Revision Guide for Chapter 9

Contents

Student's Checklist

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I can show my understanding of effects, ideas and relationships by describing and explaining:

the meaning of relative velocity Revision Notes: Relative velocity Summary Diagrams: Relative velocity graphically	
motion under constant acceleration and force Revision Notes: Acceleration; *F = m a* Summary Diagrams: Uniform acceleration; Logic of motion 1; Logic of motion 2; Graphs for constant acceleration; Graphs for realistic motion; Computing uniform acceleration	
the parabolic trajectory of a projectile that the horizontal and vertical components of the velocity of a projectile are independent that a force changes only the component of velocity in the direction of the force Revision Notes: Projectile Summary Diagrams: A parabola from steps	
that work done = force × displacement in the direction of the force power as the rate of transfer of energy (energy transferred per second) Revision Notes: Work; Kinetic energy; Potential energy; Power Summary Diagrams: Calculating kinetic energy; Kinetic and potential energy; Flow of energy; Power, force and velocity	

I can use the following words and phrases accurately when describing the motion of objects:

relative velocity Revision Notes: Relative velocity Summary Diagrams: Relative velocity graphically	
acceleration, force, mass Revision Notes: Acceleration; *F = m a*; Mass; Summary Diagrams: Uniform acceleration	
gravitational force, weight, acceleration of free fall Revision Notes: Weight; Mass; Free fall	
kinetic energy, potential energy, work done, power Revision Notes: Kinetic energy; Potential Energy; Work; Power Summary Diagrams: Calculating kinetic energy; Kinetic and potential energy; Flow of energy;	

Power, force and velocity	

I can interpret:

vector diagrams showing relative velocities Revision Notes: Relative velocity Summary Diagrams: Relative velocity graphically	
graphs of speed–time and distance–time for accelerated motion, including the area under a speed–time graph and the slope of a distance–time and speed–time graph Revision Notes: Acceleration Summary Diagrams: Graphs for constant acceleration; Graphs for realistic motion	

I can calculate:

the resultant vector produced by subtracting one vector from another Revision Notes: Relative velocity Summary Diagrams: Relative velocity graphically	
speed from the gradient (slope) of a distance–time graph distance from the area under a speed–time graph See also Revision Guide Chapter 8 Summary Diagrams: Graphs for constant acceleration; Graphs for realistic motion	
the unknown variable, when given other relevant data, using the kinematic equations: $v = u + at$; $v^2 = u^2 + 2as$; $s = ut + \frac{1}{2}at^2$; $s = \frac{(u+v)}{2}t$ Summary Diagrams: Logic of motion 1; Logic of motion 2	
the unknown quantity, given other relevant data, using the equation $F = m\,a$ Revision Notes: $F = m\,a$	
the path of a moving body acted upon by the force of gravity when the body is (a) moving vertically and (b) moving both vertically and horizontally including use of the kinematic equations and of step by step changes of velocity and displacement in short time intervals. Revision Notes: Free fall; Projectile Summary Diagrams: Uniform acceleration; Computing uniform acceleration; A parabola from steps	
the kinetic energy of a moving body using $KE = \frac{1}{2}mv^2$	

Revision Notes: Kinetic energy Summary Diagrams: Calculating kinetic energy	
the change in potential energy when a massive body changes height in a gravitational field using $\Delta PE = m\,g\,h$ Revision Notes: Potential energy Summary Diagrams: Kinetic and potential energy	
the work done (energy transferred) $\Delta E = F \Delta s$ force, energy and power: power = $\Delta E / t = F v$ Revision Notes: Work; Power Summary Diagrams: Power, force and velocity; Flow of energy; Kinetic and potential energy	

I can give and explain an example of:

a method of measuring the distance of a remote object (i.e. by methods involving timing a signal) Refer to your own notes for your own example	

Revision Notes

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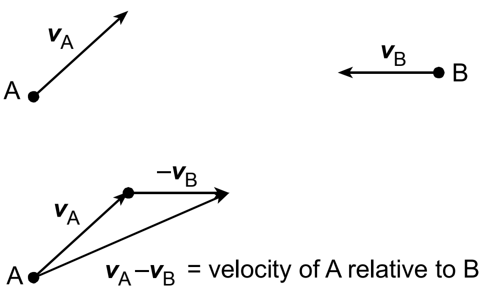
Relative velocity

If two objects A and B move at velocities $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ in a given frame of reference, the relative velocity of A with respect to B, $\boldsymbol{v}_{AB} = \boldsymbol{v}_A - \boldsymbol{v}_B$.

Relative velocity is a **vector quantity**.

A velocity vector may be represented as an arrow having a length in proportion to the speed in the appropriate direction. The relative velocity of an object A to an object B can therefore be represented as the velocity vector $-\boldsymbol{v}_B$ added on to the end of the velocity vector $\boldsymbol{v}_A$, giving a resultant velocity vector $\boldsymbol{v}_A - \boldsymbol{v}_B$ as shown below.

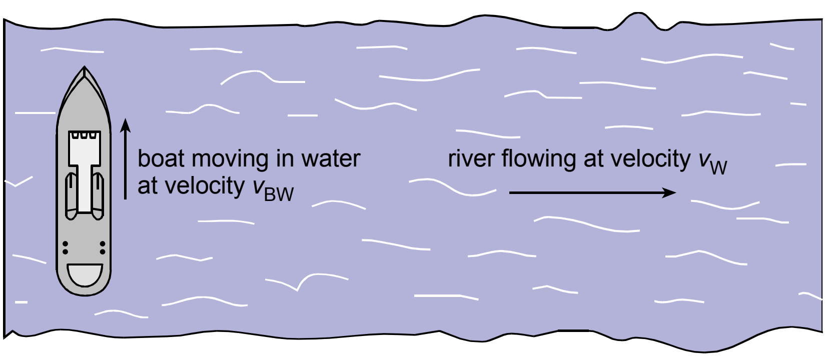
Relative velocity

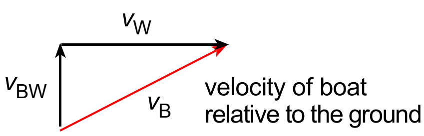


Using components

Given the velocity components of two objects, the differences of the velocity components give the components of the relative velocity.

Velocity in a flowing river





The velocity of a boat crossing a river is affected by the velocity of the water. The boat velocity relative to the ground, $\boldsymbol{v}_B = \boldsymbol{v}_{BW} + \boldsymbol{v}_W$, where $\boldsymbol{v}_{BW}$ is the velocity of the boat relative to the water and v_W is the velocity of the water relative to the ground. This is because the velocity of the water is added on to the velocity of the boat relative to the water to give the velocity of the boat relative to the ground. For example, if the boat is travelling at a speed of 5 m s^{-1} relative to the water in a direction opposite to the flow of the water which is moving at 2 m s^{-1}, the boat velocity relative to the ground is 3 m s^{-1} (= 5 – 2 m s^{-1}) in the opposite direction to the flow.

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Acceleration

Acceleration is the rate of change of velocity. The SI unit of acceleration is the metre per second per second (m s^{-2}). Acceleration is a **vector quantity**.

If in time Δt the vector velocity $\boldsymbol{v}$ changes by the vector amount $\Delta\boldsymbol{v}$, then the vector acceleration $\boldsymbol{a}$ is given by:

$$\boldsymbol{a} = \Delta\boldsymbol{v} / \Delta t$$

The acceleration is in the direction of the change of velocity.

A decelerating object, whose speed decreases, has a change in velocity opposite in direction to the velocity.

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F = *m* *a*

The acceleration $\boldsymbol{a}$ of a mass m acted on by a force $\boldsymbol{F}$ is given by:

$$\boldsymbol{F} = m\,\boldsymbol{a}$$

That is, force = mass × acceleration. Force is a **vector quantity**. The acceleration is in the direction of the force.

The SI unit of force, the newton (N), is the force that gives a 1 kg mass an acceleration of 1 m s^{-2}.

In calculations, often the acceleration is to be found:

$$a = \frac{F}{m}$$

In the case when the force F is the weight mg of the body falling under gravity alone:

$$a = \frac{mg}{m} = g$$

Thus g is both the force of gravity per unit mass (gravitational field), units N kg^{-1}, and the acceleration of free fall, units m s^{-2}.

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Mass

A massive object changes its velocity less than does a less massive object, when acted on by the same force for the same time.

If two bodies interact, their velocities change in inverse proportion to their masses

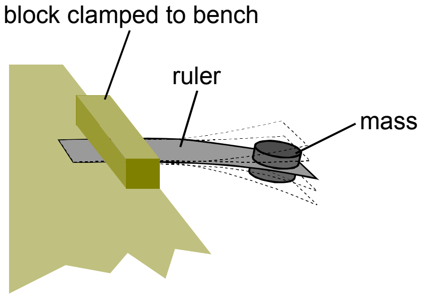
$$\frac{\Delta v_1}{\Delta v_2} = \frac{m_2}{m_1}.$$

The larger mass suffers the smaller change in velocity. This relationship only defines ratios of masses. To establish a unit, a mass has to be assigned to some particular object. Mass is measured in kilograms. The kilogram is defined as the mass of a certain quantity of platinum kept in an international standards laboratory in Paris.

Mass is a **scalar quantity**.

Masses of objects can be compared by comparing their changes of velocity under the same forces or in the same interaction. One way to do this is to compare the frequencies of oscillation of masses fixed to the same springy support.

Oscillations of a loaded rule



Mass is also the source of the gravitational field and is acted on by a gravitational field. Masses are conveniently compared by comparing the gravitational forces on them in the same gravitational field. This is what a balance does. A standard mass can be used to calibrate a spring balance.

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Weight

The weight of an object is the gravitational force acting on it. Weight is measured in newtons (N).

The strength of a gravitational field, g, at a point in a gravitational field is the force per unit mass acting on a small mass at that point.

Gravitational field strength is a **vector quantity** in the direction of the gravitational force on a mass. The SI unit of gravitational field strength is the newton per kilogram (N kg^{-1}) or (equivalently) m s^{-2}.

The force F on a point mass m at a point in a gravitational field is given by $F = m\,g$, where g is the gravitational field strength at that point. Thus the weight = mg.

If an object is supported at rest, both the object and its support will be compressed. This compression can be used in a spring balance to weigh the object.

An object that is in free fall is sometimes said to be 'weightless' even though the force of gravity still acts on it. Such an object will appear to weigh nothing if put on a spring balance falling freely with it, not because the Earth is not exerting a force on it, but because both object and balance are falling together.

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Free fall

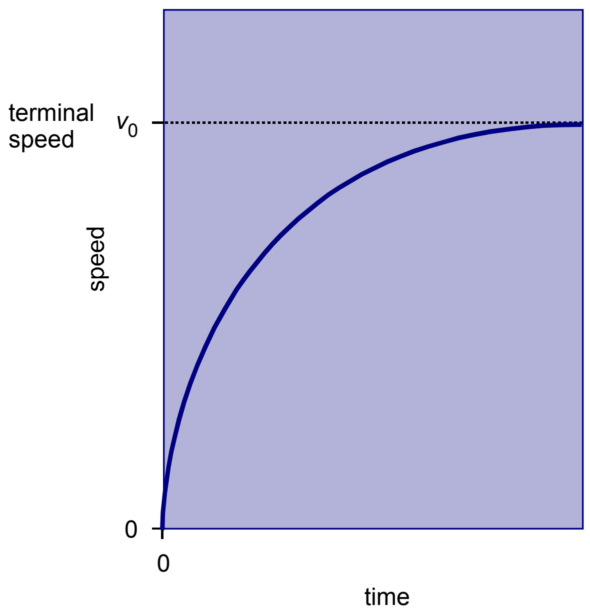
Objects acted on by gravity alone are in free fall.

For an object of mass m, its weight = $m\,g$, where m is the mass of the object and g is the gravitational field strength at the object.

The acceleration of the object, a, is the force of gravity divided by the mass = $m\,g\,/\,m = g$. Hence the acceleration of a freely falling object is equal to g. The acceleration is constant if the gravitational force is constant, and no other forces act.

If the object is acted on by air resistance as it falls, its acceleration gradually decreases to zero and its velocity reaches a maximum value known as its terminal velocity. The object is thus not in free fall as it is 'partially supported' by air resistance. The air-resistance force F_R, increases with velocity so the resultant force (= $m\,g - F_R$) and hence the acceleration decreases. At the **terminal velocity**, the force due to air resistance is equal and opposite to the weight of the object so the resultant force and hence the acceleration is zero.

Terminal speed



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Projectile

At any point on the path of a projectile, provided that air resistance is negligible:

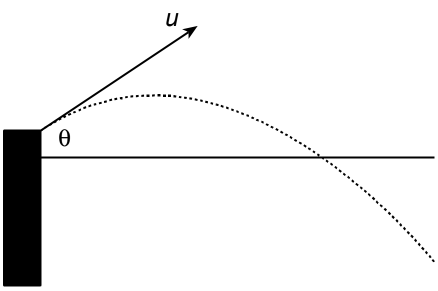
1. the horizontal component of acceleration is zero
2. the downward vertical component of acceleration is equal to g, the gravitational field strength at that point.

A projectile travels equal horizontal distances in equal times because its horizontal component of acceleration is zero. Its horizontal motion is unaffected by its vertical motion. The combination of constant horizontal velocity and constant downward acceleration leads to a parabolic path.

See A parabola from steps for a graphical calculation of the parabolic path of a projectile.

Using kinematic equations

Projection at a non-zero angle above the horizontal



The projectile is launched with an initial speed u at angle θ above the horizontal at time $t = 0$.

Its constant horizontal component of velocity $u_x = u \cos\theta$. Thus the horizontal component of its displacement at time t is
$x = u_x t$

Its initial vertical component of velocity $u_y = u \sin\theta$. Thus its vertical component of velocity at time t is $u_y - gt$. The vertical component of displacement is
$y = u_y t - \frac{1}{2} g t^2$.

The highest point is reached at time t_0 when the vertical component of velocity $u_y - g t_0 = 0$, hence $t_0 = u_y / g$.

The shape of the path is parabolic.

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Work

The work W done by a force F that moves its point of application by a distance s in the direction of the force is given by $W = F s$.

Work measures the amount of energy transferred from one thing to another.

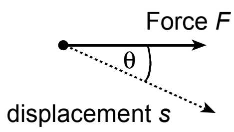
The SI unit of energy and of work is the joule (J). Work and energy are **scalar quantities**.

Work is done whenever an object moves under the action of a force with a component of the force along the displacement. If there is no movement of the object, no work is done. However, an outstretched hand holding a book does need to be supplied with energy to keep the arm muscles taut. No work is done on the book provided it is stationary. But, energy has to be supplied to repeatedly contract the muscle fibres so as to keep the muscles taut.

No work is done on an object by a force when the displacement of the object is at right angles to the direction of the force.

If an object is moved by a force F a distance s along a line that is at angle θ to the direction of the force, the work done by the force is given by $W = F\,s\cos\theta$.

Work done



Work done = $Fs\cos\theta$

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Kinetic energy

The kinetic energy of a moving object is the energy it carries due to its motion. An object of mass m moving at speed v has kinetic energy $E_k = \frac{1}{2}mv^2$.

To show this, consider an object of mass m initially at rest acted on by a constant force F for a time t. The object accelerates to speed v where $F\,t = m\,v$.

Since the average speed is $v/2$, the distance moved by the object, $s = \frac{1}{2}\,v\,t$, and the work done on the object is equal to

$$Fs = \frac{mv}{t}\frac{vt}{2} = \frac{1}{2}mv^2.$$

The work done is equal to the gain of kinetic energy. Hence the kinetic energy at speed v is $E_k = \frac{1}{2}mv^2$

See also Calculating kinetic energy

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Potential energy

The potential energy of a system is the energy associated with the position of objects relative to one another, for example a mass raised above the Earth.

The SI unit of potential energy is the joule.

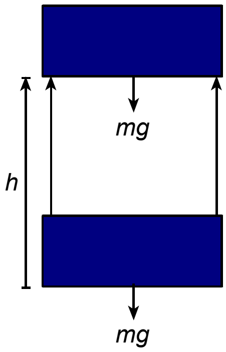
Potential energy can be thought of as stored in a field, for example a gravitational field.

The potential energy is measured by the capacity to do work if the positions of objects change.

If the height of an object above the Earth changes, the potential energy changes., The energy transferred when it is raised is equal to the force × distance moved along the line of action of the force.

Since the force of gravity on an object of mass m is equal to mg, then if the object is raised by a height h, the change of potential energy = mgh.

Gain of potential energy



$\Delta E_p = mgh$

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Power

The power of a device is the rate at which it transfers energy. The SI unit of power is the watt (W). Power is a scalar quantity.

If energy ΔE is transferred in time Δt the average power $P = \Delta E / \Delta t$.

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Summary Diagrams (OHTs)

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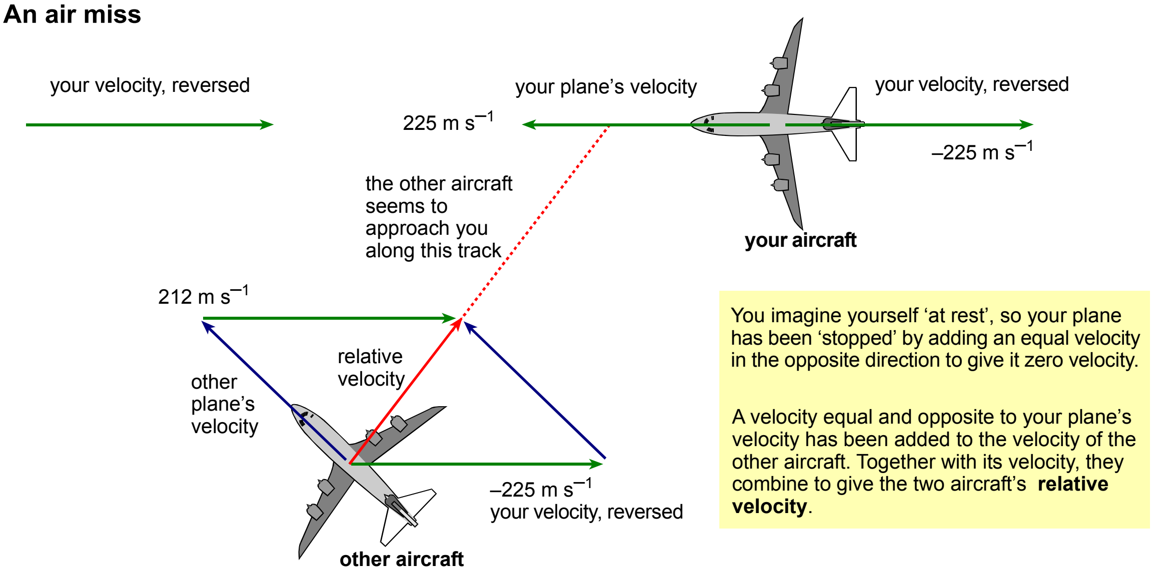
Relative velocity graphically

An air miss



Rules:
1. Add a velocity opposite to that of one plane to the velocities of both.
2. Find the resultant relative velocity, adding vectors tip to tail.
3. See if the direction of the relative velocity hits your plane.
If so, take avoiding action!

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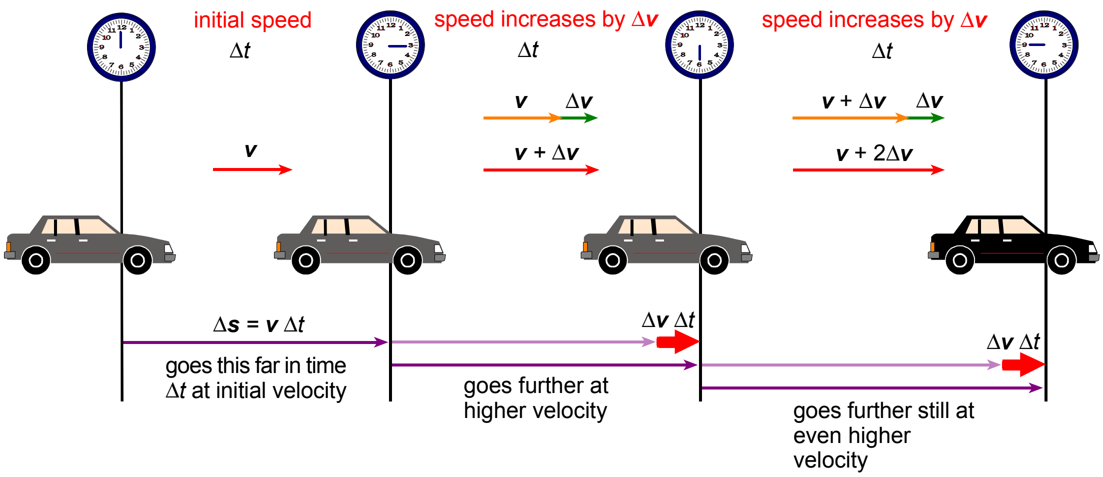
Uniform acceleration

Uniform acceleration



A uniformly accelerating object adds the same amount to its velocity in each short time interval

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Logic of motion 1

The logic of uniform acceleration 1

Velocity from time

Starting point:
The meaning of acceleration

Define:
u = initial velocity
v = final velocity
a = acceleration
t = time

Definition:

acceleration = $\frac{\text{change of velocity}}{\text{time taken}}$

$a = \frac{v - u}{t}$

multiply both sides by t:

change of velocity
= acceleration × time

$v - u = at$

rearrange

time taken = $\frac{\text{change of velocity}}{\text{acceleration}}$

$t = \frac{v - u}{a}$

add u to both sides

final velocity
= initial velocity
+ change of velocity

$v = u + at$

Use $v = u + at$ to compute velocity from acceleration and time

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Logic of motion 2

The logic of uniform acceleration 2

Distance from time; velocity from distance

Starting point: average velocity

average velocity = $\frac{\text{initial plus final velocity}}{2}$

average velocity = $\frac{u + v}{2}$

distance gone = average velocity × time

$$s = \frac{u + v}{2}\, t$$

From the meaning of acceleration:

$$t = \frac{v - u}{a} \qquad v = u + at$$

Replace *t*:

$$s = \frac{u + v}{2} \times \frac{v - u}{a}$$

since:

$$(v + u)(v - u) = v^2 - u^2$$

then:

$$s = \frac{v^2 - u^2}{2a}$$

rearrange:

$$v^2 = u^2 + 2as$$

Replace *v*:

$$s = \frac{u + u + at}{2}\, t$$

since:

$$\frac{u + u + at}{2} = u + \frac{1}{2}at$$

then:

$$s = (u + \tfrac{1}{2}at)t$$

rearrange:

$$s = ut + \frac{1}{2}at^2$$

Use $s = ut + \frac{1}{2}at^2$ to compute distance gone from acceleration and time

Use $v^2 = u^2 + 2as$ to compute final velocity from acceleration and distance

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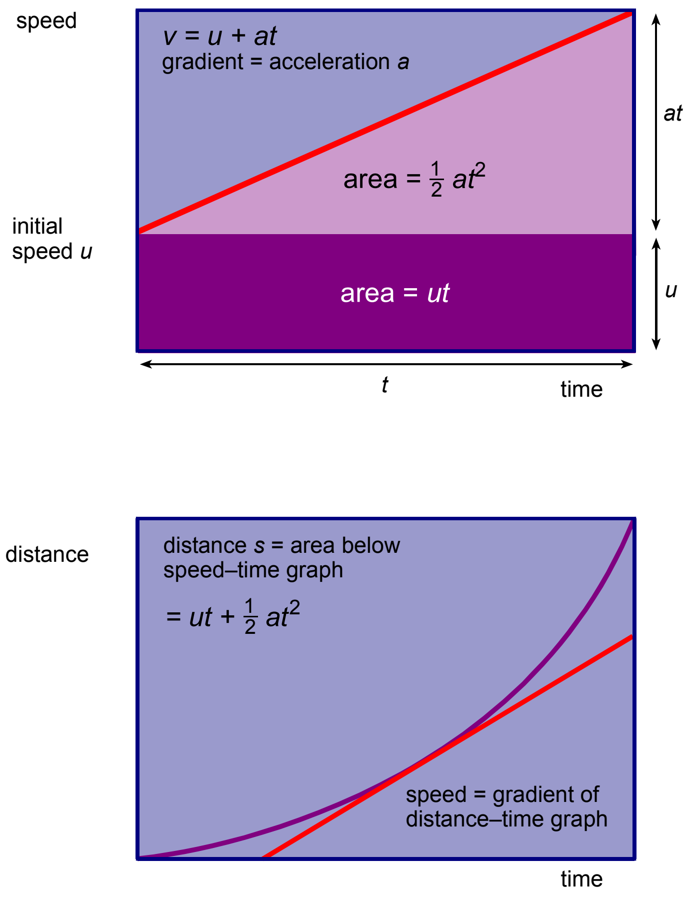
Graphs for constant acceleration

Graphs for constant acceleration



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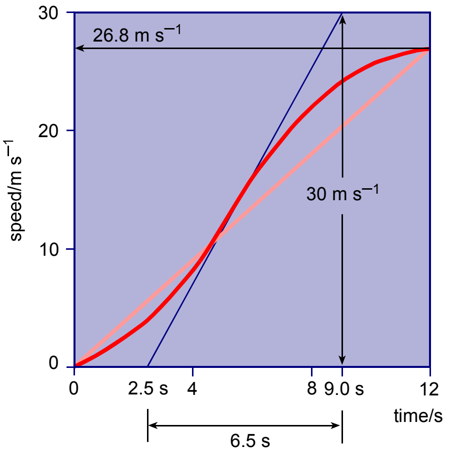
Graphs for realistic motion

Graph of a more realistic motion

Car accelerating from 0 to 26.8 m s^{-1} in 12 s



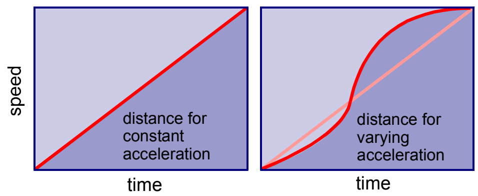
maximum acceleration
= steepest gradient of
speed–time graph

change of speed = 30 m s^{-1}
time interval = 6.5 s

acceleration = $\frac{30 \text{ m s}^{-1}}{6.5 \text{ s}}$

acceleration = 4.6 m s^{-2}

Average acceleration same but acceleration varies



distance = area below
speed–time graph

more time spent at higher
speed: larger distance

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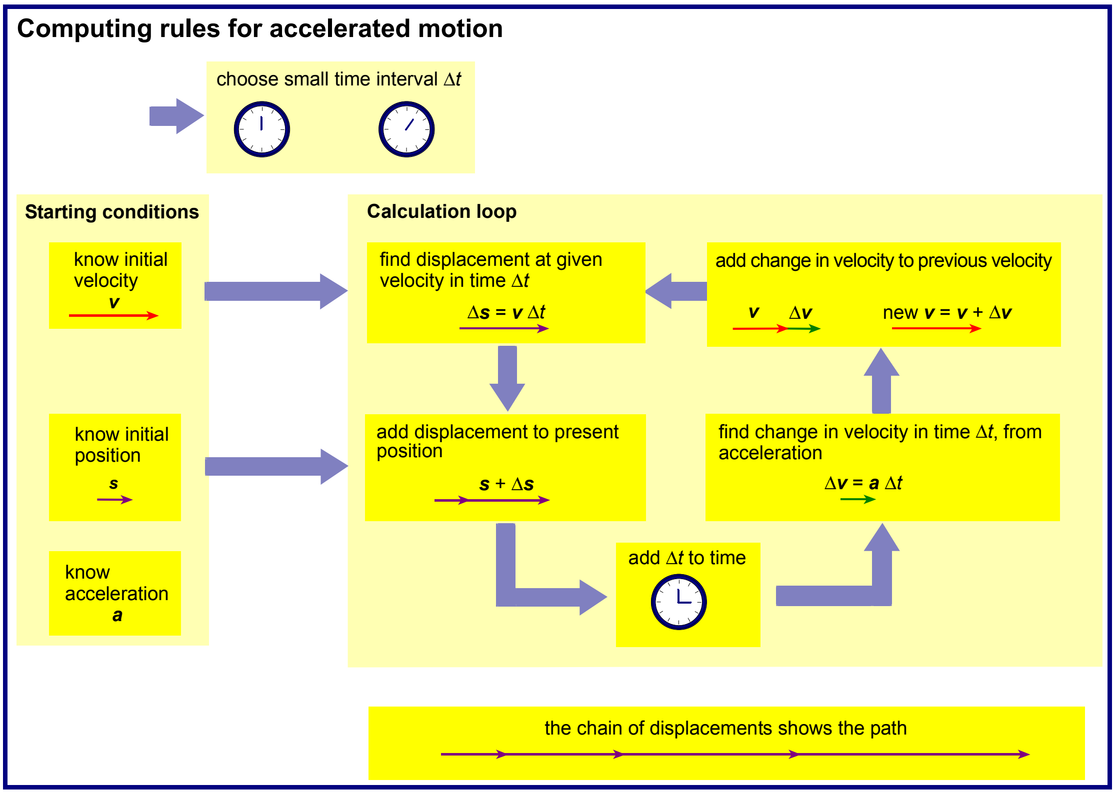
Computing uniform acceleration



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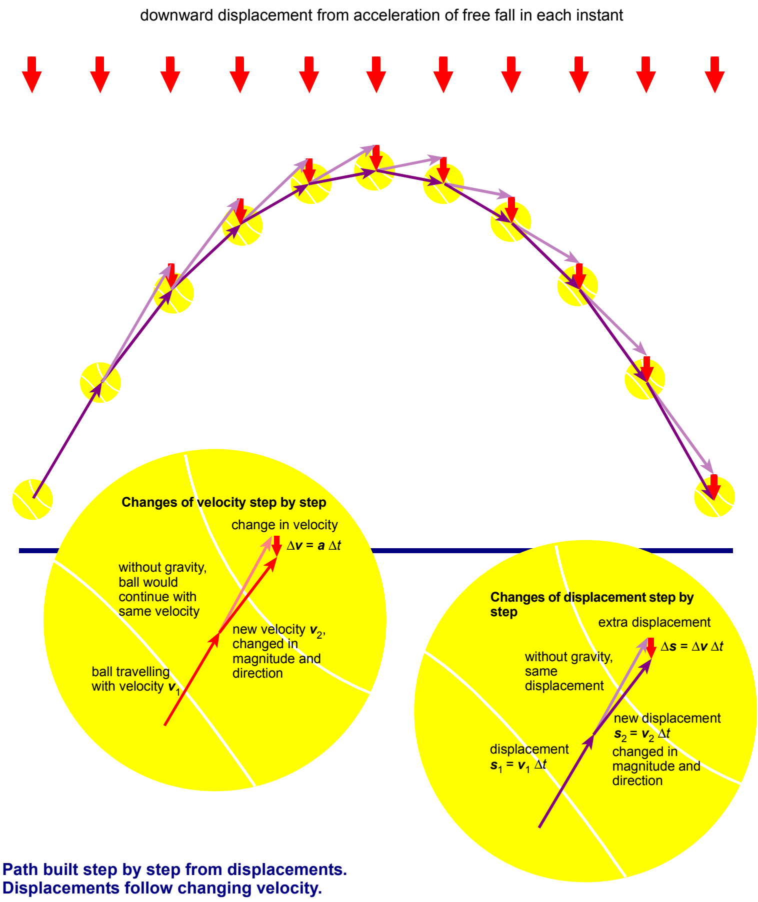
A parabola from steps

Lobbing a ball in the air



Path built step by step from displacements.
Displacements follow changing velocity.

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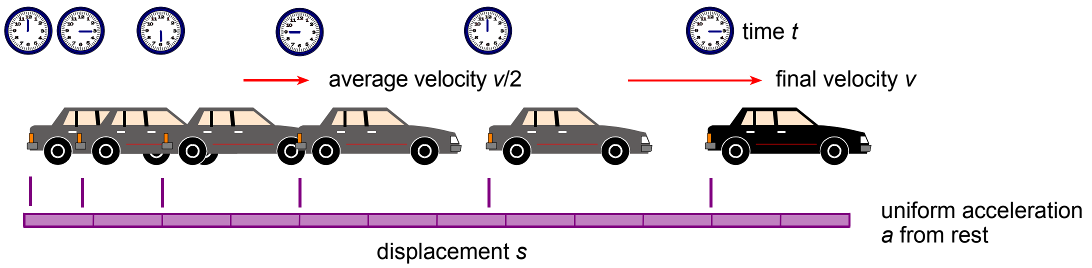
Calculating kinetic energy

Kinetic energy = $\frac{1}{2}mv^2$



Force × time

$F = ma$
$v = at$

Force × time = mass × acceleration × time
acceleration × time = velocity

Force × time = *mv*

***mv* is called momentum**

Force × displacement

distance = average velocity × time
average velocity = $v/2$

Force × distance
= force × average velocity × time
= force × time × $v/2$

force × displacement = mv × $v/2$

Force × displacement = $\frac{1}{2}mv^2$

$\frac{1}{2}mv^2$ is called kinetic energy

Momentum *mv* says
how big a force is needed to stop in a given time

Kinetic energy $\frac{1}{2}mv^2$ says
how big a force is needed to stop in a given distance

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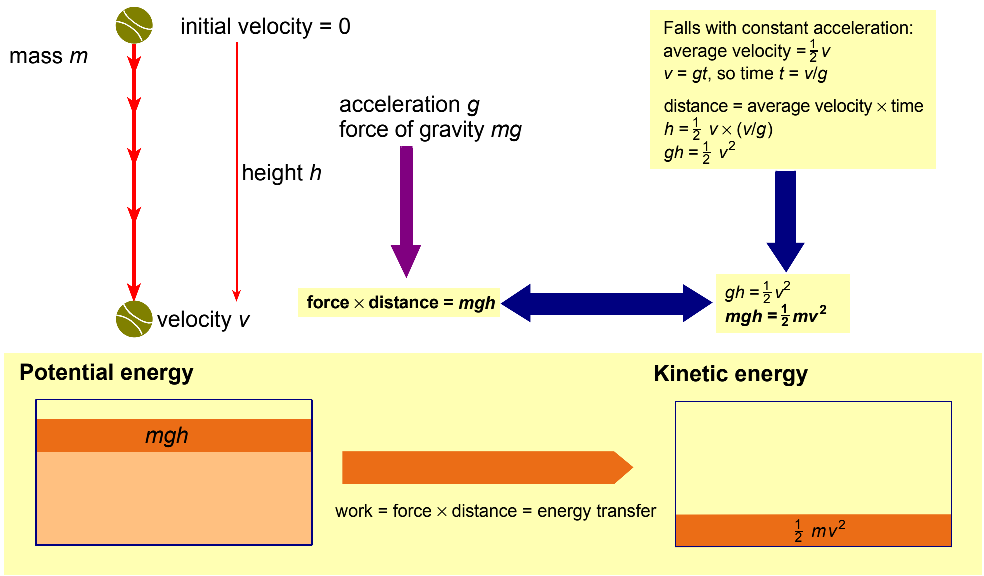
Kinetic and potential energy

Force × distance under gravity



Energy comes from the gravitational field: decrease in potential energy = mgh

Energy now carried by motion of ball: increase in kinetic energy = $\frac{1}{2}mv^2$

Decrease in potential energy = increase in kinetic energy

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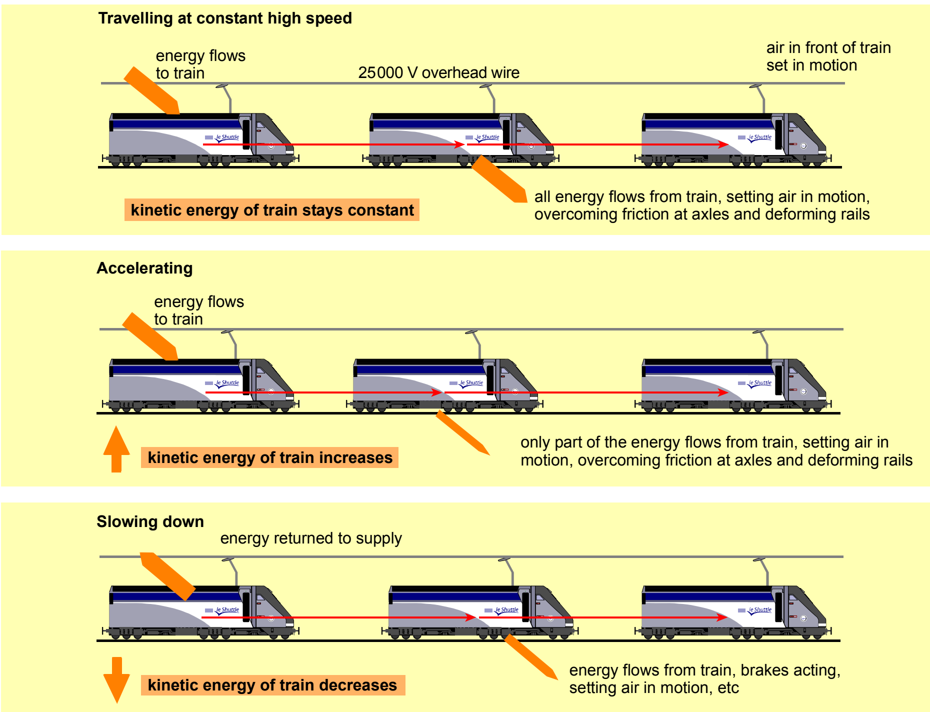
Flow of energy

Going at a steady speed means that as much energy flows in every second as flows out. If more energy flows in than out then the train speeds up. If more energy flows out than in then the train slows down.

Flow of energy to a train



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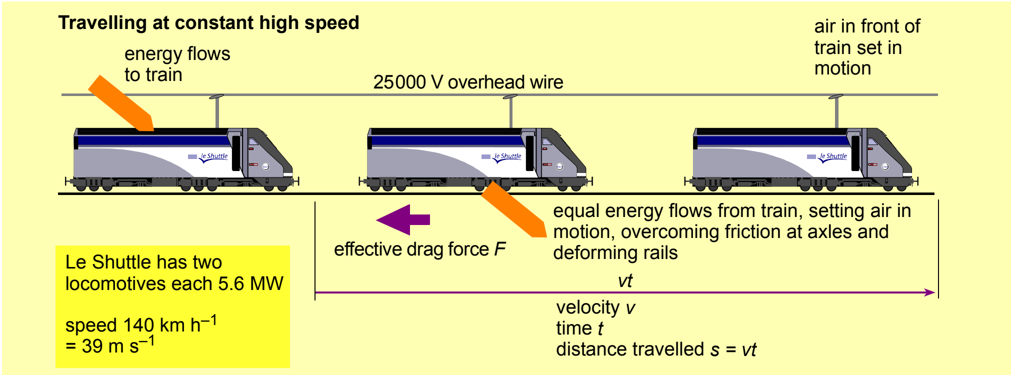
Power, force and velocity

Power = force × velocity



Power used

Energy from train to surroundings = drag force × distance moved

$E = Fs$

Power used by train = rate of dissipation of energy

$P = \frac{E}{t}$

$P = \frac{Fs}{t}$ with $s = vt$

$P = Fv$

Power used by train = drag force × velocity

power in

two locomotives each 5.6 MW

$P = 2 \times 5.6$ MW = 11.2 MW

power out

maximum speed 140 km per hour
$v = 39$ m s^{-1}

power in = power out
$P = Fv$
11.2×10^6 W = $F \times 39$ m s^{-1}

$F = 290$ kN

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