Worked Solutions for Sample Examination Questions

Question 1

Probability is proportional to $(\text{amplitude})^2$.

Thus the probability of photons arriving at X ~ $2^2 = 4$ and at Y ~ $0.5^2 = 0.25$ so the ratio

$$\frac{\text{probability of photons arriving at X}}{\text{probability of photons arriving at Y}} = \frac{4}{0.25} = 16.$$

Question 2

(a) The key fact to bear in mind is that the **speed** of the sound waves is the same for both instruments:

$v = f\lambda$ so $f\lambda$ is a constant.

By inspection the wavelength of the fundamental in the organ = $4L$, but for the flute it is $2L$. Because $f\lambda$ is a constant, halving λ requires doubling f:

2 x 130 Hz = 260 Hz.

(b) If L = 0.65 m, then λ = 4 x 0.65 m = 2.6 m.

$v = f\lambda$ = 130 Hz x 2.6 m = 338 m s^{-1} ~ 340 m s^{-1} (to 2 significant figures).

Question 3

(a)(i) The parallel beam incident on the double slits is diffracted at each slit. Where the diffracted beams from the two slits overlap constructive and destructive superposition takes place.

(a)(ii) Equivalent descriptions are:
- the two paths differ by an integral number of wavelengths
- when the diffracted waves from each slit arrive at **X** they are in phase
- the phasors of the two waves line up
- the waves from each slit superpose constructively.

(a)(iii) Equivalent descriptions are:
- the path difference is not a whole number of wavelengths
- the diffracted waves from each slit are not in phase
- the phasors of the two waves tend to cancel
- the waves from each slit superpose destructively.

(b) Using the Young's two-slit relationship

$n \lambda = d \sin \theta$

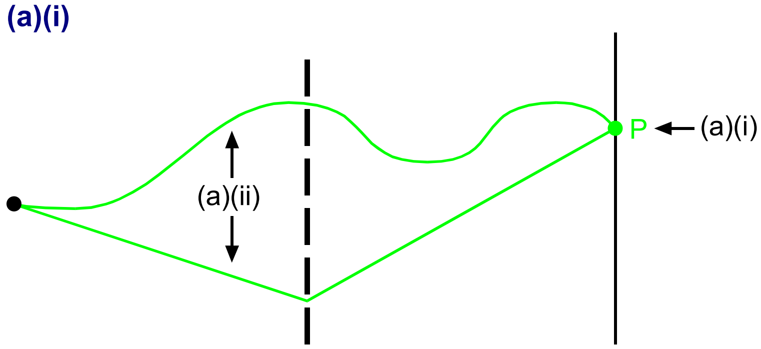
the graph shows that for $n = 1$ (the first maximum to either side of the central maximum) $\sin \theta = 1.5 \times 10^{-3}$ so

$\lambda = (0.4 \times 10^{-3}\ \text{m}) \times (1.5 \times 10^{-3}) = 0.6 \times 10^{-6}\ \text{m} = 6 \times 10^{-7}\ \text{m}$.

(c) The two most obvious differences are that the fringes are brighter and sharper. More subtle effects are 'missing orders' and a greater variation in brightness across the fringe pattern.

Question 4

(a)(i)

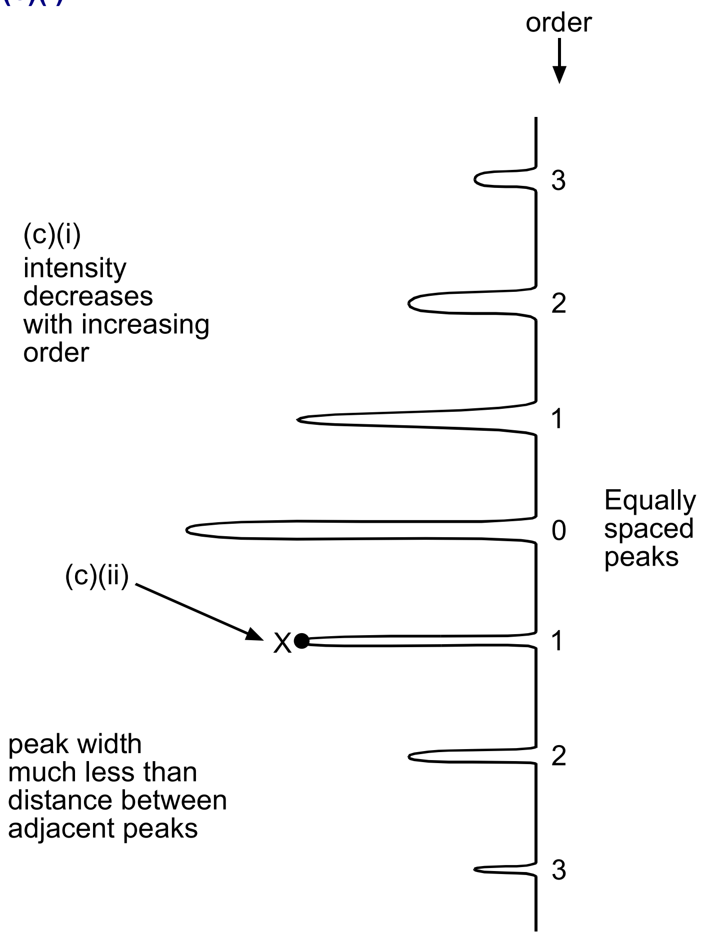


(a)(ii)
Any two unbroken paths from the source that go through a slit in the grating and intersect on the screen at the point **P** are a correct answer. The paths do not have to be straight lines.

(b)(i) The phasors rotate at the frequency of the light *f*. The difference in path length and speed give the angle between the phasors for the two paths. The phasors are combined 'tip to tail' to give the resultant.

(b)(ii) The probability of arrival is proportional to the (resultant amplitude)^2.

(c)(i)



(c)(ii) see diagram for (c)(i)
Any peak could be labelled with the **X**.

Question 5

(a)(i) The grating relationship is

$d \sin \theta = n \lambda$

for first order n = 1

$$d = \frac{5.0 \times 10^{-7}\ \text{m}}{\sin 16^\circ} = 1.814 \times 10^{-6}\ \text{m}$$

which is 1.8 x 10^{-6} m to 2 significant figures (the same as the data given).

(a)(ii)

$$d = \frac{2 \times (5.0 \times 10^{-7}\ \text{m})}{\sin 33.7^\circ} = 1.802 \times 10^{-6}\ \text{m}$$

which is 1.8 x 10^{-6} m (to 2 significant figures).

(a)(iii) The distance OX is the fringe separation:

$$\tan 16^\circ = \frac{OX}{1.2\ \text{m}}$$

thus OX = 0.34 m.

(b)(i) If the spacing of the lines (slits) is *d* metres, then the number of slits per metre = 1 / *d*.

From **(a)(i)** *d* = 1.814 x 10^{-6} m, so the number of slits per metre = 5.513 x 10^{5} slit m^{-1}. Thus the number of lines per mm = 5.5 x 10^{2} slit mm^{-1}, or 550 slit mm^{-1} to 2 significant figures.

(b)(ii) Two significant figures are used because that is the **least** number of significant figures in the data used to do the calculation. (The calculation needs to be carried out with numbers with more than 2 significant figures, and the final answer rounded down to 2 significant figures. Observe that if 1.8 x 10^{-6} m had been used for *d*, then the lines per mm comes out as 560.)

Question 6

(a)(i) $E = h f$, thus

$f = E / h$ = (5.6 x 10^{-19} J) / (6.6 x 10^{-34} J s) = 8.5 x 10^{14} Hz.

(a)(ii)

$$\frac{\text{Pulse energy}}{\text{Photon energy}} = \frac{1.8\times10^{6}\ \text{J pulse}^{-1}}{5.6\times10^{-19}\ \text{J photon}^{-1}}$$
$$= 3.214\times10^{24}\ \text{photon pulse}^{-1}$$

i.e. approximately 3.2 x 10^{24} photons in the pulse.

(a)(iii)

$$\text{Power} = \frac{\text{energy transferred}}{\text{time taken}}$$
$$= \frac{1.8\times10^{6}\ \text{J pulse}^{-1}}{5.0\times10^{-9}\ \text{s pulse}^{-1}}$$
$$= 3.6\times10^{14}\ \text{watt (W)}.$$

(b)(i)
(NB: the photoelectric effect is being described.)

Maximum kinetic energy = photon energy – minimum energy for release (‘the work function’)
= (5.6 x 10^{-19} J) – (4.8 x 10^{-19} J) = 8.0 x 10^{-20} J.

(b)(ii)

Kinetic energy $= \dfrac{mv^2}{2} = 8.0 \times 10^{-20}$ J.

Solving for

$$v^2 = \frac{2 \times 8.0 \times 10^{-20}\ \text{J}}{m}$$
$$= \frac{1.6 \times 10^{-19}\ \text{J}}{9.1 \times 10^{-31}\ \text{kg}}$$
$$= 1.758 \times 10^{11}\ \text{J kg}^{-1}\ (\text{equivalent to m}^2\ \text{s}^{-2})$$

thus

$$v = \sqrt{1.758 \times 10^{11}\ (\text{m s}^{-1})^2} = 4.2 \times 10^5\ \text{m s}^{-1}.$$

(b)(iii)

$$\lambda = \frac{6.6 \times 10^{-34}\ \text{J s}}{(9.1 \times 10^{-31}\ \text{kg}) \times (4.2 \times 10^5\ \text{m s}^{-1})}$$
$$= 1.7 \times 10^{-9}\ \text{m}.$$

Question 7

(a)(i) Number of atoms along an edge is:

$$\frac{\text{length of edge}}{\text{diameter of an atom}} = \frac{4.2 \times 10^{-2}\ \text{m}}{2.8 \times 10^{-10}\ \text{m atom}^{-1}} = 1.5 \times 10^8\ \text{atoms}.$$

(a)(ii) $(1.5 \text{ x } 10^8)^2 = 2.25 \text{ x } 10^{16} \sim 2.3 \text{ x } 10^{16}$ atoms.

(b)(i) Each second 9.0 x 10^{-7} J of energy is absorbed by 2.25 x 10^{16} atoms, thus the energy absorbed per atom per second is

$$\frac{9.0 \times 10^{-7}\ \text{J s}^{-1}}{2.25 \times 10^{16}\ \text{atom}} = 4.0 \times 10^{-23}\ \text{J s}^{-1}\ \text{atom}^{-1}.$$

(b)(ii) Time to accumulate 3.2 x 10^{-18} J at 4.0 x 10^{-23} J s^{-1} is

$$\frac{3.2 \times 10^{-18}\ \text{J}}{4.0 \times 10^{-23}\ \text{J s}^{-1}} = 8.0 \times 10^4\ \text{s}.$$

(c)

$E = h\,f$ = (6.6 x 10^{-34} J s) x (6.0 x 10^{15} Hz)

= 4.0 x 10^{-18} J

This is greater than 3.2×10^{-18} J, so photoemission is possible.

Question 8

(a) Note that you only want the **magnitude** of the resultant force. Measuring the distance between the vector arrow tips gives 4.5 cm, equivalent to a force of 4.5 N. ‘Some other method’ would use trigonometry applying Pythagoras’s theorem:

$(\text{resultant})^2 = 4^2 + 2^2 = 20$

so the length of the resultant represents $\sqrt{20}$ = 4.47 N = 4.5 N to two significant figures.

(b) The angle with the air stream = $\tan^{-1}(2 / 4) \sim 27$ degrees.

Question 9

(a) You want the velocity of **A** as seen by **B**, which considers an observer at rest with respect to **B**. To do this we need to superpose a velocity of 300 m s^{-1} to the right on the situation. Thus the velocity of **A** as seen by **B** is 200 m s^{-1} + 300 m s^{-1} = 500 m s^{-1} to the right.
(From **A**’s point of view the relative velocity of **B** is 500 m s^{-1} to the left.)

(b) To cover 40 km at a speed of 500 m s^{-1} takes $(40 \times 10^3 \text{ m}) / 500 \text{ m s}^{-1}$ = 80 s to collide.

Question 10

Either: measure the diagonal of the resultant triangle of forces = 3.6 cm.
Scale = 1 cm = 500 N, thus 3.6 cm ~ 1800 N.
Or, using Pythagoras’s theorem

$(\text{resultant})^2 = (1000 \text{ N})^2 + (1500 \text{ N})^2 = 3\,250\,000 \text{ N}^2$

thus the resultant = $\sqrt{3\,250\,000 \text{ N}^2}$ = 1800 N.

Question 11

The following information is given:

- initial speed $u = 0$ m s^{-1}
- final speed $v = 300$ m s^{-1}
- distance s over which projectile is accelerated = 2.00 m

$v^2 = u^2 + 2as$

thus

$$a = \frac{v^2}{2s} = \frac{(300\ \text{m s}^{-1})^2}{2 \times 2.00\ \text{m}} = 22{,}500\ \text{m s}^{-2}.$$

Question 12

(a)
$s = u\,t$ = 20 m s^{-1} x 0.5 s = 10 m (= 'thinking distance')

(b)(i)
(1) $F = m\,a$, thus $a = F / m$ = 7.1 x 10^3 N / 1200 kg = 5.9 m s^{-2} deceleration.

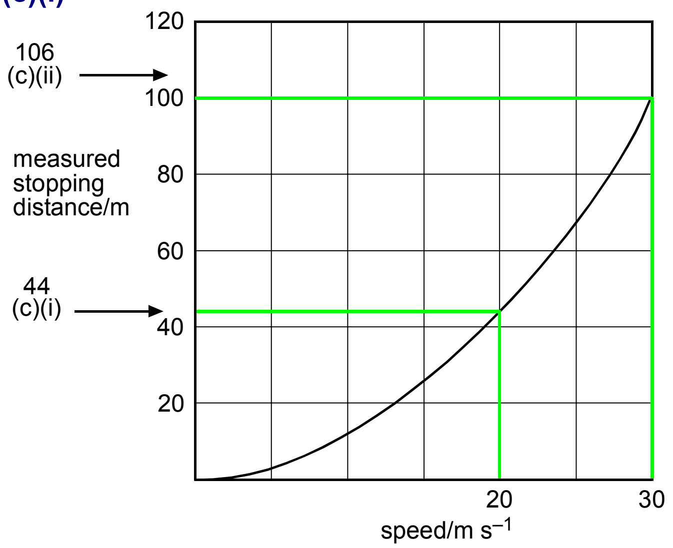
(2) $v^2 = u^2 + 2\,a\,s$

when stopped, u = 0, so

$s = v^2 / 2\,a$ = (20 m $s^{-1})^2$ / (2 x 5.9 m s^{-2}) = 33.89 m ~ 34 m.

(b)(ii) Total stopping distance = thinking distance + stopping distance
= 10 m + 34 m = 44 m.

(c)(i)



The vertical scale on the graph is 1 cm = 20 m.
At 20 m s^{-1} the stopping distance is '2.2 cm', equivalent to 2.2 x 20 = 44 m, which is consistent with the answer in part **(b)(ii)**.

(c)(ii) At 30 m s^{-1}, stopping distance is '5.3 cm', equivalent to 5.3 x 20 = 106 m.

(d) Possible reasons:
1. Braking friction could **decrease** as the brakes get hot, so that it takes longer to stop.
2. Air drag has been **ignored** in the calculation (but that would tend to **reduce** the calculated stopping distance!).

Question 13

(a)(i) While falling the kinetic energy increases at the expense of gravitational potential energy. Because of the air drag the internal energy of the mass and the air increases.

(a)(ii) Assuming the effects of air drag are negligible

$$mgh = \frac{mv^2}{2}$$

thus

$v^2 = 2\,g\,h$

so

$$\begin{aligned} v &= \sqrt{2gh} \\ &= \sqrt{2 \times 9.8\ \text{m s}^{-1} \times 5.0\ \text{m}} \\ &= 9.899\ \text{m s}^{-1} \\ &\approx 10\ \text{m s}^{-1}. \end{aligned}$$

(a)(iii) If the pole is driven 0.4 m into the ground the total distance the 220 kg mass has fallen is 5.0 m + 0.4 m = 5.4 m.
The work done driving the pole into the ground = the average F x distance
= 0.4 m x F.
By conservation of energy

0.4 m x $F = m\,g\,h$ = 220 kg x 9.8 N kg^{-1} x 5.4 m

thus

F = 29 106 N ~ 29 kN.

(b)(i) The resistive force increases with depth because more surface area of the pole is in contact with the earth and it is harder to push the earth at the lower end of the pole out of the way.

(b)(ii) The two given points on the graph show that the second drop achieved only half the original penetration, so the constant fraction is ½. Thus drops 3, 4 and 5 should be plotted at 0.1 m, 0.05 m and 0.025 m.

Question 14

(a) Ramp A: high initial acceleration which decreases smoothly (or noting that part (b) of the question gives speed time/graphs; the speed increases rapidly, then more slowly).

Ramp B: the acceleration is low at first, then higher, then gets less again
(or in terms of speed; speed increases slowly, then faster, then more slowly).

(b)(i) The area under the graph represents the distance travelled which, since both ramps have the same length, must therefore be equal. (NB: it is nothing to do with the fact that the vertical drops are the same.)

(b)(ii) (NB: for both ramps the skateboarder is accelerating **all the way** to the bottom, and have the same speed of 4 m s^{-1} at the bottom.)
There are several equivalent ways this can be accounted for:

- average acceleration for B is less than for A
- until they reach the bottom of the ramp, at all times the speed of B is less than the speed of A
- the slope of each speed–time graph represents the acceleration, for B this is less than A.

Question 15

(a) 1.33 MeV = 1.33×10^{6} eV

$= (1.33 \times 10^{6}\ \text{eV}) \times (1.6 \times 10^{-19}\ \text{J (eV)}^{-1})$

$= 2.128 \times 10^{-13}\ \text{J} \sim 2.0 \times 10^{-13}\ \text{J}.$

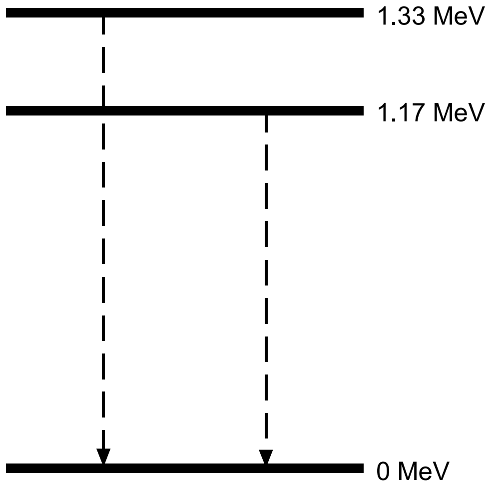
(b) $E = h f$

so

$f = E / h = 2.13 \times 10^{-13}\ \text{J} / 6.63 \times 10^{-34}\ \text{J s}$

$= 3.2 \times 10^{20}$ Hz.

(c)



Question 16

(a) Weight = $m\,g = 2.0 \times 10^{5}$ kg $\times$ 9.8 N kg^{-1} = 1.96×10^{6} N ~ 2 MN.

(b) F_A and F_B are the same size and at the same angle above and below the horizontal, so their vertical components will be equal and opposite, giving a zero vertical resultant. Thus the resultant must be horizontal.

(c) Horizontal component of $F_A = F_A \cos 30°$.
Horizontal component of $F_B = F_B \cos 30° = F_A \cos 30°$ (because $F_B = F_A$).
Resultant = $2 \times F_A \cos 30° = 2 \times (1.7 \times 10^5\ \text{N}) \times 0.866 = 2.94 \times 10^5\ \text{N} \sim 3.0 \times 10^5\ \text{N}$.

(d) $F = ma$, so:

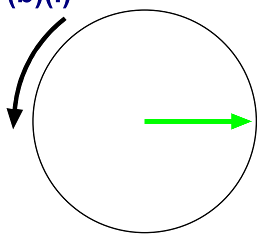
$$a = \frac{F}{m} = \frac{3.0 \times 10^5\ \text{N}}{200 \times 10^3\ \text{kg}} = 1.5\ \text{m s}^{-2}.$$

This is greater than the actual acceleration of Transrapid, because air drag has been ignored.

Question 17

(a) The wavefronts are parallel, so come from a very distant centre of curvature.

(b)(i)



Phasors rotate anti-clockwise (with the frequency of the wave they represent). The wave front arrives at B at an earlier time than at A. The diagram shows that the equivalent point on a wave front arrives at B half a wavelength before arriving at A. Half a wavelength corresponds to 180 degrees (or π) phase difference, which is half a revolution of the phasor. So the arrow should point to 3 o'clock.

(b)(ii) For the source overhead (Fig. 17.1) the two signals are in phase and superpose constructively. In Fig. 17.2 the signals are in antiphase and will superpose destructively.

(b)(iii) Consider the triangle with base AB. Because $\lambda / 2 << d$, θ is a small angle so

$\sin\theta \sim \theta = ½ \lambda / d = \lambda / 2d$.

(b)(iv) Substituting the data into the equation given in **(b)(iii)**

$\sin\theta = 0.21\ \text{m} / (2 \times 50\ \text{m}) = 0.0021$ radians

there are 2π radians in 360 degrees, so

1 radian = $360 / 2\pi$ degrees

and

0.0021 radian = (0.0021 radian x 360 degrees radian^{-1}) / 2π = 0.12 degree.