Worked Solutions for Sample Examination Questions

Question 1

(a)(i) Condenser lens: the condenser lens **adds** curvature. So the negatively curved waves become flat (or have zero curvature) to lie on the plane of the transparent negative.

(a)(ii) The lens formula is:

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}.$$

Zero curvature means the rays are parallel.
Parallel rays require v = infinity, so that u = **minus** f.
Thus to result in parallel rays, the lamp must be at the focus of the condenser lens,

f = 1 / power = 1 / 4 m

thus u = (–) 0.25 m.

(b)(i) Projecting lens:

magnification $M = 6 = v / u$

thus $u = v / M$

v = 0.35 m (see diagram)

u = 0.35 m / 6 = 0.058 m.

(b)(ii) Rearranging the thin lens formula:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}.$$

Magnification $M = 6 = v / u$, so $v = 6u$.

Cartesian sign convention: u is negative, v is positive so

$$\begin{aligned} f &= \frac{1}{v} - \frac{1}{-v/6} \\ &= \frac{1}{v} + \frac{6}{v} \\ &= \frac{7}{v}. \end{aligned}$$

and

$$\frac{1}{f} = \frac{7}{0.35\ \text{m}} = 20\ \text{D}$$

(D = dioptres, the strength of the lens) and

$$f = \frac{1}{20\ \text{D}} = 0.05\ \text{m}.$$

Question 2

(a) A 4 bit binary number allows for 2^4 alternatives = 16 digits.
0 to 9 inclusive = 10 digits which is less than 16, so 4 bits is sufficient.

(b) One hundred 12-digit numbers requires 100 x 12 x 4 = 4800 bits.
Number of bytes = bits / 8 = 600.

(c) The English alphabet has 26 letters. 4 bits can only code 2^4 = 16 alternatives.
26 alternatives requires 5 bits (2^5 = 32 > 26).

(d)(i) Highest frequency = ½ of the sample rate = 10 000 / 2 = 5000 Hz = 5 kHz.

(d)(ii) Any two of the following:

- music may contain a wider frequency spectrum than speech
- lower quality reproduction of speech is still heard correctly
- only a poor quality speaker is used
- stereo is often used for music
- music suffers more from high-frequency cut-off, aliasing, spurious frequencies.

Question 3

(a) UV is absorbed by the ozone layer.

(b) 1.2 x 10^8 m diameter is imaged using 512 pixels.
Distance per pixel = (1.2 x 10^8 m) / 512 = 2.34 x 10^5 m resolution

Question 4

(a)

	conductor	insulator
electrical conductivity	very high	very low
suitable material	copper	plastic (or rubber)

(b)(i) Conductance $G = (\sigma A) / L$.
Total length of the two cables in series = 60 m.

$$G = \frac{(5.9 \times 10^7\ \text{S m}^{-1}) \times (1.8 \times 10^{-6}\ \text{m}^2)}{60\ \text{m}} = 1.77\ \text{S}.$$

Substituting given data gives G = 1.8 S to 2 significant figures (the precision of the data).

(b)(ii) You do not need to work out the resistance from the conductance.

$R = 1 / G$

$V = I R = I / G$.

$$V = \frac{13\ \text{A}}{1.8\ \text{S}} = 7.2\ \text{V}.$$

Substituting data gives 7.2 V.

(b)(iii) Power = $I V$ = 13 A x 7.2 V = 93.6 W (~100 W)

(b)(iv) When coiled, the cooling of the cable is reduced (less surface area in contact with the air). So the power dissipated will raise the temperature of the inner coils, possibly melting the insulation and and/or becoming a fire hazard.

Question 5

(a)(i) $I = V / R$ = (200 x 10^{-3} V) / (5 x 10^{6} Ω) = 4.0 x 10^{-8} A. (NB: mV are converted to V. The equation is a re-arrangement of the definition of resistance, not an application of Ohm's law.)

(a)(ii) The total resistance in the circuit is now 10 MΩ.
$I = V /$ (total R) = 2.0 x 10^{-8} A.

(a)(iii) $V = I R$ = (2.0 x 10^{-8} V) x (5 x 10^{6} Ω) = 0.1 V. (NB: A suitable device to measure 0.1 V with a reasonable precision might be a potential divider.)

(b)(i) The best choice is the cathode ray oscilloscope.

(b)(ii) There are two reasons. The cathode ray oscilloscope has the highest internal resistance, so the output voltage is not dropped too much when connected to the external load presented by the cathode ray oscilloscope. The variable sensitivity allows a near full-scale deflection (FSD) setting to take data.

(b)(iii) Cathode ray oscilloscope:

$$\begin{aligned} V_{\text{out}} &= V_{\text{in}} \times \frac{R_{\text{LOAD}}}{R_{\text{LOAD}} + R_{\text{DEVICE}}} \\ &= 0.2\ \text{V} \times \frac{25\ \Omega}{25\ \Omega + 5\ \Omega} \\ &= 0.167\ \text{V} \\ &= 167\ \text{mV}. \end{aligned}$$

If you had chosen the digital voltmeter a similar calculation gives 57 mV.

If you had chosen the ammeter, the current I that would flow would be

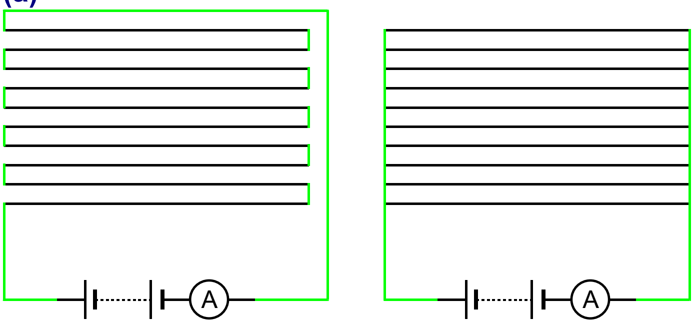
V / (total R) = 0.2 V / [15 Ω + (5 x 10^6 Ω)] = 4 x 10^{-8} A,

which when flowing through the ammeter only corresponds to a voltage drop of

$I R$ = (4 x 10^{-8} A) x 15 Ω = 0.6 microvolts!

Question 6

(a)



(b)

	series connection	parallel connection
p.d. across each track / V	1.2	12
current in each track / A	20	2
conductance of each track / S	17	0.17

The question states that **each** track must dissipate 24 W.

Power = V x I

Series connection: the total resistance in the circuit is 10 times the resistance of a single track. So (ignoring the internal resistance of the battery as instructed in the question) the p.d. across each track will be 1/10 of the total p.d. or 1.2 V.

For a series connection

current I = power / V = 24 W / 1.2 V = 20 A.

Conductance G = 1 / R = I / V = 20 A / 1.2 V = 17 S (to 2 significant figures)

Parallel connection: each track is connected directly to the battery, so the p.d. across each track = 12 V.

For a parallel connection

current I = power / 12 V = 24 W / 12 V = 2 A.

Conductance $G = I / V$ = 2 A / 12 V = 0.17 S (to 2 significant figures).

(c)(i) Start from the familiar

$$R = \frac{\rho L}{A}$$

where L = length, A = cross sectional area and ρ = resistivity.

$$G = \frac{1}{R} = \frac{A}{\rho L} = \frac{\sigma A}{L}$$

where $\sigma = 1 / \rho$ = conductivity.

Now

A = width w x thickness t.

Substituting the above for A and re-arranging for w gives

$$w = \frac{GL}{t\sigma}.$$

$$w = \frac{0.17\ \text{S} \times 1.0\ \text{m}}{(2.0 \times 10^{-4}\ \text{m}) \times (4.2 \times 10^{5}\ \text{S m}^{-1})}$$
$$= 2.0 \times 10^{-3}\ \text{m}.$$

Substituting the given data gives w = 2.0 mm (to 2 significant figures).

(c)(ii) The best choice is the parallel connection. The tracks are quite narrow and so do not obscure too much of the view. If one track breaks, the remaining nine will still work.

With the series connection, the conductance needs to be 100 times larger, which for the same thickness of track implies that the width is 100 times wider (= 20 cm each, so 10 tracks will obscure the whole view!). Furthermore, if one track fails, no track receives any current.

The parallel arrangement uses much less conducting material and should be cheaper.

Question 7

(a) Any situation where the variation in time of the frequencies present in the sound signal is needed, e.g. analysing the sound from a piece of machinery, quality control in the manufacture of musical instruments.

(b) Features of the whistle to choose from are:
- Qualitative observation – frequency falls gently then rises more rapidly. This general observation could be quantified thus: frequency starts around 1.5 kHz, falls to 1.0 kHz then ends near to 2.0 kHz.
- The whistle frequencies at any instant have a range of about ±0.2 kHz = 200 Hz bandwidth..
- Whistle frequencies are higher than those of the speech.

Question 8

(a)

angle of rotation	received signal
0°	maximum
90°	minimum (ideally zero)
180°	maximum

(b) Electromagnetic waves are transverse and thus can be plane polarised. The plane of polarisation is taken to be that of the electric field. When the metal rods of the t.v. aerial are parallel to the direction of polarisation the signal is a maximum. When at 90º there is no component of the electric field in the direction of aerial rods so the signal is zero.

(c)(i) The wave speed formula is $v = f\lambda$:

3.0×10^8 m s^{-1} = $f \times 0.14$ m

thus

$f = 2.14 \times 10^9$ Hz ~ 2 GHz.

(c)(ii) Transmission rate = frames/second x pixels/frame x bits/pixel
= $25 \times 2 \times 10^6 \times 8$

(NB: the information in the question is 1 **byte** = 8 bits)

so the rate = 400×10^6 bits s^{-1}.

(c)(iii) There are two ways to calculate this.

Either:

the time interval per bit = 1 / (bits per the time interval)

where 'bits per the time interval' is the transmission rate, so

time per bit = 1 / (400×10^6 bit s^{-1}) = 2.5×10^{-9} s bit^{-1} = 2.5 ns bit^{-1}.

Or:

the carrier wave frequency [i.e. the answer to part **(c)(i)**] is 2.14 GHz.
One bit corresponds to 5.5 cycles (count them on the diagram). Thus the time for 5.5 cycles at 2.14 GHz is

$5.5 \times (1 / 2.14 \times 10^9 \text{ Hz}) = 2.6 \text{ ns}$

(to 2 significant figures).

Question 9

(a) Advantages of digital form (answer only requires one to be discussed):

- less noise
- easy removal of noise
- can be reproduced (and thus re-transmitted easily)
- can be processed by a computer ('signal processing')
- many kinds of information can be represented (text, vision, music etc).

Disadvantages of digital form:

- sampling reduces the frequency range (equivalent to a loss of information) because of upper sampling frequency limit
- introduces low frequency aliases that distort the signal
- loss of resolution due to limited (i.e. finite number of) bits per sample.

(b)(i) Count them on the diagram: 15 levels (including zero).

(b)(ii) 4 bits are needed to encode 15 levels:

4 bits can encode $2^4 = 16$ alternatives (more than the 15 needed); 3 bits would only encode $2^3 = 8$ (less than the 15 needed).

(b)(iii) The diagram shows 16 samples in 1 ms equivalent to $16 / 10^{-3} = 16$ kHz.

(b)(iv) Information transmission rate in bits per second
= sample rate x bits per sample

= answer to **(b)(iii)** x answer to **(b)(ii)**

$= (16 \times 10^3 \text{ Hz}) \times 4 \text{ bit} = 64 \text{ kbit s}^{-1}$.

Question 10

(a) Weight = $m\,g$

$= 300 \text{ kg} \times 9.8 \text{ N kg}^{-1} = 2940 \text{ N}$

≈ 2900 N (to 2 significant figures).

(b)

$$\text{Stress} = \frac{\text{force}}{\text{area}}.$$

The force is just the weight of the freezer calculated in part **(a)**. Thus:

$$\text{stress} = \frac{2940\ \text{N}}{8.0 \times 10^{-4}\ \text{m}^2} = 3.7\ \text{MPa}\ \ \text{(to 2 significant figures)}$$

(to be consistent with the precision of the data given in the question) which is greater than the stated yield stress of 3.5 MPa and can thus cause permanent damage.

Question 11

(a) Any of the following statements would get the mark:

- extension is directly proportional to the force
- extension varies linearly through the origin
- as the force doubles so does the extension.

(b)(i) The 'area under the graph' is shaped like a triangle.

The area of a triangle = ½ base x height.

Base = the extension = 4 mm = 4 x 10^{-3} m.

Height = force = 90 N.

Thus:

energy stored = ½ (4 x 10^{-3} m) x 90 N = 0.18 N m (or J).

(b)(ii) Young modulus E = stress / strain:

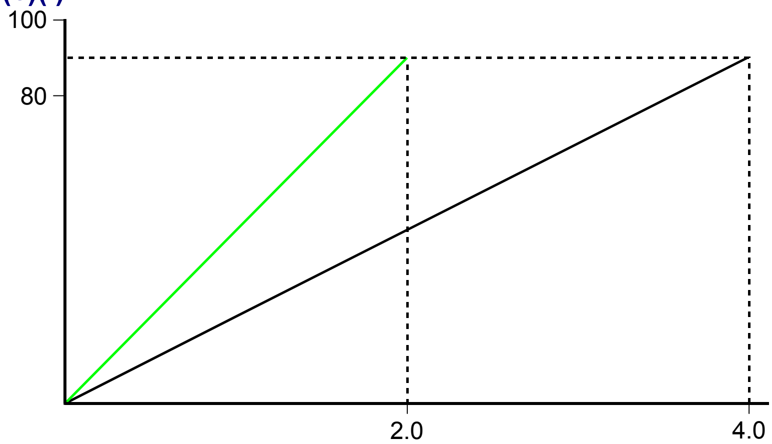
$$\begin{aligned}\text{stress} &= \frac{\text{force}}{\text{area}} \\ &= \frac{90\ \text{N}}{2.5 \times 10^{-7}\ \text{m}^2} \\ &= 3.6 \times 10^{8}\ \text{N m}^{-2}\ (=\text{Pa})\end{aligned}$$

$$\begin{aligned}\text{strain} &= \frac{\text{extension}}{\text{original length}} \\ &= \frac{4 \times 10^{-3}\ \text{m}}{2.0\ \text{m}} \\ &= 2 \times 10^{-3}.\end{aligned}$$

Thus

$$\begin{aligned}E &= \frac{3.6 \times 10^{8}\ \text{N m}^{-2}}{2 \times 10^{-3}} \\ &= 1.8 \times 10^{11}\ \text{N m}^{-2}.\end{aligned}$$

(c)(i)



A straight line graph from the origin to the point (2.0, 90), i.e. with double the gradient of the first wire.

(c)(ii) The Young modulus is a property of the material and does not depend upon the shape of a particular sample, so it is the same. [Note the question is illustrated by a force–extension graph and **not** a stress–strain graph.]
Half the original length under the same stress will result in half the original extension, so the strain remains the same.

Question 12

(a) **A** shows ductile or plastic fracture: possible materials are metals, some plastics.
B shows brittle fracture: possible materials are glass, ceramics, some alloys (e.g. cast iron).

(b) Material **A** has close-packed planes of atoms. Single atoms can slide over each other when a relatively low stress is applied, thus giving rise to 'dislocation' movement.
Material **B** is amorphous like glass and cannot break and re-form bonds easily to allow slip of whole planes of atoms. Brittle materials fail by small surface cracks or imperfections that concentrate stress at the crack tip allowing the crack to grow.

Question 13

$$\text{Refractive index } n = \frac{\sin i}{\sin r} = \frac{\sin 50^\circ}{\sin 35^\circ} = \frac{0.766}{0.574} = 1.34.$$

Question 14

(a) Frequency is the number of (any one of the following will do) oscillations / waves / cycles / vibrations / samples per second.

(b) Digital samples are samples of the signal taken at discrete intervals, and quantised on a digital scale composed of a sequence of binary digits.

(c) A bit of information is a binary digit (the 0 or 1); one bit is 1 / 8th of a byte

Question 15

(a)(i) Each pixel only needs 2 alternative values (black or white). A single bit encodes 2^1 alternatives: $2^1 = 2$.

(ii) The letter M is 7 pixels tall, so each pixel covers
2.6 mm / 7 = 0.37 mm ~ 4×10^{-4} m.

(iii) (Converting mm to m) the area of page = $(300 \times 10^{-3}$ m$) \times (200 \times 10^{-3}$ m$)$
= 6.00×10^{-2} m^2

Area per pixel = $(3.7 \times 10^{-4}$ m$)^2 = 1.37 \times 10^{-7}$ m^2 pixel^{-1}.

Thus pixels per covered page = 6.00×10^{-2} m^2 / 1.37×10^{-7} m^2 pixel^{-1}
= 4.3×10^5 pixels.

One pixel only needs one bit, so bits / page = 4.3×10^5 bits.

(b) At this lower resolution the M and the y may be confused with letters of a similar shape (e.g. N and H, q and g). A block of 24 pixels would not allow for a clear column between the letters.

Question 16

(i) 80 A h = 80 C s^{-1} for 60 x 60 seconds = 288 000 C.

(ii) 10 batteries in parallel can deliver 10 x 80 A for 1 hour = 800 A h.
Thus in 4 hours they will deliver 800 A h / 4 h = 200 A.

(iii) Power = $I V$ = 200 A x 24 V = 4.8 kW.

(iv) Delivering the power will also heat the wiring, and result in heating of the batteries themselves due to their internal resistance. (Also, electric motors are not 100% efficient, and work must be done against air drag and friction between all moving parts in the gear box etc etc.)