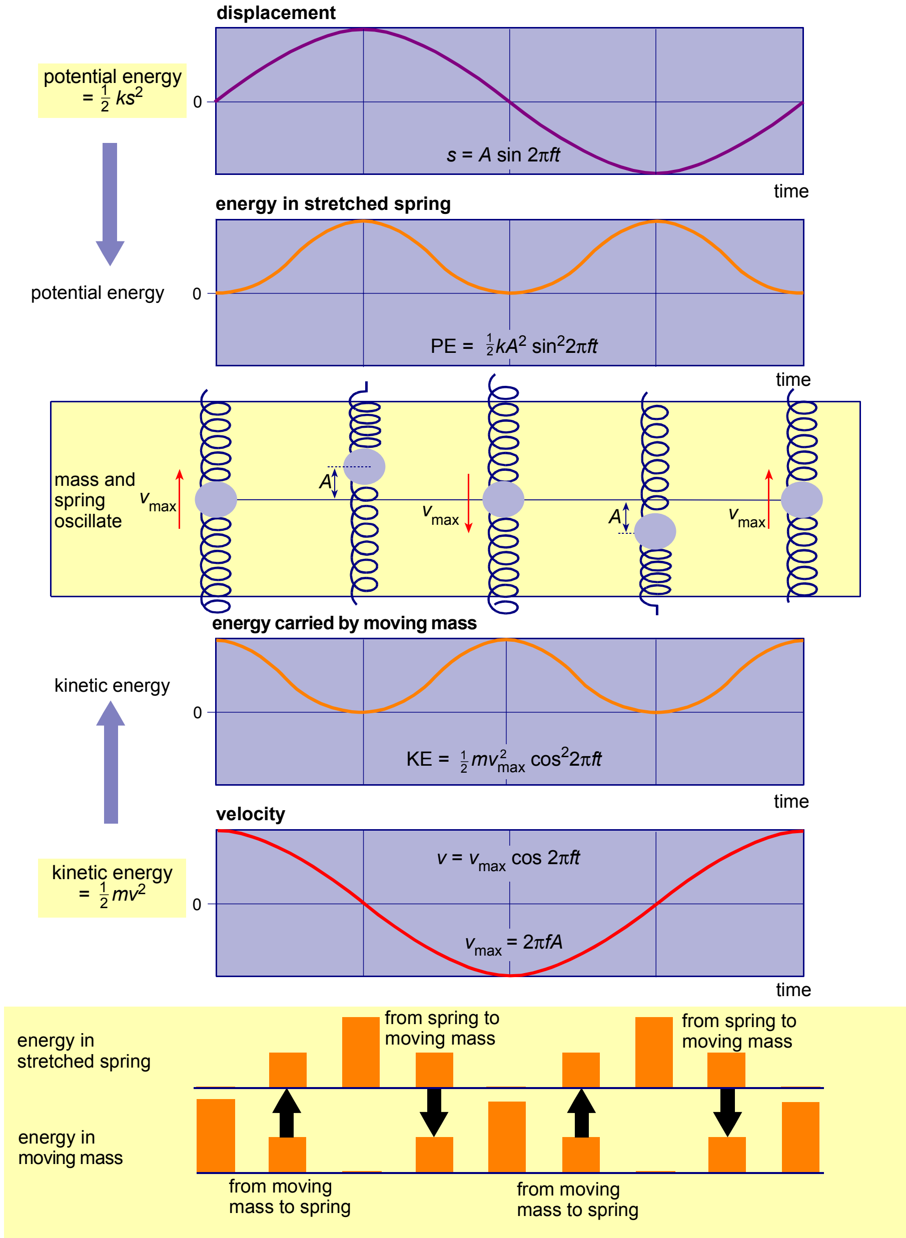
Energy flows in an oscillator

The energy sloshes back and forth between being stored in a spring and carried by the motion of the mass.

Energy flow in an oscillator



The energy stored in an oscillator goes back and forth between stretched spring and moving mass, between potential and kinetic energy

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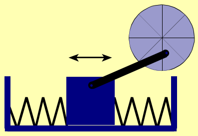
Resonance

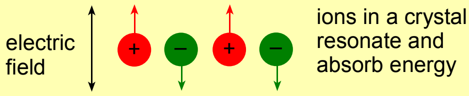
Resonance occurs when driving frequency is equal to natural frequency. The amplitude at resonance, and just away from resonance, is affected by the damping.

Resonant response

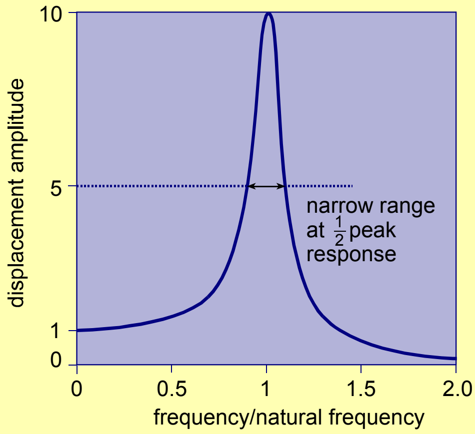
Oscillator driven by oscillating driver



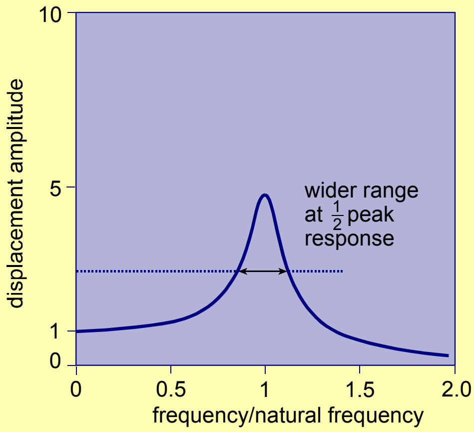
Example: ions in oscillating electric field



low damping:
large maximum response
sharp resonance peak



more damping:
smaller maximum response
broader resonance peak



Resonant response is a maximum when frequency of driver is equal to natural frequency of oscillator

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